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ADVERTISING AND PRICING POLICIES IN A TWO-ECHELON SUPPLY CHAIN WITH A CAPITAL-CONSTRAINED RETAILER

Honglin Yang[1,*], Lingling Chu[1] and Hong Wan[2]

Abstract. We consider a two-echelon supply chain consisting of one supplier and one capital-constrained retailer. The supplier can offer the retailer trade credit to fund his orders. To boost sales, the retailer invests part or all of initial capital exclusively in advertising at the beginning of the sales season. Demand is sensitive to both retail price and advertising expenses of the retailer. With a wholesale price contract, we analytically derive the Stackelberg equilibrium with respect to pricing by both parties and advertising by the retailer. Our results show that the retailer with less initial capital prefers to invest full initial capital in advertising irrespective of the advertising elasticity or the interest rate charged by the supplier. The retailer with more initial capital only invests part of initial capital in advertising. The retailer's advertising policy under different initial capital levels always benefits the supply chain and the supplier. We further identify the effects of the advertising elasticity and the interest rate on the pricing policies. Numerical simulations and sensitivity analysis are given to elaborate our theoretical results.

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1. Introduction

In today's business environments, supply chain parties often face two crucial problems: limited capital and insufficient demand. The former greatly constrains the parties when attempting to make optimal decisions, particularly small and medium-sized entrepreneurs. The latter requires the parties to invest in a greater number of promotional initiatives. For example, a survey of 3561 representative U.S. small firms reports that approximately 60% of them often resort to financial institutions [3]. Due to the lack of collateral or credit, it is quite difficult for SMEs to borrow from banks smoothly [26]. To improve supply chain performance, the dominant party with full capital offers trade credit to the capital-constrained party. More recently, trade credit has been widely employed as a short-term financing source to address capital constraints in supply chains (*e.g.,* see [4, 11, 16, 20, 33, 35]). Goyal [10] initially develops an EOQ model with the permissible delay in payment. In this vein, many studies extend Goyal's model to inventory management (*e.g.,* see [6, 14]), ordering policies (*e.g.,* see

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[1] School of Business Administration, Hunan University, Changsha, Hunan Province 410082, PR China.
[2] School of Business, State University of New York at Oswego, Oswego, NY 13126, USA.
*Corresponding author: ottoyang@126.com

[15, 24]), deteriorating items (*e.g.,* see [13, 21, 25]), partial trade credit (*e.g.,* see [27, 30–32]), and two-part trade credit (*e.g.,* see [7]). It is well documented in the literature that joint financing and ordering decisions create greater profit in the supply chain.

In practice, the parties invest in considerable promotion initiatives, which lead to heavier capital pressure of the parties (*e.g.,* see [18, 34]). For example, Zenith Optimedia reports that the global advertising markets have steadily grown at 4%–5% since 2011. Total advertising expenses worldwide are estimated at above $592 billion at the end of 2018. By increasing advertising investment, many firms build up their long-term brand names and successfully lure many customers. Furthermore, product sales and market share are boosted (*e.g.,* see [18, 34]). Those studies related to advertising policy can be roughly divided into two main categories according to the demand assumption: advertising-linked demand and advertising-and-price-linked demand. Specifically, Balcer [2] presents an additive demand function that solely depends upon the retailer's advertising expenses. Seyedesfahaniaab [22] develops Balcer's model and investigates coordination pricing and vertical co-op advertising strategies in a manufacturer-retailer supply chain when demand depends upon the supplier's and retailer's advertising expenses. Many scholars extend the above advertising demand setting to more-practical situations in which demand is sensitive to both advertising expenses and retail price (*e.g.,* see [9, 12]). More specifically, Szmerekovsky and Zhang [23] identify the effects of advertising and retail price on optimal decisions and obtain valuable insights into the cooperative advertising of the supply chain. Based on the advertising-price-linked demand, several recent studies have examined different aspects: supply chain coordination (*e.g.,* see [5, 19]), cooperative advertising (*e.g.,* see [22, 29]), and an advertising game (*e.g.,* see [1, 18]).

The advertising and pricing decisions are closely interactive in a supplier and a capital-constrained supply chain. In particular, the advertising expenses are incurred by the retailer with limited capital alone. The more the retailer advertises, the less the retailer orders unless the supplier defrays part of his advertising expense. However, the literature on advertising implicitly assumes that the parties are endowed with full capital. By advertising, the parties can create greater profit in the supply chain. In practice, a shortage of capital complicates decisions with respect to advertising and pricing. Specifically, the capital-constrained retailer often faces the significant challenge of how to efficiently allocate his limited capital between product ordering and advertising expenses.

Our work differs from existing studies because we consider an EOQ model in a two-echelon supply chain with a capital-constrained retailer. The supplier can offer the retailer trade credit to satisfy demand. The retailer invests part or all of the initial capital exclusively in advertising to lure customers and boost sales. In a decentralized system, we analyze the relationship of pricing and advertising policies under the retailer's different initial capital levels. We further clarify how the advertising elasticity and the interest rate charged by the supplier affect the pricing and advertising policies.

