RAIRO-Oper. Res. 53 (2019) 1937–1965
https://doi.org/10.1051/ro/2018109

RAIRO Operations Research
www.rairo-ro.org

MODELLING PRICING, VERTICAL CO-OP ADVERTISING AND QUALITY IMPROVEMENT IN A NON-COOPERATIVE THREE-ECHELON SUPPLY CHAIN USING GAME THEORY APPROACH

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Abstract. Vertical cooperative (co-op) advertising is one of the well-known mechanisms for coordination of supply chains. Vertical co-op advertising is a financial agreement in which a member of the chain pays certain percentage (*i.e.* cooperation rate) of a subsequent member's advertisement cost. Since increasing the number of echelons and decision variables in supply chain problems increase the modelling and computational complexity, most researchers study vertical co-op advertising in a two-level supply chain including a manufacturer and a retailer. This paper investigates the problem by considering price and quality levels as additional decision variables in a three-echelon supply chain consisting of one supplier, one manufacturer, and one retailer. The ultimate goal is to show supply chain managers the importance of product quality as well the role of local advertisement in positively influencing market demand on top of the traditional approach of speed and efficiency optimization. Using game theory approach, power of the manufacturer is assumed to be higher than or equal to those of others in the chain. Five different relationships between players are considered in five non-cooperative games (named as G1–G5) and equilibrium solutions are extracted for each. The results show that the manufacturer prefers to play Stackelberg with the retailer and the supplier rather than be in conflict with them in Nash game. Such preference can lead manufacturer towards high quality and cost-efficient product/service via efficient advertisement in our complex network of business firms.

Mathematics Subject Classification. 91A10.

Received November 6, 2017. Accepted November 25, 2018.

1. Introduction

In today's competitive and global market where many pricing and non-pricing factors influence demand, price and advertisement are not the only factors which affect customers' choice. Many other non-pricing factors undoubtedly affect their choice, among which the quality of product/service is of high importance. In fact, the key sustainable competitive advantage of a manufacturer is to work in a supply chain framework and be able to provide high quality and cost-efficient product/service through efficient advertisement. Hence, prosperity of

Keywords. Supply chain management, pricing theory, vertical cooperative advertising, quality improvement, game theory.

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Article published by EDP Sciences

a manufacturer depends, to a large extent, on coordination with other members of a supply chain in a complex network of commercial communications.

Supply chain management involves an optimization system that enables companies to deliver their products to end consumers in an efficient manner. Traditional approach involves optimization of speed and efficiency. The most effective ones, however, can deliver products as fast and as cheap as possible without sacrificing quality. This can be achieved by means of complicated logistics tools that implements some computer algorithms which maintain productivity, quality, and efficiency of operations. In a supply chain with independent members, the manufacturer can apply various mechanisms for coordination. Vertical cooperative (co-op) advertising is one of the well-known mechanisms for coordination of the supply chain of manufacturer–retailer which has been extensively studied. By definition, vertical co-op advertising is a financial agreement in which a manufacturer pays a certain percentage (cooperation rate) of a retailer's advertisement cost [1]. Accordingly, the first cooperative advertising agreement was issued by Warner Brothers in 1903 and afterwards, applying vertical co-op advertising was developed in supermarkets, fashion stores, and other non-consuming and capital products [2]. The first mathematical model was published on cooperative advertising by Berger in 1972 and since then, there have been a lot of researches which have addressed the issue [3].

Most studies in this regard consider one or two-level supply chain including one manufacturer and one retailer. They normally consider the optimal level of advertisement as the primary decision variable. Price is usually considered as the second decision variable in studies which consider more decision variables; and the same applies to the level of advertisement. Other decision variables such as inventory and quality level are also considered in a few studies such as [4]. However, one of the influential factors on customer's demand in today's markets is product quality. Since quality improvement is quite costly, making decision on product quality in a supply chain is of great importance and can influence other decision variables such as price and advertisement levels. Depending on the type of product, quality can be influenced by both the quality of raw material as well as production quality (product design, packaging, etc.), which are of different importance to different industries. For instance, design quality is considered more important in fashion industry than the quality of raw materials; while in food industry, the quality of the end products mainly depends on the quality of raw materials.

The rest of the paper is organized as follows: Section 2 reviews the literature on supply chain coordination considering vertical co-op advertising and quality improvement. Section 3 presents the proposed model, which is subsequently discussed in the framework of five non-cooperative games in Section 4. Section 5 gives numerical analysis; sensitivity analysis of the parameters is given in Section 6. Finally, Section 7 presents concluding remarks and some ideas on further research in this area.

2. Literature review

In this section, relevant previous studies that have considered vertical co-op advertising in the pricing problem in supply chains are initially reviewed. After presenting their findings, a tabular form their demand function, supply chain structures and utilized games along with those of the current research are shown. Finally, contribution of the current research and its importance are highlighted. It is to be noted that most of the researches in this area have considered a two-echelon manufacturer–retailer supply chain, though there are a few which have considered more than one retailer. There are also a number of researches that have considered product quality factor with pricing problem.

Huang and Li presented power demand model in a manufacturer–retailer supply chain to study vertical co-op advertising for both a complete cooperative game as well as for a Stackelberg game in which the manufacturer was the leader and the retailer was the follower [5]. In contrast to the researches that had addressed Stackelberg game with the manufacturer and retailer playing the role of leader and follower, respectively, Huang *et al.* considered a state wherein the retailer and the manufacturer played inverse roles [6]. Quilliot *et al.* examined a pricing model which was an extension of the cooperative game concept considering elastic demand [7]. They modeled the problem utilizing network pricing concept and proposed a solution algorithm. SeyedEsfahani *et al.* [8] extended the demand function of Xie and Wei [9] and examined four games in a manufacturer–retailer supply chain similar to those of Xie and Neyret [10]. Aust and Buscher [11] utilized the demand function of SeyedEsfahani *et al.* [8] and studied four games which relaxed the restrictive assumption of identical margins for both players in the Nash and the Stackelberg retailer games that were used by Xie and Wei [9] and SeyedEsfahani *et al.* [8]. Naoum-Sawaya and Elhedhli [12] analyzed a multi-period oligopolistic market where each period was a Stackelberg game between one member as the leader and multiple other members as followers; initially, the leader determined its production level, and then the followers decided on their production levels. The leader had the power to make the followers out of business by preventing them from achieving a predetermined sales level within a given time period. The leader could also reduce the market prices to even lower than the Stackelberg's equilibrium level in order to push the followers to sell less and eventually drive them out of business. They proved that there exists a predatory pricing strategy where the market price is above the average cost and consumer welfare is preserved.

Unlike many studies which only used bargaining games to divide the profit in cooperative games, Marchi and Cohen [13], and Ghadimi *et al.* [14] utilized Shapley value so as to characterize it by a collection of desirable properties. In contrast to previous researches which were based on the assumption that the retailer and the manufacturer decided simultaneously on their parameters (*i.e.* price and the level of advertisement), Karray [15] assessed the optimal sequence of decisions on price and advertising for nine non-cooperative games which were categorized into three cases of: manufacturer being the leader, retailer being the leader and no leardership scenario. The first three games were solved for the case where the manufacturer was leader. In the first game, the manufacturer decided its price and the level of advertisement and, then in the next stage, the retailer decided its price and the level of advertisement. The second and third games were modelled on the basis of two Stackelberg games such that in the first stage of the second game, players decided on their prices, then in next stage, on their level of advertisement while in the first stage of the third game, players initially competed for the advertisement and then for the price. In the next case, the retailer was the leader and three games similar to those in the first case were solved. In the third case, there was no leader (vertical Nash) and both the manufacturer and the retailer simultaneously selected their prices and advertisement policies. Ang *et al.* [16] studied a game model of multi-leader and one-follower in a supply chain including a number of suppliers competing to provide a single product for a manufacturer. The selling price of each supplier was regarded as a pre-determined parameter and suppliers were assumed to compete on the basis of delivery frequency to the manufacturer. Each supplier's profit depended not only on its own delivery frequency, but also on other suppliers' frequencies through their impact on manufacturer's quota allocation to the suppliers. They formulated the given game as a generalized Nash game. For the special case that the selling prices of all suppliers were identical, a sufficient and necessary condition for the existence and uniqueness of the Nash equilibrium was given. Zhao *et al.* [17] studied pricing and vertical coop advertising decisions in a manufacturer–retailer supply chain using a Stackelberg game model where the manufacturer acted as the game leader and the retailer as the game follower; they presented closed-form equilibrium solution and explicitly showed how pricing and advertising decisions were made. When market demand decreased exponentially with respect to the retail price and increased with respect to national and local advertising expenditures in an additive way, the manufacturer benefited from providing partial reimbursement for the retailer's local advertising expenditure when demand price elasticity was large enough. Studying price effect on demand in the addressed research is similar to Szmerekovsky and Zhang [18], and studying advertising effect on demand is similar to Xie and Wei [9]. Chaeb and Rasti-Barzoki [19] combined advertising-sales response function proposed by Huang and Li [5] and price demand function similar to the approach proposed by SeyedEsfahani *et al.* [8] in a manufacturer–retailer supply chain.

The channel structure with more than one retailer was considered in some other studies. Wang *et al.* [20] considered a supply chain with one manufacturer and two retailers taking four types of games into account. The impact of multiple retailers on members' decisions and on total channel efficiencies was studied in [21]. The addressed research expanded the demand function of Xie and Wei [9] in a channel structure with one manufacturer and multiple retailers. It assumed that there were no intra-brand competitions in this channel and each retailer could only affect its own demand. This structure was modeled considering a cooperative game and two two-stage Stackelberg games with symmetric and asymmetric retailers. Aust and Buscher [22] considered a supply chain with one manufacturer and two retailers with two Stackelberg games; in the first game the retailers played Nash game and in the second they cooperated. Giri and Sharma [23] considered four manufacturer Stackelberg games with one manufacturer and two competing retailers with different sales costs. The manufacturer used two different pricing strategies; setting same or different wholesale prices for retailers in two models, and participating or not participating in retailer's advertising cost. Karray and Amin [24] studied a supply chain with one manufacturer and two competing retailers considering three games. In this research, global advertising expenditures was not considered as decision variable for manufacturer and only coop participation rate and the wholesale price were considered. The comprehensive reviews of the models used in advertising cooperative games can be found in [4, 25, 26].