The main contributions of this paper to the literature are summarized as follows. First, we analytically derive the Stackelberg equilibrium with respect to pricing by both parties and pricing by the retailer when the supplier is the leader and can offer trade credit to the capital-constrained retailer. Second, when demand depends upon both advertising and retail price, we identify the effects of advertising elasticity and interest rate on advertising, pricing and profits of both parties and the supply chain under the retailer's different initial capital levels. We characterize the conditions under which the retailer prefers to invest part or all of his initial capital to advertising.

The remainder of this paper is organized as follows. Section 2 describes the basic model. Section 3 analytically derives the Stackelberg equilibrium with respect to pricing and advertising policies. Section 4 presents numerical examples. Section 5 concludes. An appendix collects the proofs of the propositions.

2. Model setup

We consider a two-echelon supply chain consisting of an upstream supplier and a downstream capital-constrained retailer. With a wholesale price contract, as a Stackelberg leader, the supplier charges the retailer as a follower w per unit purchased. The retailer charges p per unit sold to consumers. When the retailer's initial capital level B is insufficient to cover his orders, the dominant supplier can offer the retailer trade credit at a

TABLE 1. Notations and explanations.

Notation	Explanation
p	Unit retail price (retailer decision variable)
w	Unit wholesale price (supplier decision variable)
c	Supplier unit production cost
A	Retailer advertising expenses
γ	Advertising elasticity
$D(p, A)$	Demand function depending upon both p and A
I_r	Interest rate charged by the supplier
B	Retailer initial capital level
θ	Advertising fraction of B (retailer decision variable)
π_r	Retailer profit
π_s	Supplier profit

fixed interest rate I_r. To boost sales, the retailer invests part or all of his initial capital level B exclusively in advertising expenses $A = \theta B$ at the beginning of the sales season, where $\theta \in (0, 1]$ is the advertising fraction of the retailer's initial capital level. Demand is assumed to be a function of the retail price p and the advertising expenses A. We define the demand function $D(p, A) = \alpha - p + \gamma\sqrt{A}$, where $\alpha > 0$ is primary demand and $2 > \gamma > 0$ is advertising elasticity on the retailer's advertising expenses (*e.g.,* see [8, 28]). The above demand function has been widely applied to many studies on advertising [17]. In a decentralized system, the supplier decides the wholesale price w, and the retailer decides the advertising fraction θ and the retail price p to maximize individual profit. We discuss the Stackelberg equilibrium with respect to pricing by both parties and advertising by the retailer.

We further assume that both parties are risk-neutral and aim to maximize individual profits. The retailer will satisfy all customer demand. For simplicity, we also assume the supplier produces at a constant and identical marginal cost.

3. Decision analysis

3.1. Retailer's problem

In the Stackelberg game in which the supplier is the leader and can offer the capital-constrained retailer trade credit, we initially consider the retailer's problem. Given the supplier's decision on the wholesale price w, the retailer with limited capital sets the retail price p and the advertising fraction θ maximize his own profit. The retailer's profit function consists of the following elements:

(i) Sales income over sales season $pD(p, A)$;
(ii) Advertising expenses $A = B\theta$;
(iii) Retailer's immediate payment $B(1 - \theta)$;
(iv) Retailer's delayed payment $(1 + I_r)\,[wD(p, A) - B(1 - \theta)]$.

The retailer's profit function is therefore given by

$$\pi_r = pD\left(p, A\right) - B\theta - wD\left(p, A\right)\left(1 + I_r\right) + BI_r\left(1 - \theta\right). \tag{3.1}$$

The following proposition establishes the retailer's optimal retail pricing and advertising policies in a decentralized system when the supplier is the leader and can offer the retailer trade credit.

Proposition 3.1. *(i) When* $w \leq \Delta$, *then* $p^* = \frac{\alpha+\gamma\sqrt{B}+w(1+I_r)}{2}$ *and* $\theta^* = 1$; *(ii) When* $w > \Delta$, *then* $p^* = \frac{\gamma^2 w1+I_r-2(1+I_r)[w1+I_r+\alpha]}{\gamma^2-41+I_r}$ *and* $0 < \theta^* = \frac{\gamma^2[w1+I_r-\alpha]^2}{B[\gamma^2-41+I_r]^2} < 1$, *where* $\Delta = \frac{\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r}$.

Proposition 3.1 shows that for any given wholesale price w, there always exist optimal policies with respect to retail pricing and advertising by the retailer. The optimal retail price is independent of the advertising expenses. Because the advertising expenses come from the retailer alone, the retailer will not attempt to raise advertising expenses to compensate for the reduced demand by increasing the retail price. This finding is consistent with the result in [23]. When the supplier charges a low wholesale price, i.e., $w \leq \frac{\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r}$, the retailer's purchase costs are less. The retailer can invest more initial capital in advertising promotions to lure customers and boost sales. When the supplier charges a high wholesale price, i.e., $w > \frac{\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r}$, the retailer must pay more purchase costs. Due to limited capital, the retailer can only invest part of his initial capital in advertising.

3.2. Supplier's problem

The supplier as the Stackelberg leader predicts the retailer's optimal policies in Proposition 3.1. Based on the backward induction approach, the supplier decides the wholesale price w to maximize her own profit. The supplier's profit includes the following elements:

(i) Sales revenue from the retailer's immediate payment $B(1-\theta)$;
(ii) Sales revenue from the retailer's delayed payment $(1+I_r)\left[wD(p,A) - B(1-\theta)\right]$;
(iii) Production costs $cD(p,A)$.

The supplier's profit function is given by

$$\pi_s = w(1+I_r)D(p,A) - BI_r(1-\theta) - cD(p,A). \tag{3.2}$$

Substituting the retailer's optimal p^* and θ^* into equation (3.2) obtains the following supplier's profit function

$$\pi_s = \begin{cases} \frac{\left[\alpha+\gamma\sqrt{B}-w(1+I_r)\right][w(1+I_r)-c]}{2}, & w \leq \Delta \\ \frac{2(1+I_r)[w1+I_r-\alpha][w(1+I_r)-c]}{\gamma^2-41+I_r} - BI_r\left\{1 - \frac{\gamma^2[w1+I_r-\alpha]^2}{B[\gamma^2-4(1+I_r)]^2}\right\}, & w > \Delta. \end{cases} \tag{3.3}$$

The following proposition establishes the supplier's optimal wholesale price given the retailer's optimal policies in Proposition 3.1.

Proposition 3.2. *(i) When* $B \leq \left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2$*, then* $w^* = w_1$*; (ii) When* $\left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 < B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$*, then* $w^* = \Delta$*; (iii) When* $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$*, then* $w^* = w_2$*, where* $w_1 = \frac{\alpha+c+\gamma\sqrt{B}}{2(1+I_r)}$ *and* $w_2 = \frac{(\alpha+c)\left[4(1+I_r)^2-\gamma^2\right]-\gamma^2 I_r(2\alpha+c)}{8(1+I_r)^3-\gamma^2 1+I_r(2+3I_r)}$.

From Proposition 3.2, for any given retailer initial capital level B, there always exists the equilibrium of the wholesale price. When $B \leq \left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2$, we refer to the retailer as a "poor" retailer. The supplier charges a relatively low wholesale price $w^* = w_1 < \Delta$ to encourage the retailer's additional advertising expenses and boost sales (Fig. 1). When $\left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 < B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, we refer to the retailer as a "moderately rich" retailer. The supplier charges a moderate wholesale price $w^* = \Delta$ to earn more profit (Fig. 2). When $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, we refer to the retailer as a "rich" retailer. The supplier charges a relatively high wholesale price $w^* = w_2 > \Delta$ to limit the retailer's excess advertising expenses and receive more-immediate payment (Fig. 3).

Proposition 3.3. *(i) When* $B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$*, then* $\frac{\partial\theta^*}{\partial\gamma} = 0, \frac{\partial w^*}{\partial\gamma} > 0$ *and* $\frac{\partial p^*}{\partial\gamma} > 0$*; (ii) When* $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$*, then* $\frac{\partial\theta^*}{\partial\gamma} > 0, \frac{\partial w^*}{\partial\gamma} < 0$ *and* $\frac{\partial p^*}{\partial\gamma} > 0$.

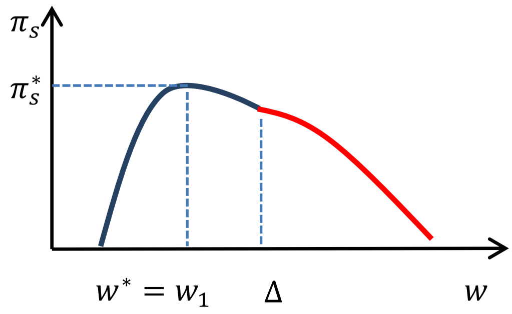

FIGURE 1. $w_2 < w_1 < \Delta$.

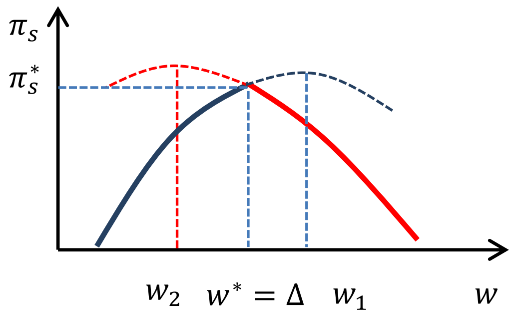

FIGURE 2. $w_2 < \Delta < w_1$.

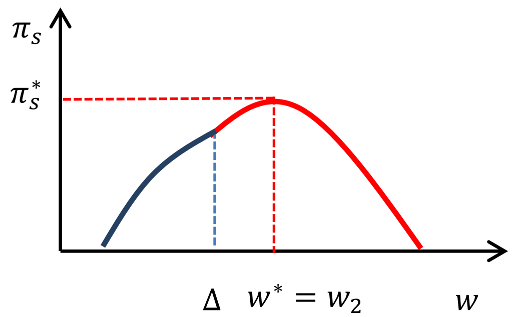

FIGURE 3. $\Delta < w_2 < w_1$.