One of the early studies which considered both quality as well as price was [27] in which two firms competed on price and quality level using two-stage Stackelberg game. Quality level of both design and conformance were considered as decision variables. In the current study, definitions of quality, demand function and quality costs are similar to the addressed research. Following portion of the review addresses a few related articles in this field. Xie *et al.* [28] studied quality improvement in a similar market segment divided between two supply chains. In this research, it was assumed that price and other parameters were fixed and the two chains only competed on quality. Hong and Chen [29] modelled quality control in a supplier–manufacturer supply chain using Stackelberg game. They found that the profit was significantly higher while considering cooperative game (using the Nash bargaining model to divide profit) rather than the Stackelberg game. Aust [30] examined pricing to determine the quality of supplier and manufacturer, and retailer's service level in a three echelon supply chain by three different games. Xie *et al.* [31] considered quality and price decisions in a supplier–manufacturer supply chain. Initially, they examined their model with manufacturer's Stackelberg and supplier's Stackelberg; then, they investigated the impacts of two quality improvement policies on equilibrium solutions: Coordination and Manufacturer's involvement in quality improvement of the supplier. Zheng *et al.* [32] compared a normal and a reverse supply chain so that each supply chain consisted of a retailer and an exclusive supplier with stable partnership. The two chains competed with each other in three competition structures of: the centralized competition game, the hybrid competition game and the decentralized competition game. In the different competition structures, the degree of competition intensity between the two chains was examined. Jafari *et al.* [33] investigated pricing and ordering decisions on a dual-channel supply chain consisting of monopolistic manufacturer and duopolistic retailers. The market was assumed to be controlled by the manufacturer. The manufacturer was considered as the leader and the two retailers acted as followers. Different game-theoretic models including Bertrand, Collusion, and Stackelberg were developed to analyse pricing strategies under various interactions between the two retailers and the equilibrium decisions were compared under different scenarios.

From among researches which have considered the quality factor in the basic pricing problem, we investigated two recent ones. Liu and Yi [34] considered the role of big data concept and targeted advertising in improving marketing accuracy and success. Meanwhile, products green degree is also an important factor in influencing sale. They studied the pricing policies of a green manufacturer–retailer supply chain considering targeted advertising input and products greening costs in the Big Data environment. Furthermore, four Game situations were proposed based on the Stackelberg game and Nash Equilibrium and the change trends of prices with the green degree along with the input level of targeted advertising were analyzed. Taleizadeh *et al.* [35] investigated the pricing strategies along with the quality level and effort decisions of the manufacturer, retailer, and third party operating in two types of closed-loop supply chains including a single-channel forward supply chain with a dual-recycling channel and a dual-channel forward supply chain with a dual-recycling channel. They utilized Stackelberg game models to find the best values for prices, quality levels, sales, and collection efforts and gave corresponding equilibrium solutions of the two model structures.

In Table 1, the demand functions, supply chain structures and various games used in the reviewed researches are illustrated along with those of the current research. As can be seen from Table 1, there are limited researches which have considered quality with vertical co-op advertising since increasing decision variables can significantly increase the complexity of the problem. Only a few studies have considered quality or quality along with price. Taking into account the importance of considering quality and vertical co-op advertising together, the current research in this paper considers price, advertising and quality levels as decision variables in a three-echelon supply chain consisting of one supplier, one manufacturer and one retailer using game theory approach. Since each channel member has different power, five different relationships among them are considered in the framework of five non-cooperative games and equilibrium solutions for each game are determined.

As can be seen from Table 1, this study has the largest number of decision variables among the relevant studies in the literature. It is also the only study which consider quality and vertical co-op advertising (*i.e.* q, t) together. Our findings shows that one of the games (G4) results in the best value across almost all the parameters something which can reveal the importance of product quality and the role of local advertisement in influencing demand sensitivity. It is only in this game that the manufacturer is the leader, determines quality level when dealing with the supplier and participates in the retailer's advertising cost.

3. Model framework

Numerous parameters and decision variables that we have used for modelling the problem are as follows:

Parameters

R	Index of the retailer.
M	Index of the manufacturer.
S	Index of the supplier.
D	Demand of the product.
Π_s, Π_R, Π_M	Profit function of the supplier, the retailer and the manufacturer, respectively.
α	Baseline demand.
β	Demand sensitivity to price.
χ	Fix constant.
λ_M, λ_S	Demand sensitivity to quality level of the manufacturer and the supplier, respectively.
c_s, c_R, c_M	The supplier's unit production cost of raw material, the retailer's unit handling and sales cost and the manufacturer's unit production cost of the product, respectively.
k_M, k_R	Demand sensitivity to local and national advertising, respectively.
ν_M, ν_S	Measure of the responsiveness of quality investment cost to quality level which is selected by manufacturer and the supplier, respectively.
f_M, f_S	Fix costs of investment on quality improvement program by the manufacturer and the supplier, respectively.
θ_S, θ_M	Impact of quality level on unit material cost and unit production cost, respectively.

Decisions Variables are as follows:

a_M	Global advertising expenditures of the manufacturer.
a_R	Local advertising expenditures of the retailer.
q_M	Design quality level selected by the manufacturer for the product.
q_S	Quality level selected by the supplier for the raw material.
t	Manufacturer 's participation rate in retailer's advertising cost.
P_M	Manufacturer's price for the product (wholesale price).
P_R	Retailer's price for the product.
P_S	Supplier's price for the raw material.

As indicated in Figure 1, a three-echelon supply chain including one supplier, one manufacturer, and one retailer is considered. The supplier provides raw materials at quality level of q_S and unit price of p_S for the manufacturer and the manufacturer processes the raw materials and sells the ultimate product only through the retailer at unit price of p_M. The retailer only sells the manufacturer's product to ultimate customers at retailer price of p_R. It is assumed that each unit of final product needs one unit of raw material. The manufacturer participates in the retailer's local advertising cost with a known participation rate. This model follows the assumption of previous researchers that the retailer orders from the manufacturer exactly what consumers will demand, and the manufacturer fully provide the retailer's order.

TABLE 1. The demand function, supply chain structures and games used in the vertical co-op advertising articles.

Ref.	Demand Function[a]	No. of suppliers	No. of manufactures	No. of retailers	Decision Variables[b]	Games[c]
[9]	$D=(1-\beta p_R)(k_M\sqrt{a_M}+k_R\sqrt{a_R})$	–	1	1	p, a, t	$S(\underset{a_M,p_M,t}{M} \rightarrow \underset{p_R,a_R}{R})$ $C(\underset{a_M,p_M}{M}, \underset{p_R,a_R}{R})$
[10]	$D=(\alpha-\beta p_R)(A-Ba_M^{-\mu_1}a_R^{-\mu_2})$	–	1	1	p, a, t	$S(\underset{a_M,p_M,t}{M} \rightarrow \underset{p_R,a_R}{R})$ $S(\underset{p_R,a_R}{R} \rightarrow \underset{a_M,p_M,t}{M})$ $N(\underset{a_M,p_M,t}{M}, \underset{p_R,a_R}{R})$ $C(\underset{a_M,p_M}{M}, \underset{p_R,a_R}{R})$
[18]	$D=p_R^{-e}(A-Ba_M^{-\mu_1}a_R^{-\mu_2})$	–	1	1	p, a, t	$S(\underset{a_M,p_M,t}{M} \rightarrow \underset{p_R,a_R}{R})$
[8]	$D=D_0(\alpha-\beta p_R)^{\frac{1}{\nu}}(k_M\sqrt{a_M}+k_R\sqrt{a_R})$	–	1	1	p, a, t	$S(\underset{a_M,p_M,t}{M} \rightarrow \underset{p_R,a_R}{R})$ $S(\underset{p_R,a_R}{R} \rightarrow \underset{a_M,p_M,t}{M})$
[11]	$D=(\alpha-\beta p_R)^{\frac{1}{\nu}}(k_M\sqrt{a_M}+k_R\sqrt{a_R})$	–	1	1		$N(\underset{a_M,p_M,t}{M}, \underset{p_R,a_R}{R})$ $C(\underset{a_M,p_M}{M}, \underset{p_R,a_R}{R})$
[15]	$D=\alpha-p_R+k_M\sqrt{a_M}+k_R\sqrt{a_R}$	–	1	1	p, a, t	$S(\underset{a_M,p_M,t}{M} \rightarrow \underset{p_R,a_R}{R})$ $S(S(\underset{p_M}{M} \rightarrow \underset{p_R}{R}) \rightarrow S(\underset{a_M,t}{M} \rightarrow \underset{a_R}{R}))$ $S(S(\underset{a_M,t}{M} \rightarrow \underset{a_R}{R}) \rightarrow S(\underset{p_M}{M} \rightarrow \underset{p_R}{R}))$ $S(\underset{p_R,a_R}{R} \rightarrow \underset{a_M,p_M,t}{M})$ $S(S(\underset{p_R}{R} \rightarrow \underset{p_M}{M}) \rightarrow S(\underset{a_R}{R} \rightarrow \underset{a_M,t}{M}))$ $S(S(\underset{a_R}{R} \rightarrow \underset{a_M,t}{M}) \rightarrow S(\underset{p_R}{R} \rightarrow \underset{p_M}{M}))$ $N(\underset{a_M,p_M,t}{M}, \underset{p_R,a_R}{R})$ $S(N(\underset{p_M}{M}, \underset{p_R}{R}) \rightarrow N(\underset{a_M,t}{M}, \underset{a_R}{R}))$ $S(N(\underset{a_M,t}{M}, \underset{a_R}{R}) \rightarrow N(\underset{p_M}{M}, \underset{p_R}{R}))$
[17]	$D=p_R^{-e}(k_M\sqrt{a_M}+k_{R_j}\sqrt{a_{R_j}})$	–	1	1	$p, a, t,$	$S(\underset{a_M,p_M,t}{M} \rightarrow \underset{p_R,a_R}{R})$
[19]	$D=D_0(\alpha-\beta p_R)^{\frac{1}{\nu}}(A-Ba_M^{-\gamma}a_R^{-\delta})$ $D=(\alpha-\beta p_R)^{\frac{1}{\nu}}(A-Ba_M^{-\mu_1}a_R^{-\mu_2})$	–	1	1	p, a, t	$N(\underset{a_M,p_M,t}{M}, \underset{p_R,a_R}{R})$ $S(\underset{p_R,a_R}{R} \rightarrow \underset{a_M,p_M,t}{M})$ $S(\underset{a_M,p_M,t}{M} \rightarrow \underset{p_R,a_R}{R})$ $C(\underset{a_M,p_M}{M}, \underset{p_R,a_R}{R})$
[20]	$D_i=A_i-Ba_{R_i}^{-\mu_1}a_{R_j}^{\mu_2}(1+a_M)^{-\mu_3}$ $i,j=1,2, i\neq j$	–	1	2	a, t	$S(\underset{a_M}{M} \rightarrow N(\underset{a_{R_i}}{R_i}, \underset{a_{R_j}}{R_j}))$ $N(\underset{a_M}{M}, \underset{a_{R_i}}{R_i}, \underset{a_{R_j}}{R_j})$ $N(\underset{a_M}{M} \rightarrow C(\underset{a_{R_i}}{R_i}, \underset{a_{R_j}}{R_j}))$
[21]	$D_i=\alpha_i+\sqrt{a_M}+k_{R_i}\sqrt{a_{R_i}}$ $i=1,\ldots,n$	–	1	n	a, t	$C(\underset{a_M}{M}, \underset{a_{R_1}}{R_1}, \ldots, \underset{a_{R_J}}{R_J})$ $S(\underset{a_M,t_1,\ldots,t_J}{M} \rightarrow N(\underset{a_{R_1}}{R_1}, \ldots, \underset{a_{R_J}}{R_J}))$ with symmetric retailers $S(\underset{a_M,t_1,\ldots,t_J}{M} \rightarrow N(\underset{a_{R_1}}{R_1}, \ldots, \underset{a_{R_J}}{R_J}))$ with asymmetric retailers
[22]	$D_i=(\alpha_i-\beta_1 p_{R_i}+\beta_2 p_{R_j})\times\left(k_M\sqrt{a_M}+k_R(\sqrt{a_{R_1}}+\sqrt{a_{R_2}})\right)$ $i,j=1,2, i\neq j$	–	1	2	p, a, t	$S(\underset{a_M,p_M,t}{M} \rightarrow N(\underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{R_j}}{R_j}))$ $S(\underset{a_M,p_M,t}{M} \rightarrow C(\underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{R_j}}{R_j}))$