When $B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, the "poor or moderately rich" retailer fully invests in advertising to boost sales as much as possible. As the advertising elasticity increases, advertising becomes more effective at increasing demand. The retailer therefore invests in more advertising. The supplier must offer more trade credit to fund the retailer's orders. To compensate for the costs of trade credit, the supplier charges a relatively high wholesale price. Correspondingly, the retailer increases the retail price. When $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, the "rich" retailer has sufficient initial capital to invest in advertising. As the advertising elasticity increases, it is more productive for the retailer to invest in more advertising. To encourage additional retailer orders, the supplier decreases the wholesale price to share part of the retailer's advertising expenses. In such a high demand environment, the retailer earns more sales income by increasing the retail price.

Proposition 3.4. *(i) When* $B \leq \left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2$*, then* $\frac{\partial\theta^*}{\partial I_r} = 0$*,* $\frac{\partial w^*}{\partial I_r} < 0$*, and* $\frac{\partial p^*}{\partial I_r} = 0$*; (ii) When* $\left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 < B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$*, then* $\frac{\partial\theta^*}{\partial I_r} = 0$*,* $\frac{\partial w^*}{\partial I_r} < 0$ *and* $\frac{\partial p^*}{\partial I_r} < 0$*; (iii) When* $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$*, then* $\frac{\partial\theta^*}{\partial I_r} < 0$*,* $\frac{\partial w^*}{\partial I_r} < 0$ *and* $\frac{\partial p^*}{\partial I_r} < 0$*.*

When $B \leq \left[\frac{(\alpha-\mathrm{C})\gamma}{8(1+I_r)-\gamma^2}\right]^2$, the "poor" retailer fully invests in advertising irrespective of the interest rate charged by the supplier. Increasing the interest rate adds to the retailer's purchase costs and further decreases order quantity. To encourage additional retailer orders, the supplier decreases the wholesale price. The low wholesale price actually compensates for the retailer's purchase costs at a high interest rate and leads to an unchanged retail price.

When $\left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 < B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, the "moderately rich" retailer conducts an identical advertising policy with the "poor" retailer. As the interest rate increases, the retailer bears more purchase costs. To raise the retailer's orders, the supplier decreases the wholesale price. Correspondingly, the retailer decreases the retail price to boost sales.

When $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, the "rich" retailer considers a balance between his purchase costs and advertising expenses. As the interest rate increases, the retailer shifts more initial capital into the immediate payment to reduce purchase costs. The reduction of advertising expenses directly leads to demand decreasing. To stimulate the retailer to invest in more advertising, the supplier decreases the wholesale price and, correspondingly, the retailer decreases the retail price.

4. Numerical simulations

In this section, we present numerical simulations to illustrate our theoretical results. The basic settings with respect to the main parameters are as follows: $\alpha = 100$, $c = 10$, $\gamma = 1$ and $I_r = 0.05$ (*e.g.*, see [28]). The numerical results are as follows:

Case 1: Given $B = 100$, then $B \leq \left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 = 147.918$. From Proposition 3.2(i), we have $w^* = \frac{\alpha+c+\gamma\sqrt{B}}{2(1+I_r)} = 57.143$. Based on Proposition 3.1(i), when $w^* < \Delta = \frac{\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r} = 64.762$, we have $p^* = 85$ and $\theta^* = 1$ (Fig. 4). The retailer's optimal profit is $\pi_r^* = 525$, and the supplier's optimal profit is $\pi_s^* = 1250$ (Fig. 5). Without advertising expenses, the profits of the retailer and supplier are $\pi_r^*(\theta = 0) = 511.273$ and $\pi_s^*(\theta = 0) = 1007.48$, respectively. An advertising policy at a low initial capital level benefits both parties.

Case 2: Given $B = 150$, then $\left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 = 147.918 < B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2 = 200.730$. From Proposition 3.2(ii), we have $w^* = \frac{\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r} = 57.913$. Based on Proposition 3.1(i), when $w^* = \Delta$, we have $w^* = \Delta = 57.913$, $p^* = 86.528$, and $\theta^* = 1$ (Fig. 6). The retailer's optimal profit is $\pi_r^* = 511.5$, and the supplier's optimal profit is $\pi_s^* = 1306.768$ (Fig. 7). Without advertising expenses, the profits of the retailer and supplier are $\pi_r^*(\theta = 0) = 513.773$ and $\pi_s^*(\theta = 0) = 1004.978$, respectively. Although the retailer's profit with advertising expenses $\pi_r^* = 511.5$ is less than $\pi_r^*(\theta = 0) = 513.773$, the advertising policy at a moderate initial capital level benefits the supply chain and supplier, for whom $\pi(\theta = 0) = 1518.751 < \pi(\theta \neq 0) = 1818.268$and $\pi_s^*(\theta = 0) = 1004.978 < \pi_s^*(\theta \neq 0) = 1306.768$ but hurts the retailer, for whom $\pi_r^*(\theta = 0) = 513.773 > \pi_r^*(\theta \neq 0) = 511.5$.

Case 3: Given $B = 250$, then B $> \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2 = 200.730$. From Proposition 3.2(iii), we have $w^* = \frac{(\alpha+c)\left[4(1+I_r)^2-\gamma^2\right]-\gamma^2 I_r(2\alpha+c)}{8(1+I_r)^3-\gamma^2 1+I_r(2+3I_r)} = 52.060$. Based on Proposition 3.1(ii), when $w^* > \Delta = 47.051$, we have

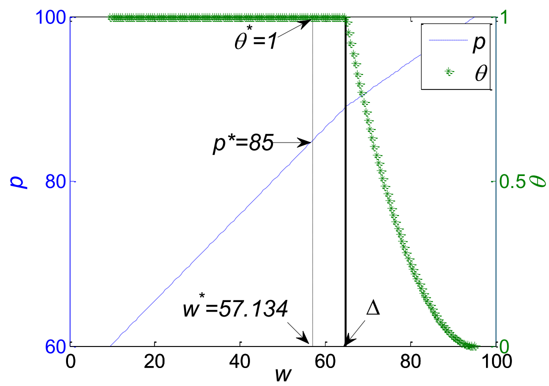

FIGURE 4. w, p and θ in Case 1.

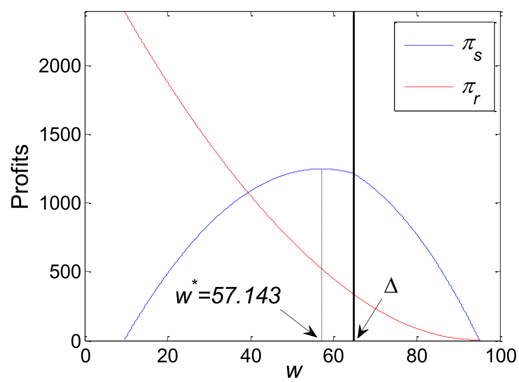

FIGURE 5. w, π_r and π_s in Case 1.

$w^* > \Delta = 47.051$. Furthermore, we obtain $p^* = 84.415$ and $\theta^* = 0.803$ (Fig. 8). The retailer's optimal profit is $\pi_r^* = 655.164$, and the supplier's optimal profit is $\pi_s^* = 1326.368$ (Fig. 9). Without advertising expenses, the profits of the retailer and supplier are $\pi_r^*(\theta = 0) = 518.773$ and $\pi_s^*(\theta = 0) = 999.978$, respectively. An advertising policy at a high initial capital level benefits both parties.

Cases 1–3 show that under different initial capital levels, advertising by the retailer alone always benefits the supply chain and the supplier. Although the retailer is hurt in Case 2 under a moderate initial capital level, the supplier's appropriate profit sharing can reduce the retailer's advertising costs and further push the retailer to invest in more advertising.

To make our theoretical results clearer, we analyze the sensitivity of w^*, p^* and θ^* with respect to γ and I_r. All of the settings are identical to those in Case 3. The results are listed in Table 2. We find that as γ increases, w^* is decreasing, p^* is increasing and θ^* is non-decreasing. The higher values of γ lead to correspondingly greater π_r^*, π_s^* and π^* because as the advertising elasticity increases, the advertising becomes more effective. The retailer is willing to invest in more advertising to enlarge demand, which further creates greater profits for both parties. By setting a low wholesale price, the supplier bears part of the retailer's costs and pushes the retailer to adopt this advertising policy. Thus, the profits of the supplier and retailer are increasing in γ. Hence, when advertising elasticity is greater, the beneficial supplier should consider sharing part of the retailer's advertising costs to encourage the retailer to engage in more sales promotions.

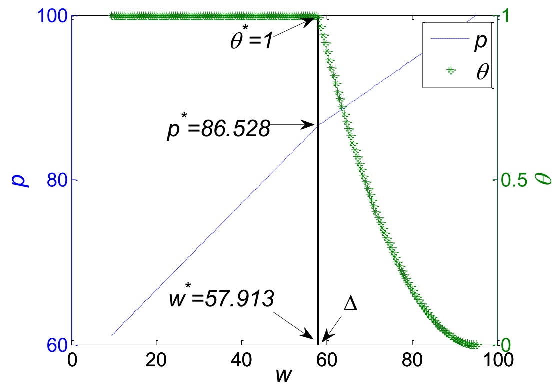

FIGURE 6. w, p and θ in Case 2.

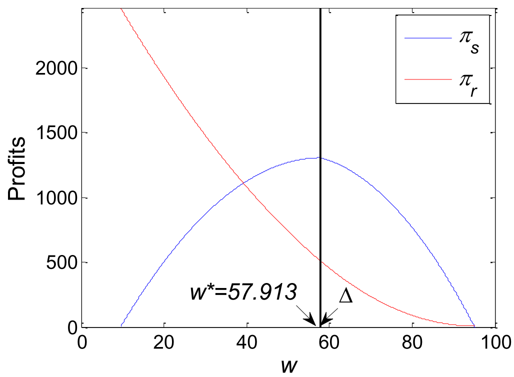

FIGURE 7. w, π_r and π_s in Case 2.

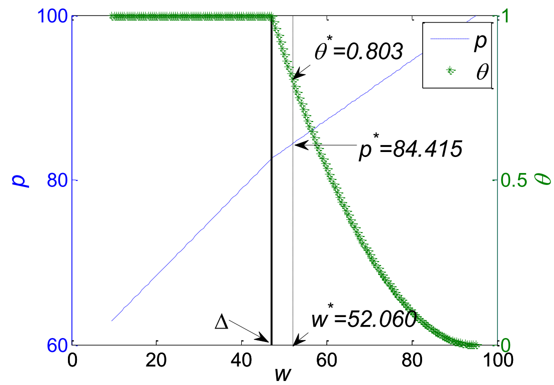

FIGURE 8. w, p and θ in Case 3.