Table 1. Continued.

Ref.	Demand Function[a]	No. of suppliers	No. of manufactures	No. of retailers	Decision Variables[b]	Games[c]
[23]	$D_i = \alpha + (k_1\sqrt{a_{R_i}}$ $-k_2\sqrt{a_{Rj}})$ $i, j = 1, 2, i \neq j$	–	1	2	p, a, t	$S(\underset{a_M,p_M,t}{M} \to N(\underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{Rj}}{R_j})$ $S(\underset{a_M,p_{M_1},p_{M_2},t}{M} \to N(\underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{Rj}}{R_j})$ $S(\underset{a_M,p_M}{M} \to N(\underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{Rj}}{R_j})$ $S(\underset{a_M,p_{M_1},p_{M_2}}{M} \to N(\underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{Rj}}{R_j})$
[24]	$D_i = \alpha - p_{R_i} + \beta p_{R_j}$ $+ (\kappa_1\sqrt{a_{R_i}} - \kappa_2\sqrt{a_{R_j}})$ $i, j = 1, 2, i \neq j$	–	1	2	p, a_R, t	$S(\underset{p_M}{M} \to N(\underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{Rj}}{R_j}))$ $S(\underset{p_M,t}{M} \to N(\underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{Rj}}{R_j}))$ $C(\underset{p_M}{M}, \underset{p_{R_i},a_{R_i}}{R_i}, \underset{p_{R_j},a_{Rj}}{R_j})$
[27]	$D_i = \alpha_i - \beta_1 p_{M_i} + \beta_2 p_{M_j}$ $+\lambda_1 q_{M_i} - \lambda_2 q_{M_j}$ $i, j = 1, 2, i \neq j$	–	2,N	1	p, q	$S(N(\underset{q_M}{M^i}, \underset{q_M}{M^j}) \to N(\underset{P_M}{M^i}, \underset{P_M}{M^j}))$
[28]	$D_i = \alpha_i + \lambda_1 q_i - \lambda_2(q_j - q_i)$ $i, j = 1, 2, i \neq j$	–	2 supply chains		q	$S(\underset{q_i}{SC^i} \to \underset{q_j}{SC^j})$ SC : supply chains
[29]	$D = \alpha + \log_b c q_M^{\lambda_m} q_S^{\lambda_s}$	1	1	–	q_M, q_S	$S(\underset{q_M}{M} \to \underset{q_S}{S})$
[31]	$D_i = \alpha + \lambda_S q_S - \beta p_M$	1	1	–	p_M, q	$S(\underset{p_M}{M} \to \underset{q}{S})$ $S(\underset{q}{S} \to \underset{p}{M})$ $C(\underset{q}{S}, \underset{p}{M})$
[30]	$D = \alpha - \beta(p_S + p_M + p_R)$ $+\lambda(q_S + q_M) + \lambda_R q_R$	1	1	1	q, p	$N(\underset{p_M,q_M}{M}, \underset{p_S,q_S}{S}, \underset{p_R,q_R}{R})$ $S(\underset{p_M,q_M}{M} \to N(\underset{p_S,q_S}{S}, \underset{p_R,q_R}{R}))$ $C(\underset{p_M,q_M}{M}, \underset{p_S,q_S}{S}, \underset{p_R,q_R}{R})$
Proposed model	$D =$ $(\alpha - \beta p_R + \chi\ln(q_M^{\lambda_M} q_S^{\lambda_S}))$ $\times(\kappa_M\sqrt{a_M} + \kappa_R\sqrt{a_R})$	1	1	1	p, a, t, q	$N(\underset{q_S}{S}, \underset{a_M,q_M,t}{M}, \underset{p'_R,a_R}{R})$ $S(N(\underset{q_S \cdot p_S}{S}, \underset{q_M}{M}) \to N(\underset{t,a_M,p_M}{M}, \underset{p_R,a_R}{R}))$ $S(N(\underset{q_S \cdot p_S}{S}, \underset{q_M}{M}) \to \underset{a_M,p_M,t}{M} \to \underset{p_R,a_R}{R})$ $S(\underset{q_M}{M} \to \underset{q_S \cdot p_S}{S} \to \underset{t,a_M,p_M}{M} \to \underset{p_R,a_R}{R})$ $S(\underset{q_M}{M} \to \underset{q_S \cdot p_S}{S} \to N(\underset{t,a_M,p_M}{M}, \underset{p_R,a_R}{R}))$

Notes. [(a),(b)] α: Represents the base demand; A: Represents the sales saturate asymptote; B: Represents the Advertising sensitivity; e: Represents the price-elasticity; k, μ: Represents the demand responsiveness to advertising λ: Represents the demand responsiveness to quality; β: Represents the demand responsiveness to price; ν: Represents the shape parameter; p: Price; a: Advertising expenditures; q: Quality, and other parameters and variables are defined in Section 3. [(c)] $S(\cdot \to \cdot)$: Represents the Stackelberg game; $C(\cdot, \cdot)$: Represents the cooperative game; $N(\cdot, \cdot)$: Represents The Nash game.
Ex1: Game $N(\underset{a_M,p_M,t}{M}, \underset{p_R,a_R}{R})$ implies that this is a Nash Game and shows the decisions that the manufacturer and the retailer make through their decision variables.
Ex2: Game $S(\underset{p_M,q_M}{M} \to N(\underset{p_S,q_S}{S}, \underset{p_R,q_R}{R}))$ implies that this a two stage Stakelberg Game. In the first stage the manufacture make decisions on q_M and p_M. Afterward in stage 2, the supplier and the retailer make decisions on q_s, p_s, p_R and q_R simultaneously in a Nash Game.

In this paper, demand is indicated in terms of a_M, a_R, q_M, q_S and, p_R as in shown equation (3.1).

$$D(p_R, a_R, a_M, q_M, q_S) = f(p_R, q_M, q_S)g(a_R, a_M) \tag{3.1}$$

The first part of the demand function, $f(p_R, q_M, q_S)$, gives the impact of quality and price while the second part, $g(a_R, a_M)$, represents the impact of advertising.

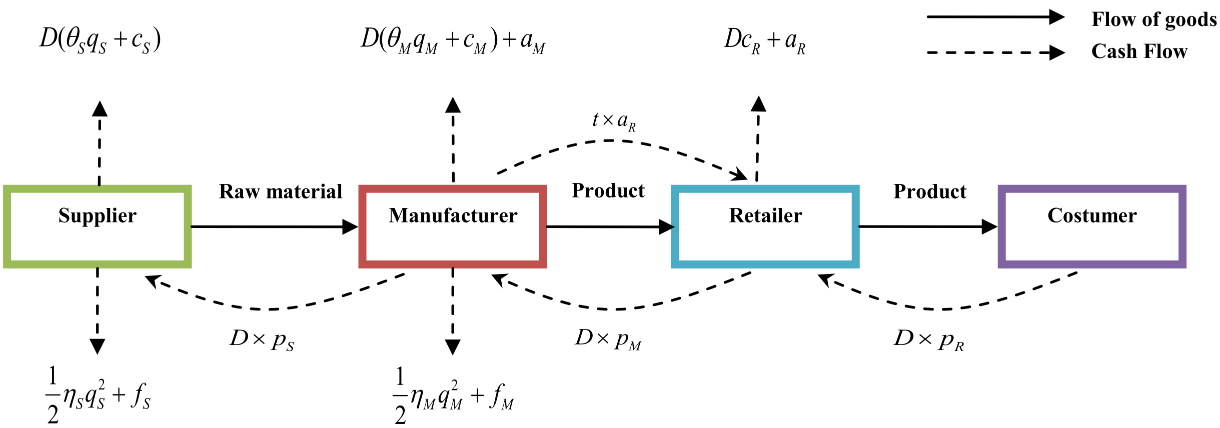

FIGURE 1. The understudy supply chain structure.

An overview of various functions that have been used to evaluate the impact of cooperative advertising is shown in Table 1. For additional information, refer to [4, 36]. In this article, the effect of advertising is considered as square root advertising function (*i.e.* $\kappa_M\sqrt{a_M} + \kappa_R\sqrt{a_R}$), which was first introduced by Xie and Wei [9] and later used by other researchers [8, 15]. The effect of quality is considered as a logarithmic function (*i.e.* $\chi\ln(q_M^{\lambda_M} q_S^{\lambda_S})$), used by Hong and Chen [29] in the form of $\log_b cq_M^{\alpha} q_S^{\beta}$ and the effect of price is considered as a liner classic demand function (*i.e.* $\alpha - \beta p_R$). The demand function used in this model is given in equation (3.2):

$$D(p_R, a_R, a_M, q_M, q_S) = (\alpha - \beta p_R + \chi\ln(q_M^{\lambda_M} q_S^{\lambda_S}))(\kappa_M\sqrt{a_M} + \kappa_R\sqrt{a_R}). \tag{3.2}$$

The advertising cost of the manufacturer and the retailer are a_m and a_r, respectively, and the quality costs for the manufacturer and the supplier are considered in two parts similar to [27]. The first part shows the investment costs for selecting or buying the desired equipment for improving quality level which is $f_M + 1/2\eta_M q_M^2$ and $f_S + 1/2\eta_S q_S^2$, respectively for the manufacturer and the supplier. This in turn consists of fixed and variable costs based on the selected quality level and it is incremental and convex in the quality level. The second part shows quality costs per unit which adds $\theta_S q_S$ and $\theta_M q_M$ to raw materials and production costs, respectively. Therefore, the profit functions of the manufacturer, supplier and retailer can be given as equations (3.3)–(3.5), respectively:

$$\Pi_M = (p_M - p_S - c_M - \theta_M q_M)\, D - \left(f_M + \frac{1}{2}\eta_M q_M^2\right) - (a_M + t a_R) \tag{3.3}$$

$$\Pi_S = (p_S - c_S - \theta_S q_S) D - (f_S + \frac{1}{2}\eta_S q_S^2) \tag{3.4}$$

$$\Pi_R = (p_R - p_M - c_R)\, D - (1 - t) a_R. \tag{3.5}$$

To simplify the model and reduce its complexity, we have adopted the variables used in [9] and [8] as indicated in equations (3.6a)–(3.6j):

$$\alpha' = \alpha - \beta(c_M + c_R + c_S) \tag{3.6a}$$

$$p'_R = \frac{\beta}{\alpha'}(p_R - (c_M + c_R + c_S)) \tag{3.6b}$$

$$p'_M = \frac{\beta}{\alpha'}(p_M - (c_M + c_S)) \tag{3.6c}$$

$$p'_S = \frac{\beta}{\alpha'}(p_S - c_S) \tag{3.6d}$$

$$\theta'_S = \theta_S \frac{\beta}{\alpha'} \tag{3.6e}$$

$$\kappa'_M = \frac{\alpha'^2}{\beta}\kappa_M \tag{3.6f}$$

$$\lambda'_S = \lambda_S \frac{\chi}{\alpha'} \tag{3.6g}$$

$$\theta'_M = \theta_M \frac{\beta}{\alpha'} \tag{3.6h}$$

$$\kappa'_R = \frac{\alpha'^2}{\beta}\kappa_R \tag{3.6i}$$

$$\lambda'_m = \lambda_M \frac{\chi}{\alpha'}. \tag{3.6j}$$

By substituting equations (3.6a)–(3.6j) into equations (3.2)–(3.5), the profit functions of the manufacturer, supplier and retailer can be given as in equations (3.7)–(3.10), respectively:

$$D(p_r, a_s, a_m, q_m, q_s) = \frac{\beta}{\alpha'}(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) \tag{3.7}$$

$$\begin{aligned}\Pi'_M = {} & (p'_M - p'_S - \theta'_M q_M)(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) \\ & - \left(f_M + \frac{1}{2}\eta_M q_M^2\right) - (a_M + t a_R)\end{aligned} \tag{3.8}$$

$$\Pi'_S = (p'_S - \theta'_S q_S)(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) - \left(f_S + \frac{1}{2}\eta_S q_S^2\right) \tag{3.9}$$

$$\Pi'_R = (p'_R - p'_M)(1 - p'_r + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) - (1-t)a_R. \tag{3.10}$$

Since demand must be non-negative, so we have equation (3.11).

$$D \geq 0 \Rightarrow p_R \leq \frac{\alpha + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S})}{\beta} \Rightarrow \alpha' p' \leq \alpha + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}) - \beta(c_M + c_R + c_S) \Rightarrow p'_R \leq 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}). \tag{3.11}$$

In order to avoid the non-negativity of profit for all members, the following conditions must be satisfied for the manufacturer, the supplier, and the retailer, respectively:

$$p'_M \geq p'_S + \theta'_M q_M \tag{3.12}$$

$$p'_S \geq \theta'_S q_S \tag{3.13}$$

$$p'_R \geq p'_M. \tag{3.14}$$

Equations (3.11)–(3.14) result in equation (3.15):

$$\theta'_S q_S + \theta'_M q_M \leq p'_S + \theta'_M q_M \leq p'_M \leq p'_R \leq 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}). \tag{3.15}$$

Thus, the manufacturer's decision problem can be stated by equations (3.16)–(3.18).

$$\begin{aligned}\text{Max}\quad \Pi'_M &= (p'_M - p'_S - \theta'_M q_M)(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) \\ &\quad - \left(f_M + \frac{1}{2}\eta_M q_M^2\right) - (a_M + t a_R) \end{aligned} \tag{3.16}$$

$$\text{s.t.}\quad \theta'_S q_S + \theta'_M q_M \le p'_S + \theta'_M q_M \le p'_M \le p'_R \le 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}), \tag{3.17}$$

$$0 \le t \le 1, \quad a_M \ge 0 \quad \text{and} \quad q_M \ge 0. \tag{3.18}$$

The retailer's decision problem can be stated by equations (3.19)–(3.21).

$$\text{Max}\quad \Pi'_R = (p'_R - p'_M)(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) - (1-t)a_R \tag{3.19}$$

$$\text{s.t.}\quad \theta'_S q_S + \theta'_M q_M \le p'_S + \theta'_M q_M \le p'_M \le p'_R \le 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}), \tag{3.20}$$

$$a_R \ge 0. \tag{3.21}$$

Finally, the supplier's decision problem can be stated by equations (3.22)–(3.24).

$$\text{Max}\quad \Pi'_S = (p'_s - \theta'_s q_s)(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) - (f_S + \frac{1}{2}\eta_S q_S^2) \tag{3.22}$$

$$\text{s.t.}\quad \theta'_S q_S + \theta'_M q_M \le p'_S + \theta'_M q_M \le p'_M \le p'_R \le 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}), \tag{3.23}$$

$$q_S \ge 0 \quad \text{and} \quad p'_S \ge \theta'_S q_S. \tag{3.24}$$

In the next section, we investigate several relationships between the channel members within the framework of the game theory.

4. Game models investigated in this article

Assuming that the power of the manufacturer is higher than or equal to the power of other players (*i.e.* the manufacturer is the leader of the Stackelberg games), five different relationships between the players are considered in the form of five non-cooperative games; equilibrium solutions for each game are extracted and the results are compared.

4.1. Game 1 (G1): $N(\underset{q_S}{S}, \underset{a_M,q_M,t}{M}, \underset{p'_R,a_R}{R})$

In the first non-cooperative game (G1), it is assumed that all members have identical decision power and try to maximize their own profits independently and simultaneously. In other words, this situation can be modelled as a Nash game (*i.e.* $N(\underset{q_S}{S}, \underset{a_M,q_M,t}{M}, \underset{p'_R,a_R}{R})$), and the solution is Nash equilibrium. Its pertinent model and optimal solution are shown Appendix A (Eqs. (A.1)–(A.11)).

4.2. Game 2 (G2): $S(N(\underset{q_S,p_S}{S}, \underset{q_M}{M}) \to N(\underset{t,a_M,p_M}{M}, \underset{p_R,a_R}{R}))$

In this game, the supplier, the manufacturer and the retailer play a two stage non-cooperative game. In the first stage, the manufacturer and the supplier make decisions on q_M, q_S and p'_S simultaneously in a nash game. Afterward in stage 2, the manufacturer and the retailer make decisions on a_M, a_R, p'_M, p_R and t simultaneously in a nash game. This game is solved using backward induction scheme, details of which are shown in Appendix B (Eqs. (B.1)–(B.14)).

Table 2. Summary of the optimal solutions of game models.

	p'_s	p'_m	p'_r	t	a_M	a_R
G1	$\frac{1}{4}(Z-\theta'_M q_M+3\theta'_S q_S)$	$\frac{1}{2}(Z+\theta'_S q_S+\theta'_M q_M)$	$\frac{1}{4}(3Z+\theta'_M q_M+\theta'_S q_S)$	0	$\left(\frac{\kappa'_M}{32}\right)^2 Y^4$	$\left(\frac{\kappa'_R}{32}\right)^2 Y^4$
G2	$\frac{1}{4}(Z-\theta'_M q_M+3\theta'_S q_S)$	$\frac{1}{2}(Z+\theta'_S q_S+\theta'_M q_M)$	$\frac{1}{4}(3Z+\theta'_M q_M+\theta'_S q_S)$	0	$\left(\frac{\kappa'_M}{32}\right)^2 Y^4$	$\left(\frac{\kappa'_R}{32}\right)^2 Y^4$
G3	$\frac{1}{4}(Z-\theta'_M q_M+3\theta'_S q_S)$	$Z-c\frac{3}{4}Y$	$Z-c\frac{3}{8}Y$	$\frac{4-5c}{4-3c}$	$\left(\frac{9c(c-1)}{64}\kappa'_M\right)^2 Y^4$	$\left(\frac{9c(4-3c)}{256}\kappa'_R\right)^2 Y^4$
G4	$\frac{1}{4}(Z-\theta'_M q_M+3\theta'_S q_S)$	$Z-c\frac{3}{4}Y$	$Z-c\frac{3}{8}Y$	$\frac{4-5c}{4-3c}$	$\left(\frac{9c(c-1)}{64}\kappa'_M\right)^2 Y^4$	$\left(\frac{9c(4-3c)}{256}\kappa'_R\right)^2 Y^4$
G5	$\frac{1}{4}(Z-\theta'_M q_M+3\theta'_S q_S)$	$\frac{1}{2}(Z+\theta'_S q_S+\theta'_M q_M)$	$\frac{1}{4}(3Z+\theta'_M q_M+\theta'_S q_S)$	0	$\left(\frac{\kappa'_M}{32}\right)^2 Y^4$	$\left(\frac{\kappa'_R}{32}\right)^2 Y^4$

4.3. Game 3 (G3): $S(N(\underset{q_S\cdot p_S}{S},\underset{q_M}{M})\rightarrow\underset{a_M,p_M,t}{M}\rightarrow\underset{p_R,a_R}{R})$

In this game model, it is assumed that the manufacturer and the supplier have identical decision powers but the manufacturer has higher decision power than the retailer. Therefore, the manufacturer and the supplier initially decide on q_M, q_S, p'_S in a Nash Game; then, the manufacturer decides on p'_M, a_M, t and finally, the retailer decides on p'_R, a_R. The game is played at three stages and solved using backward induction scheme, details of which are shown in Appendix C (Eqs. (C.1)–(C.17)).