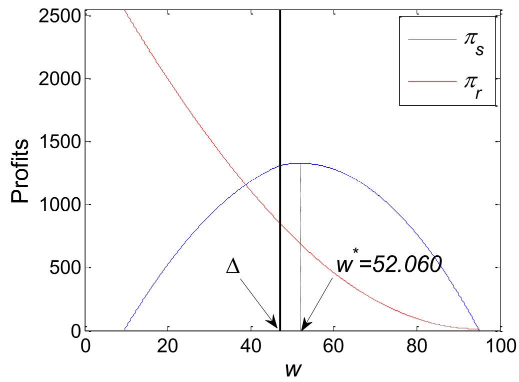

FIGURE 9. w, π_r and π_s in Case 3.

TABLE 2. Sensitivity analysis.

Parameter	Value	w^*	p^*	θ^*	π_r^*	π_s^*	π^*
γ	0.20	52.371	77.711	0.019	499.754	1009.970	1509.724
	0.60	52.285	79.564	0.199	542.479	1097.399	1639.878
	1.00	52.060	84.415	0.803	655.164	1326.368	1981.532
	1.40	51.469	88.089	1.000	909.177	1499.503	2408.680
	1.80	48.636	89.764	1.000	1247.401	1589.177	2836.578
I_r	0.03	53.193	84.640	0.840	662.970	1335.789	1998.759
	0.04	52.620	84.526	0.821	659.023	1331.073	1990.096
	0.05	52.060	84.415	0.803	655.164	1326.368	1981.532
	0.06	51.513	84.307	0.785	651.391	1321.676	1973.067
	0.07	50.978	84.202	0.768	647.705	1316.997	1964.702

As I_r increases, w^*, p^* and θ^* are decreasing. Higher values of I_r lead to correspondingly smaller π_r^*, π_s^* and π^*. The findings are straightforward because increasing I_r increases purchase costs and decreases advertising expenses. Hence, the beneficial supplier should lower her interest rate for trade credit to fund the retailer's orders. Due to decreasing interest costs, the retailer can invest in more advertising to boost sales.

5. CONCLUSION

In this paper, we consider a two-echelon supply chain consisting of one upstream supplier and one capital-constrained downstream retailer. The supplier can offer trade credit to fund the retailer's orders. To boost sales, the retailer invests part or all of his initial capital exclusively in advertising activities at the beginning of the sales season. Demand is sensitive to both the retail price and advertising expenses of the retailer. The supplier plays the dominant role in the relationship with the retailer. We analytically derive the Stackelberg equilibrium with respect to pricing by both parties and advertising by the retailer. The results show that the "poor" or "moderately rich" retailer always prefers to invest full initial capital in advertising regardless of the advertising elasticity or the interest rate charged by the supplier. Comparatively, the "rich" retailer only invests partial initial capital in advertising. The effects of the advertising elasticity and the interest rate on pricing and advertising policies depend upon the retailer's initial capital level. Numerical simulations reveal that the retailer's advertising policies under different initial capital levels always benefit the supply chain and the supplier. Although the "moderately rich" retailer is hurt by his advertising policy, the supplier's appropriate profit sharing can reduce the retailer's advertising costs and further push the retailer to invest in advertising.

Our work suggests two important management insights. First, the retailer's advertising policies under different initial capital levels always benefit the supply chain and the supplier. Therefore, the beneficial supplier should design an appropriate profit-sharing mechanism with the retailer to reduce the retailer's advertising costs. Such sharing will encourage the retailer's advertising investment. Second, the beneficial supplier should lower her interest rate for trade credit to reduce the retailer's purchase costs, thus ensuring that the retailer has more capital to invest in advertising.

Two possible directions exist for future research. First, in our analysis, the capital-constrained retailer bears the advertising costs alone. It would be interesting to discuss cooperative advertising by the supplier and the capital-constrained retailer by constructing a revenue-sharing contract. Second, setting of the interest rate charged by the supplier as a decision might yield different and interesting results in equilibrium analyses of pricing and advertising in a supply chain with a capital-constrained retailer.

Appendix

Proof of Proposition 3.1. From equation (3.1), $\frac{\partial \pi_r}{\partial p} = \alpha - 2p + \gamma\sqrt{B\theta} + w(1+I_r)$ and $\frac{\partial^2 \pi_r}{\partial p^2} = -2 < 0$. Utilizing $\frac{\partial \pi_r}{\partial p} = 0$ yields $p = \frac{\alpha+\gamma\sqrt{B\theta}+w(1+I_r)}{2}$. Substituting p into π_r yields $\pi_r = \frac{\left[\alpha-w(1+I_r)+\gamma\sqrt{B\theta}\right]^2}{4} - B\theta(1+I_r) + BI_r$. According to $\frac{\partial \pi_r}{\partial \theta} = \frac{B\gamma[\alpha-w(1+I_r)]+B\sqrt{B\theta}\left[\gamma^2-4(1+I_r)\right]}{4\sqrt{B\theta}}$ and $\alpha - w(1+I_r) > \alpha - p > 0$, $\frac{\partial^2 \pi_r}{\partial \theta^2} = -\frac{B^2\gamma[\alpha-w(1+I_r)]}{8(B\theta)^{\frac{3}{2}}} < 0$. Solving $\frac{\partial \pi_r}{\partial \theta} = 0$ obtains $\theta = \frac{\gamma^2[w1+I_r-\alpha]^2}{B[\gamma^2-41+I_r]^2}$. Because $\theta \in (0,1]$,