4.4. Game 4 (G4): $S(\underset{q_M}{M}\rightarrow\underset{q_S\cdot p_S}{S}\rightarrow\underset{t,a_M,p_M}{M}\rightarrow\underset{p_R,a_R}{R})$

In this game model, it is assumed that the manufacturer has higher decision power than the supplier and the retailer. The manufacturer and the supplier first play Stackelberg with each other to determine q_M, q_S, p_S; then, the manufacturer and the retailer play Stackelberg to determine p_M, a_M, t, p_R, a_R. The game is played at four stages and solved using backward induction scheme. This game is similar to G3 except for the last stage, details of which are shown in Appendix D (Eqs. (D.1)–(D.10)).

4.5. Game 5 (G5): $S(\underset{q_M}{M}\rightarrow\underset{q_S\cdot p_S}{S}\rightarrow N(\underset{t,a_M,p_M}{M},\underset{p_R,a_R}{R}))$

In this game model, it is assumed that the manufacturer and the retailer have identical decision powers but the manufacturer has a higher decision power than the supplier. Therefore, the manufacturer and the supplier first play Stackelberg with each other to determine q_M, q_S, p'_S, then the manufacturer and the retailer play with each other a Nash game in order to determine p'_M, a_M, t, p'_R, a_R. The game is played at three stages and solved using backward induction scheme, details of which are shown in Appendix E (Eqs. (E.1)–(E.12)).

4.6. Summary of optimal solutions for the five games

The optimal solutions for the five games (G1–G5) are summarized in Table 2. It is to be noted that all the decision variables are functions of q_S and q_M, obtained from the equations shown in Table 3. Upon classifying the equilibrium solutions, the following results are obtained:

- Optimal solutions of G1, G2 and G5 are identical as well as those of G3 and G4.
- The participation rate in G1, G2 and G5 is equal to zero and identical in G3 and G4. The value depends on only the ration κ, which reflects the effectiveness of local advertising *vs.* national advertising and denoted by $\kappa=\kappa_R'^2/\kappa_M'^2$.

As can be judged from Table 3, obtaining closed-type solutions of q_S and q_M are quite complicated. Hence, our results are illustrated by means of a numerical example in the next section.

Table 3. The optimal solutions of q_S and q_M.

	h_1	h_2	q_S, q_M
G1	$\frac{1}{128}(\kappa'^2M + \kappa'^2R)$	$\frac{1}{128}(\kappa'^2M + \kappa'^2R)$	$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow h_1 Y^3 \left(\frac{\lambda'_M}{q_M} - \theta'_M\right) - \eta_M q_M = 0$ $\frac{\partial \Pi'_M}{\partial q_S} = 0 \Rightarrow h_2 Y^3 \left(\frac{\lambda'_S}{q_S} - \theta'_S\right) - \eta_S q_S = 0$
G2	$\frac{1}{96}\left(1 - \frac{\kappa'^2M}{2}\right)$	$\frac{1}{512}(\kappa'^2M + \kappa'^2R)$	
G3	$(2c(1-c)d - 4e)\left(\frac{3}{4}\right)^3$	$\frac{1}{2}cd\left(\frac{3}{4}\right)^3$	
G4	$(2(1-c)cd - 4e)\,(\frac{3}{4})^4$	$\frac{1}{2}\left(\frac{3}{4}\right)^3 dc$	$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow h_1 Y^3 \left(\frac{\lambda'_M}{q_M} - \theta'_M + (\frac{\lambda'_S}{q_S} - \theta'_S)\frac{\partial q_S}{\partial q_m}\right) - \eta_M q_M = 0$ $\frac{\partial \Pi'_M}{\partial q_S} = 0 \Rightarrow h_2 Y^3 \left(\frac{\lambda'_S}{q_S} - \theta'_S\right) - \eta_S q_S = 0$
G5	$(\frac{3}{4})^4\left(\frac{1}{162}(\kappa'^2M + \kappa'^2R) - \left(\frac{\kappa'_m}{18}\right)^2\right)$	$\frac{1}{48}(\frac{3}{4})^3(\kappa'^2M + \kappa'^2R)$	

where

$$\frac{\partial q_S}{\partial q_M} = \frac{3q_S\left(\frac{\lambda_M}{q_M} - \theta'_M\right)\left(\frac{\lambda'_S}{q_S} - \theta'_S\right)}{Y\left(2\frac{\lambda'_S}{q_S} - \theta'_S\right) - 3q_S\left(\frac{\lambda'_S}{q_S} - \theta'_S\right)^2}, \quad c = \frac{8(1+\kappa)}{12 + 9\kappa + \sqrt{16 + 16\kappa^2 + 9\times\kappa^4}}, \kappa = \frac{{\kappa_R}'^2}{{\kappa_M}'^2},$$

$$d = \left(\frac{1}{4}\kappa'^2_M\,|(c-1)c| + \frac{{\kappa_M}'^2c^2}{8(1-t)}\right)$$

$$e = \left(\frac{1}{16}{\kappa_M}'^2(c-1)^2c^2 + t\frac{\kappa_R c^4}{64(1-t)^2}\right) Y = 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}) - \theta'_S q_S - \theta'_M q_M Z = 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}).$$

Table 4. The value of the parameters.

α	β	c_S	c_R	c_M	λ_M	λ_S	χ	k_M	k_R	η_M	η_S	f_M	f_S	θ_M	θ_S
30000	100	15	5	8	0.3	0.5	1200	.0003	.0005	20	30	500	750	2	1

5. Numerical analysis

Due to the complexity of computing the closed-type solutions of q_S and q_M in G1–G5, this section presents a numerical example to illustrate the optimal solution of the five games presented in the previous section. The numerial analysis is based on some typical parameters of a traditional clothing industry which heavily relies on natural fabric.

Table 4 shows the typical parameters of the clothing industry consisting of one supplier which provides essential natural fabric, one manufacturer with which the first author of the manuscript was associated in making the end product and one retailer which markets the product. Fixed cost of this industry, is around three-folds for supplier (which includes overhead of machineries that are required for high quality natural fabric production), around two-folds for manufacturer (for cutting, sewing, etc.) and one-fold for the retailer which sells the product. Despite the higher fixed cost of the supplier, its variable cost is less than that of the manufacturer's (for design, embroidery, etc.). Hence, product quality becomes highly dependent on the quality of raw material as well as the quality of end product. Table 5 shows pertinent optimal results for the five games.

According to the results, it can be seen that the profits of all players have the highest values for G4 and the lowest values for G2. Furthermore, customers' demand has the highest value in G4. The raw material quality, q_S, and design quality, q_M, have their highest values in G4 and G2, respectively. Through a close examination of Table 5, one can easily note that except for the design quality, q_M, all other parameters have their highest values in G4, in which the manufacturer takes the leadership and plays the Stackelberg with the retailer and

Table 5. The optimal results of the five games.

	Structure	D	p_R($)	p_M($)	p_S($)	a_R	a_M	t	q_S	q_M	Π_M	Π_S	Π_R
G1	$N(S_{q_S}, M_{a_M,q_M,t}, R_{p_R,a_R})$	565.31	242.12	239.12	89.07	14399	5183.60	0	4.79	1.75	33451	38071	24766
G2	$S(N(S_{q_S\cdot p_S}, M_{q_M}) \rightarrow N(M_{t,a_M,p_M}, R_{p_R,a_R}))$	560.59	241.44	238.44	87.54	14239	5126.00	0	3.45	2.37	33048	37801	24491
G3	$S(N(S_{q_S\cdot p_S}, M_{q_M}) \rightarrow M_{t,a_M,p_M} \rightarrow R_{p_R,a_R})$	729.55	246.47	243.47	89.04	32135	5772.00	0.41	4.76	1.74	37377	49453	28471
G4	$S(M_{q_M} \rightarrow S_{q_S\cdot p_S} \rightarrow M_{t,a_M,p_M} \rightarrow R_{p_R,a_R})$	729.96	246.73	243.73	89.27	32159	5776.30	0.41	4.98	1.74	37405	49458	28492
G5	$S(M_{q_M} \rightarrow S_{q_S\cdot p_S} \rightarrow N(M_{t,a_M,p_M}, R_{p_R,a_R}))$	565.30	241.90	238.90	89.16	14398	5183.40	0	4.88	1.59	33455	38057	24765

Table 6. The optimal results of the five games in normalized percentage values.

%	Structure	D	p_R($)	p_M($)	p_S($)	a_R	a_M	t	q_S	q_M	Π_M	Π_S	Π_R
G1	$N(S_{q_S}, M_{a_M,q_M,t}, R_{p_R,a_R})$	77.44	98.13	98.11	99.78	44.77	89.74	0.00	96.18	73.84	89.43	76.98	86.92
G2	$S(N(S_{q_S\cdot p_S}, M_{q_M}) \rightarrow N(M_{t,a_M,p_M}, R_{p_R,a_R}))$	76.80	97.86	97.83	98.06	44.28	88.74	0.00	69.28	100.00	88.35	76.43	85.96
G3	$S(N(S_{q_S\cdot p_S}, M_{q_M}) \rightarrow M_{t,a_M,p_M} \rightarrow R_{p_R,a_R})$	99.94	99.89	99.89	99.74	99.93	99.93	100.00	95.58	73.42	99.93	99.99	99.93
G4	$S(M_{q_M} \rightarrow S_{q_S\cdot p_S} \rightarrow M_{t,a_M,p_M} \rightarrow R_{p_R,a_R})$	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	73.42	100.00	100.00	100.00
G5	$S(M_{q_M} \rightarrow S_{q_S\cdot p_S} \rightarrow N(M_{t,a_M,p_M}, R_{p_R,a_R}))$	77.44	98.04	98.02	99.88	44.77	89.74	0.00	97.99	67.09	89.44	76.95	86.92

the supplier. Table 6, which is transformation of Table 5 into normalized percentage form (*i.e.* percentage of each parameter value in each of the five games with respect to the respective parameter's maximum value of the five games), reveals this fact at one glance. G3 has the next highest parameter values, which indicates that when the manufacturer takes the leadership or has at least higher decision making power compared with the retailer, the results are far superior.

Based on the results shown in Tables 5 and 6, when the manufacturer plays Stackelberg game with other players and is the leader, its profit is higher than the situations where it plays Nash game, $\Pi_M^{\mathrm{G4}} > \Pi_M^{\mathrm{G3}}$ and $\Pi_M^{\mathrm{G5}} > \Pi_M^{\mathrm{G2}}$. This indicates that the manufacturer prefers to be the leader.