(i) When $\frac{\gamma^2[\alpha-w(1+I_r)]^2}{B[\gamma^2-41+I_r]^2} \geq 1$, i.e., $w \leq \Delta = \frac{\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r}$, then $\theta^* = 1$ and $p^* = \frac{\alpha+\gamma\sqrt{B}+w(1+I_r)}{2}$;

(ii) When $\frac{\gamma^2[\alpha-w(1+I_r)]^2}{B[\gamma^2-41+I_r]^2} < 1$, i.e., $\frac{\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r} < w < \frac{-\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r}$, then $\frac{\sqrt{B}[\gamma^2-4(1+I_r)]}{\gamma(1+I_r)} + \frac{\alpha}{1+I_r} < w < \frac{\alpha}{1+I_r}$. Hence, when $w > \Delta$, then $\theta^* = \frac{\gamma^2[w1+I_r-\alpha]^2}{B[\gamma^2-41+I_r]^2}$ and $p^* = \frac{\gamma^2 w1+I_r-2(1+I_r)[w1+I_r+\alpha]}{\gamma^2-41+I_r}$.

Proof of Proposition 3.2. When $w \leq \Delta$ and $\pi_s = \frac{\left[\alpha+\gamma\sqrt{B}-w(1+I_r)\right][w(1+I_r)-c]}{2}$, then $\frac{\partial^2 \pi_s}{\partial w^2} < 0$. Solving $\frac{\partial \pi_s}{\partial w} = 0$ obtains $w = \frac{\alpha+c+\gamma\sqrt{B}}{2(1+I_r)}$. Let $w_1 = \frac{\alpha+c+\gamma\sqrt{B}}{2(1+I_r)}$. When $w > \Delta$ and $\pi_s = \frac{2(1+I_r)[w1+I_r-\alpha][w(1+I_r)-c]}{\gamma^2-41+I_r} - BI_r\left\{1 - \frac{\gamma^2(w1+I_r-\alpha)^2}{B(\gamma^2-4(1+I_r))^2}\right\}$, then $\frac{\partial^2 \pi_s}{\partial w^2} = \frac{2(1+I_r)^2[(2+3I_r)\gamma^2-8(1+I_r)^2]}{[\gamma^2-4(1+I_r)]^2} < \frac{2(1+I_r)^2[4(2+3I_r)-8(1+I_r)^2]}{[\gamma^2-4(1+I_r)]^2} < 0$. Solving $\frac{\partial \pi_s}{\partial w} = 0$ obtains $w = \frac{(\alpha+c)\left[4(1+I_r)^2-\gamma^2\right]-\gamma^2 I_r(2\alpha+c)}{8(1+I_r)^3-\gamma^2 1+I_r(2+3I_r)}$. Let $w_2 = \frac{(\alpha+c)\left[4(1+I_r)^2-\gamma^2\right]-\gamma^2 I_r(2\alpha+c)}{8(1+I_r)^3-\gamma^2 1+I_r(2+3I_r)}$. $w_1 - w_2 > \frac{\alpha+c}{2(1+I_r)} - \frac{(\alpha+c)\left[4(1+I_r)^2-\gamma^2\right]-\gamma^2 I_r(2\alpha+c)}{8(1+I_r)^3-\gamma^2 1+I_r(2+3I_r)} = \frac{\gamma^2 I_r(\alpha-c)}{2(1+I_r)\left[8(1+I_r)^2-\gamma^2(2+3I_r)\right]} > 0$.

(i) When $w_1 \leq \Delta$, then $B \leq \left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2$. When $w < w_1$, then $\frac{\partial \pi_s}{\partial w} > 0$. When $w > w_1$, then $\frac{\partial \pi_s}{\partial w} < 0$. Hence, $w^* = w_1$;

(ii) When $w_2 \leq \Delta < w_1$, then $\left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 < B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$. When $w < \Delta$, then $\frac{\partial \pi_s}{\partial w} > 0$. When $w > \Delta$, then $\frac{\partial \pi_s}{\partial w} < 0$. Hence, $w^* = \Delta$;

(iii) When $\Delta < w_2$, then $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$. When $w < w_2$, then $\frac{\partial \pi_s}{\partial w} > 0$. When $w > w_2$, then $\frac{\partial \pi_s}{\partial w} < 0$. Hence, $w^* = w_2$.

Proof of Proposition 3.3.