The results of some games are close to each other. For example, the results of games G3 and G4 are nearly the same, because stages 2 and 3 in G3 are the same as stages 3 and 4 in G4. Their only difference is that, while in G3 the manufacturer and the supplier have the same powers for determining the quality levels and deciding on q_M and q_S simultaneously (*i.e.* Nash Game), in G4 the manufacturer as the leader first determines q_M, and then the supplier determines q_S.

As illustrated in Table 2, the participation rate in G1, G2 and G5 is equal to zero and identical in G3 and G4 with its value depending on only the ration κ. This indicates that the manufacturer participates in the retailer's advertising cost (*i.e.* local advertising) only if he is the leader when dealing with the supplier to determine the quality level. Due to the participation of the manufacturer in local advertising cost, the retailer's advertising cost has the highest value in G3 and G4. The sensitivity analysis of the key parameters is performed in the next

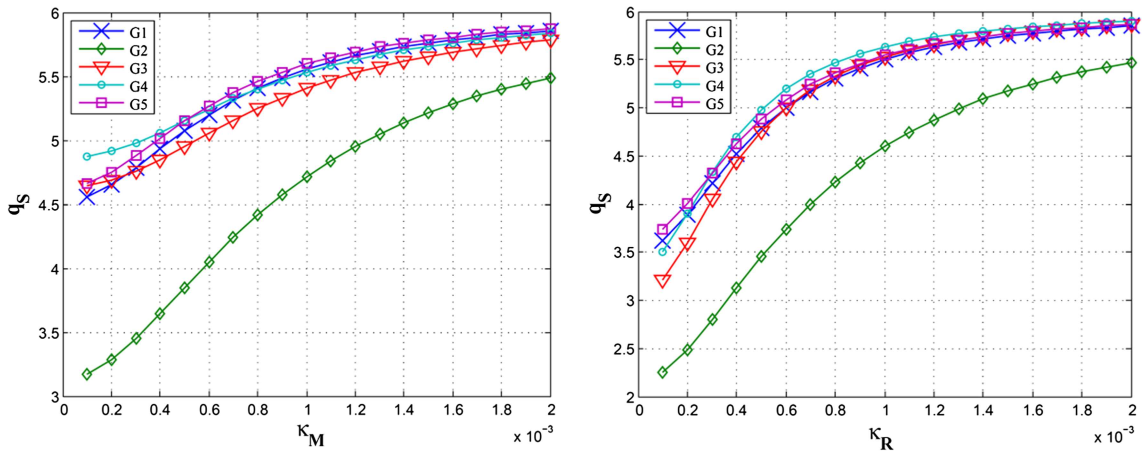

FIGURE 2. Variations in the quality level with respect to (*panel a*) κ_M, (*panel b*) κ_R.

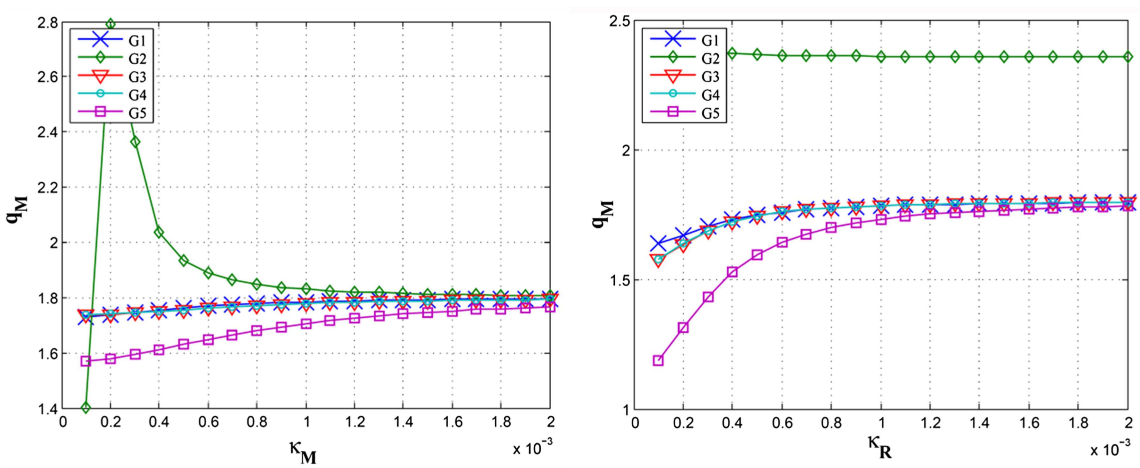

FIGURE 3. Variations in the quality level with respect to (*panel a*) κ'_M, (*panel b*) κ'_R.

section. Sensitivity analysis can help the players to determine which parameters are the key drivers and can be of significant importance in appropriate decision making.

6. Sensitivity analysis

This section studies the variations in the main decision variables as well as optimal profit of the supplier, the manufacturer and the retailer while varying the key parameters of the games (*i.e.* κ_M, κ_R, λ_S and λ_M).

6.1. Sensitivity analysis with respect to κ_M and κ_R

The sensitivity of κ_M and κ_R are demonstrated in Figures 2–7. It can be seen that with the increasing values of advertising-sensitive parameters, κ_M and κ_R, the following results are obtained.

– The optimal raw material quality level, q_S, increases in all game models and has the lowest value in G2.

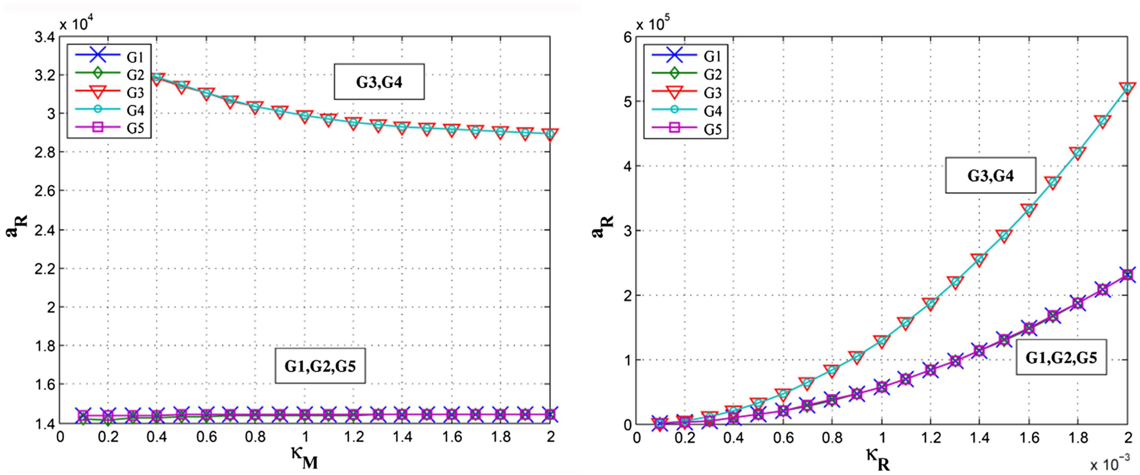

FIGURE 4. Variations in the advertising with respect to κ_M and κ_R.

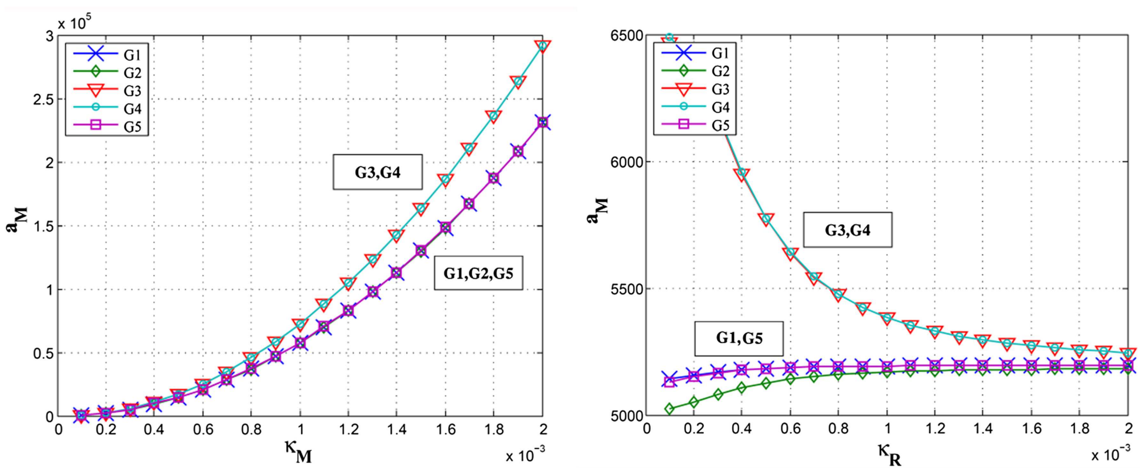

FIGURE 5. Variations in the advertising with respect to κ_M and κ_R.

– The optimal design quality level, q_M, increases in G1, G3, G4 and G5, but decreases in G2. q_M has the highest value in G2 and the lowest value in G5 (Fig. 3).
– The optimal advertising by the retailer, a_R, decreases in G3, G4 and there is no high sensitivity in G1, G2, G5 with respect to κ_M(overlapped left-hand side graphs) and increases in all games with respect to κ_R(overlapped right-hand side graphs) as can be depicted in Figure 4. The optimal advertising of the manufacturer, a_M, increases in all games with respect to κ_M(overlapped left-hand side graphs) and decreases in G3, G4 and increases in others (overlapped right-hand side graphs) with respect to κ_R as can be depicted in Figure 5. a_R and a_M have the highest values in G3 and G4 in all game models.

As discussed in the previous sections, the participation rate is equal to zero in G1, G2 and G5, and identical with its value depending on only the ration $\kappa = \kappa_R^2/\kappa_M^2$ in G3 and G4. Figures 6 and 7 illustrate the behaviour of the optimal participation rate, t. It is clear that with increasing value of κ, the value of t decreases. Based on this result, it is obvious that when the effectiveness of local advertising is higher than that of global advertising,

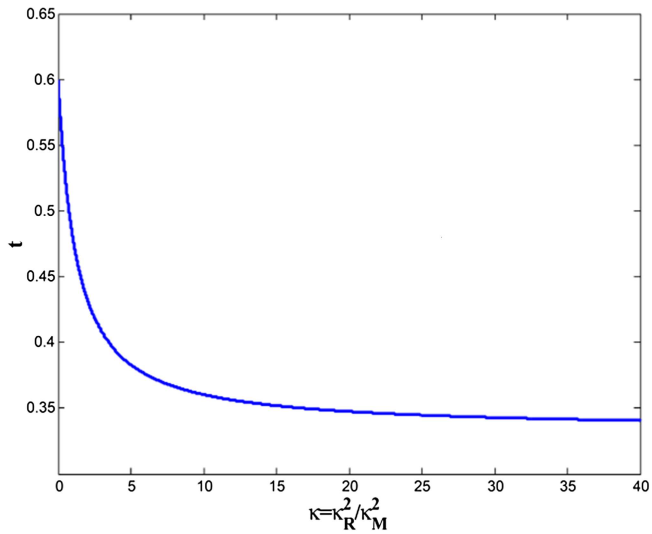

FIGURE 6. The variations of t w.r.t. $\kappa = \kappa_R^2$ / $\kappa = \kappa_M^2$.

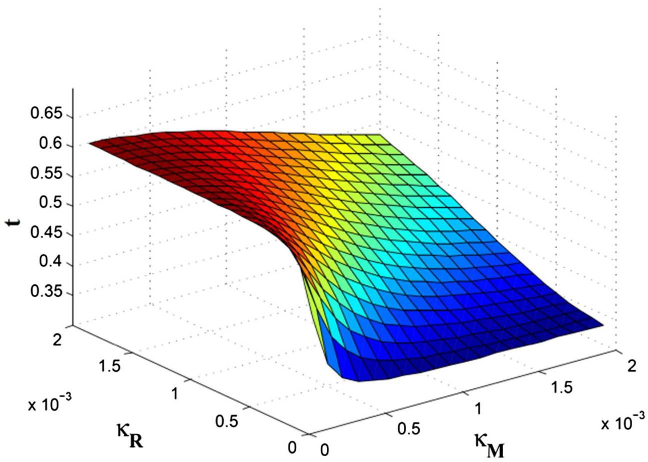

FIGURE 7. The variations of t w.r.t. κ_M and κ_R.

the manufacturer prefers to decrease the participation rate. This result is consistent with what was reported by SeyedEsfahani *et al.* [8].

The profit is the most important measurement factor for players in a supply chain when selecting their decision variables for profit maximization. As shown in Figures 8 and 9, the optimal profits of the retailer, the manufacturer and the supplier increase with respect to κ_M and κ_R for all game models.

Based on the graphical results of the five non-cooperative games of Figures 8 and 9 and the numerical example shown in the previous section, it can be said that there is no significant difference between the profit values in G3 and G4 and also among G1, G2, and G5. By increasing the value of λ_S, the profits of all the players in G2 are different from those in G1 and G5, while the result of G1 and G2 remain the same. This implies that the proposed model for demand function does not have a considerable impact on the optimal profit of the players in some games.

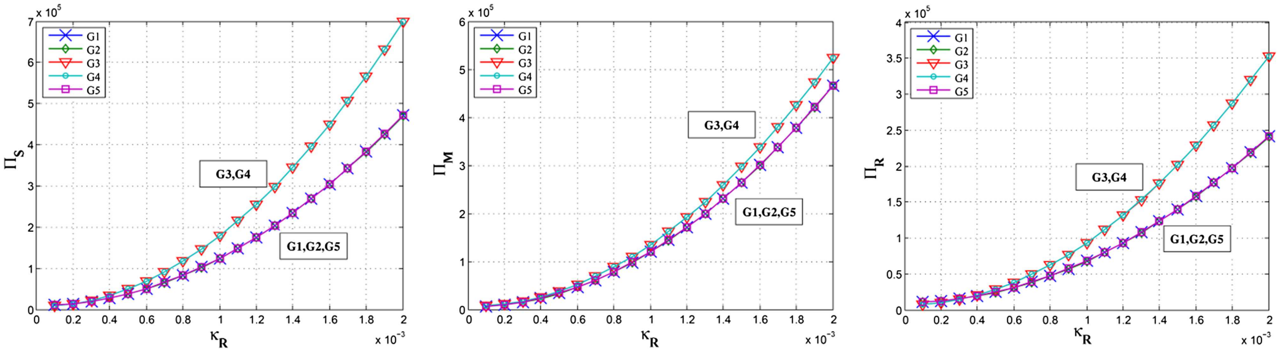

FIGURE 8. Variations in the profit of retailer, the manufacturer and supplier w.r.t. κ_R.

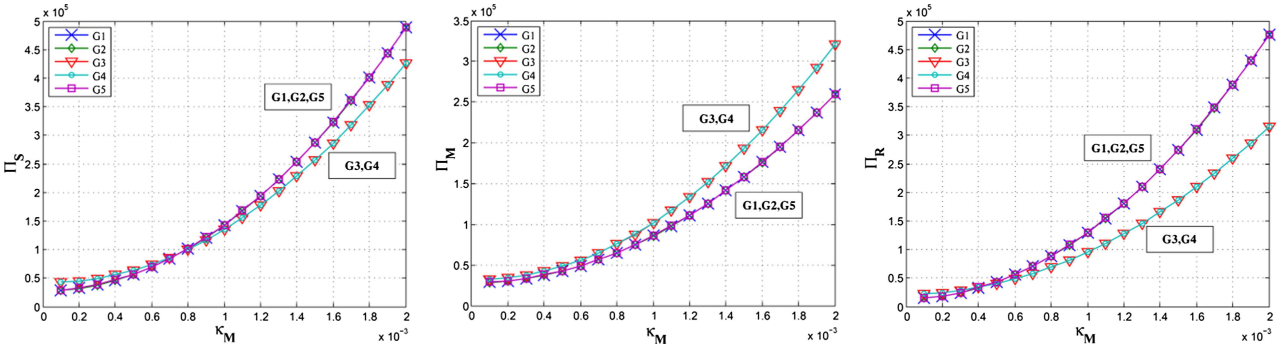

FIGURE 9. Variations in the profit of retailer, the manufacturer and supplier w.r.t. κ_M.

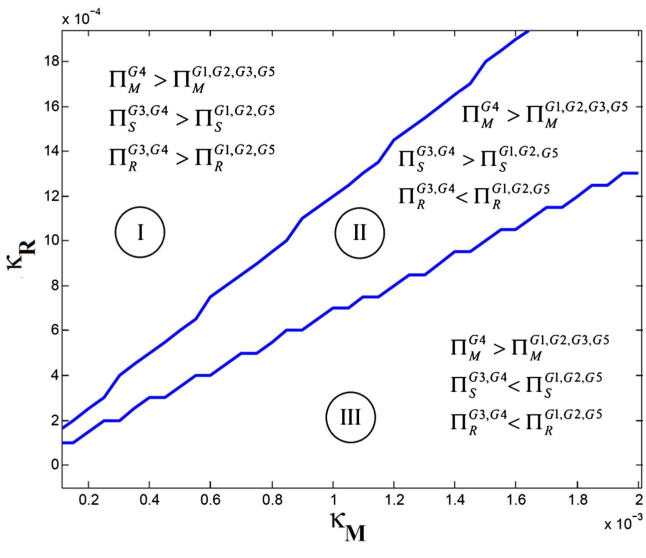

FIGURE 10. Optimal profit comparison of the players in three regions.

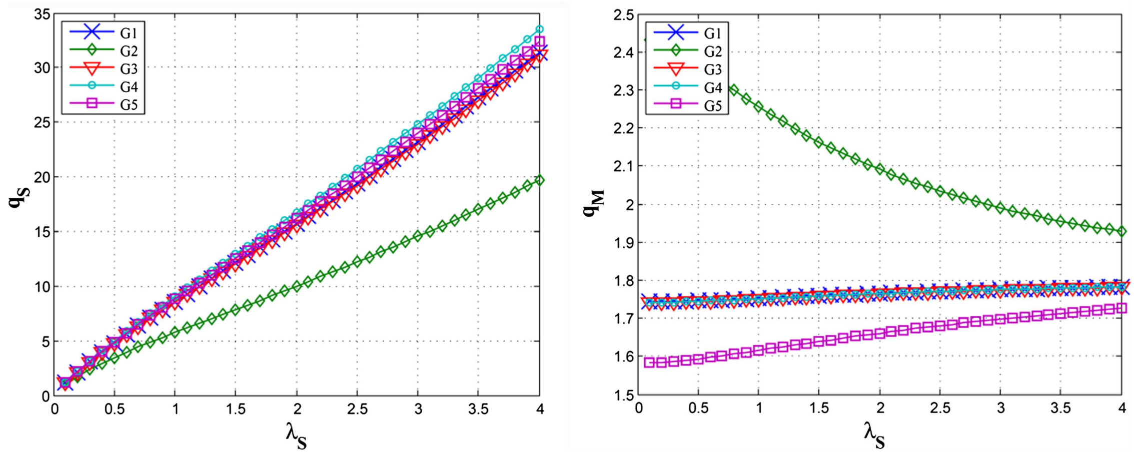

FIGURE 11. Variations in quality level for the manufacturer and supplier w.r.t. λ_S.

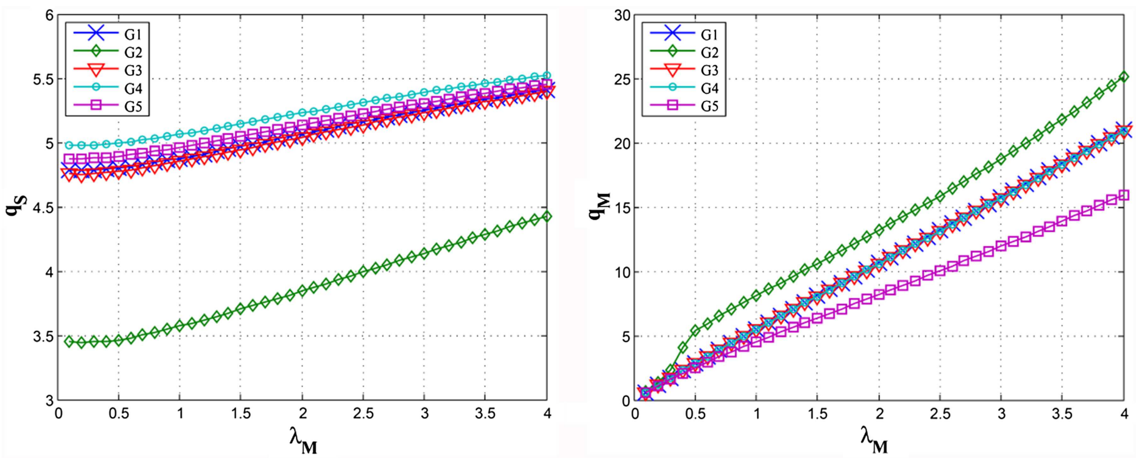

FIGURE 12. Variations in quality level for the manufacturer and supplier w.r.t. λ_M.