(i) When $B \leq \left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2$, then $w^* = w_1$, $\theta^* = 1$ and $p^* = \frac{\alpha+\gamma\sqrt{B}+w^*(1+I_r)}{2}$. Hence, $\frac{\partial w^*}{\partial \gamma} = \frac{\sqrt{B}}{2(1+I_r)} > 0, \frac{\partial \theta^*}{\partial \gamma} = 0$ and $\frac{\partial p^*}{\partial \gamma} = \frac{\sqrt{B}}{2} + \frac{(1+I_r)}{2}\frac{\partial w^*}{\partial \gamma} > 0$. When $\left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 < B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, then $w^* = \Delta$, $\theta^* = 1$ and $p^* = \frac{\alpha+\gamma\sqrt{B}+w^*(1+I_r)}{2}$. Hence, $\frac{\partial w^*}{\partial \gamma} = \frac{\sqrt{B}\left[\gamma^2+4(1+I_r)\right]}{\gamma^2(1+I_r)} > 0, \frac{\partial \theta^*}{\partial \gamma} = 0$ and $\frac{\partial p^*}{\partial \gamma} = \frac{\sqrt{B}}{2} + \frac{(1+I_r)}{2}\frac{\mathrm{d}w^*}{\mathrm{d}\gamma} > 0$. Therefore, when $B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, then $\frac{\partial w^*}{\partial \gamma} > 0, \frac{\partial \theta^*}{\partial \gamma} = 0$ and $\frac{\partial p^*}{\partial \gamma} > 0$.

(ii) When $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, then $w^* = w_2$, $= \frac{\gamma^2[w^*1+-\alpha]^2}{B[\gamma^2-41+I_r]^2}$and $p^* = \frac{\gamma^2 w^*1+I_r-2(1+I_r)[w^*1+I_r+\alpha]}{\gamma^2-41+I_r}$. Hence, $\frac{\partial w^*}{\partial\gamma} = -\frac{8\gamma I_r(\alpha-c)(1+I_r)}{[8(1+I_r)^2-\gamma^2(2+3I_r)]^2} < 0$, $\frac{\partial\theta^*}{\partial\gamma} = \frac{2\gamma(\alpha-c)^2(1+I_r)^2\left[8(1+I_r)^2+\gamma^2(2+3I_r)\right]}{B[8(1+I_r)^2-\gamma^2(2+3I_r)]^3} > 0$ and $\frac{\partial p^*}{\partial\gamma} = \frac{4\gamma(\alpha-c)(1+I_r)^2(2+I_r)}{[8(1+I_r)^2-\gamma^2(2+3I_r)]^2} > 0$.

Proof of Proposition 3.4.

(i) When $B \leq \left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2$, then $w^* = w_1$, $\theta^* = 1$ and $p^* = \frac{\alpha+\gamma\sqrt{B}+w^*(1+I_r)}{2}$. Hence, $\frac{\partial w^*}{\partial I_r} = -\frac{\alpha+c+\gamma\sqrt{B}}{2(1+I_r)^2} < 0$, $\frac{\partial\theta^*}{\partial I_r} = 0$ and $\frac{\partial p^*}{\partial I_r} = \frac{w^*}{2} + \frac{(1+I_r)}{2}\frac{\mathrm{d}w^*}{\mathrm{d}I_r} = 0$;

(ii) When $\left[\frac{(\alpha-c)\gamma}{8(1+I_r)-\gamma^2}\right]^2 < B \leq \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, then $w^* = \Delta$, $\theta^* = 1$ and $p^* = \frac{\alpha+\gamma\sqrt{B}+w^*(1+I_r)}{2}$. Hence, $\frac{\partial w^*}{\partial I_r} = -\frac{\alpha+\gamma\sqrt{B}}{(1+I_r)^2} < 0$, $\frac{\partial\theta^*}{\partial I_r} = 0$ and $\frac{\partial p^*}{\partial I_r} = -\frac{2\sqrt{B}}{\gamma} < 0$;

(iii) When $B > \left[\frac{(\alpha-c)(1+I_r)\gamma}{8(1+I_r)^2-\gamma^2(2+3I_r)}\right]^2$, then $w^* = w_2$, $\theta^* = \frac{\gamma^2[w^*1+I_r-\alpha]^2}{B[\gamma^2-41+I_r]^2}$ and $p^* = \frac{\gamma^2 w^*1+I_r-2(1+I_r)[w^*1+I_r+\alpha]}{\gamma^2-41+I_r}$. Thus, $\frac{\partial\theta^*}{\partial I_r} = -\frac{2\gamma^2(\alpha-c)^2(1+I_r)\left[8(1+I_r)^2-\gamma^2\right]}{B\left[8(1+I_r)^2-\gamma^2(2+3I_r)\right]^3} < 0$, $s\frac{\partial p^*}{\partial I_r} = -\frac{\gamma^2(\alpha-c)\left(2I_r^2+8I_r-\gamma^2+6\right)}{[8(1+I_r)^2-\gamma^2(2+3I_r)]^2} < 0$ and $\frac{\partial w^*}{\partial I_r} = \frac{8(1+I_r)[\alpha+c-3w^*(1+I_r)]+\gamma^2[w^*(5+6I_r)-2\alpha-c]}{8(1+I_r)^3-\gamma^2(2+3I_r)(1+I_r)} < \frac{4(c-w^*)}{8(1+I_r)^2-\gamma^2(2+3I_r)} < 0$.

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