Figure 10 illustrates the optimal profit comparison of the players in the five game models with respect to κ_M and κ_R. It reveals that the parameters space is divided into three distinct regions. Note that the optimal profit of the manufacturer has the highest value in G4 in all regions, which means that it prefers to play the Stackelberg with the retailer and the supplier rather than to be in conflict with them in the Nash game. In region (I), both the supplier and retailer prefer that the manufacturer and retailer play Stackelberg, $\Pi_S^{\text{G3,G4}} > \Pi_S^{\text{G1,G2,G5}}$ and $\Pi_R^{\text{G3,G4}} > \Pi_R^{\text{G1,G2,G5}}$, whereas in region (III) both the supplier and the retailer prefer that the manufacturer and the retailer play Nash, $\Pi_S^{\text{G3,G4}} < \Pi_S^{\text{G1,G2,G5}}$ and $\Pi_R^{\text{G3,G4}} < \Pi_R^{\text{G1,G2,G5}}$. In region (II), the supplier prefers that the manufacturer and the retailer play Stackelberg, $\Pi_S^{\text{G3,G4}} > \Pi_S^{\text{G1,G2,G5}}$, but retailer prefers to play with the manufacturer in the Nash game, $\Pi_R^{\text{G3,G4}} < \Pi_R^{\text{G1,G2,G5}}$.

6.2. Sensitivity analysis with respect to λ_S and λ_M

The sensitivity of λ_S and λ_M are demonstrated in Figures 11–16. It can be seen that with increasing value of λ_S and λ_M, the following results are obtained.

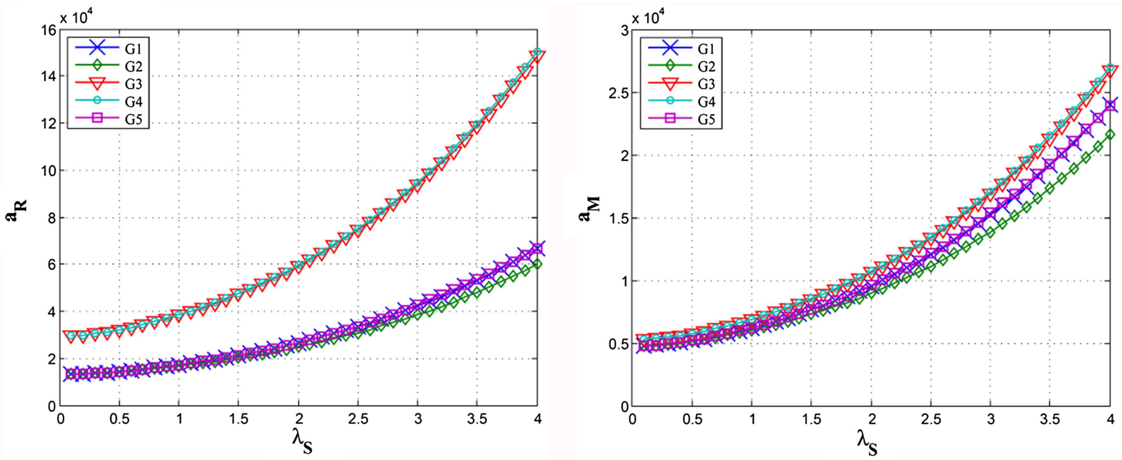

FIGURE 13. Variations in the manufacturer and retailer with respect to λ_S.

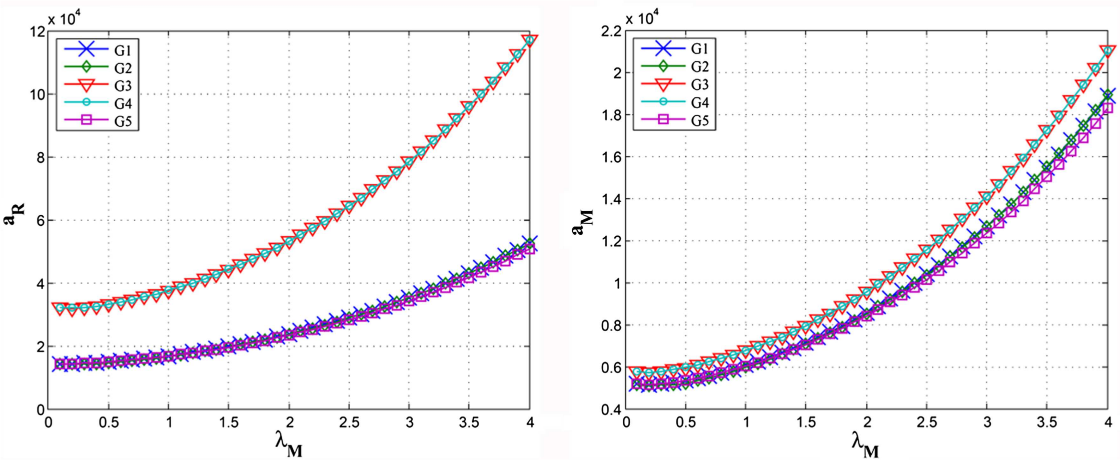

FIGURE 14. Variations in the manufacturer and retailer with respect to λ_M.

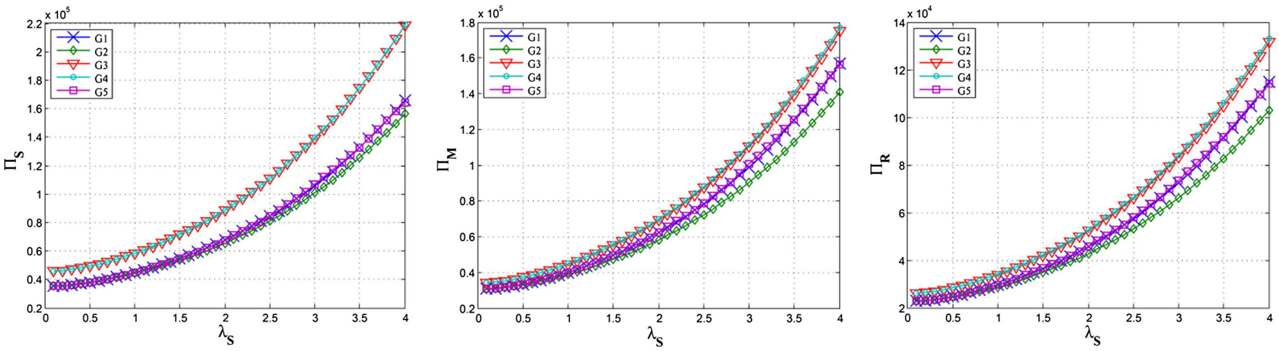

FIGURE 15. Variations in the profit of retailer, the manufacturer and supplier w.r.t. λ_S.

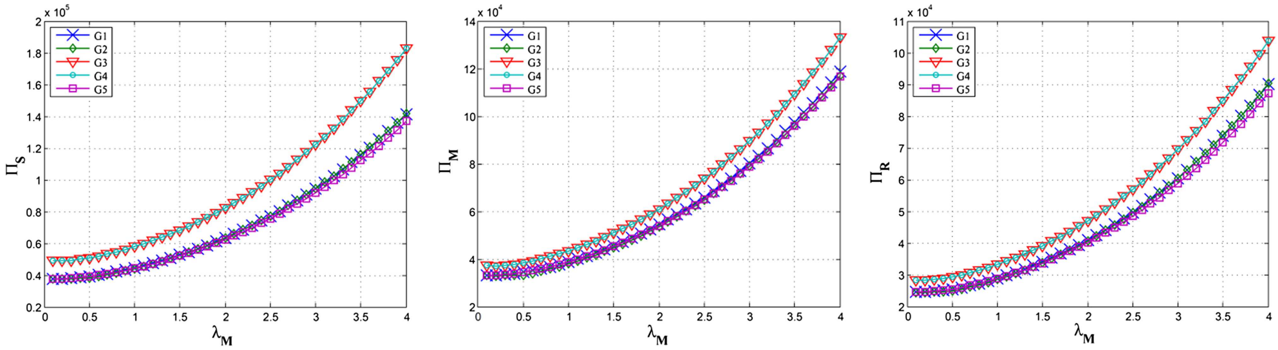

FIGURE 16. Variations in the profit of retailer, the manufacturer and supplier w.r.t. λ_M.

– As shown in Figures 11 and 12, the optimal raw material quality level, q_S, and the optimal design quality level, q_M, increase in all games while increasing the value of λ_M. If λ_S increases, q_S increases in all games and q_M also increases in all except for G2.
– By increasing λ_M and λ_S, q_S has the highest value in G4 and the lowest value in G2 and q_M has the highest value in G4 and the lowest value in G5.
– As shown in Figures 13 and 14, the optimal local advertising, a_R, and global advertising, a_M increase in all the game models.
– By increasing the value of λ_S and λ_M, there is no significant difference between the results of a_R in G3 and G4 and G1 and G5.
– It can be seen that by increasing the value of λ_S, a_R has the highest value in G3 and G4 and the lowest value in G2 and by increasing the value of λ_M, a_R has the highest value in G3 and G4 and the lowest value occurs in G5.
– The result of a_M is the same as a_R

Figures 15 and 16 illustrates the optimal profit comparison of the players in the five game models with respect to λ_S and λ_M respectively. As can be observed, the optimal profit increase in all of the game models.

7. Conclusions

Many non-pricing factors undoubtedly affect customers' choice among which the quality of product is one of the most significant factors. Decision making on quality level based on pricing and vertical co-advertising has been rarely investigated since the approach significantly increases the number of supply chain variables and computational complexity of the problem. This paper investigated such approach by considering the problem of pricing, vertical cooperative advertising and quality level as decision variables in a three-echelon supply chain consisting of one supplier, one manufacturer and one retailer. It assumed that the power of the manufacturer was higher than or equal to that of other players, thus making the manufacturer as the leader in Stackelberg games. Under this assumption, five non-cooperative games between the players were considered and the equilibrium solutions for each game were presented.

Considering the mathematical complexity of the obtained optimal solutions and its configuration for closed-type solutions of q_S and q_M, we also demonstrated the obtained optimal solutions via numerical examples. Our findings clearly showed that the scenario of G4, in which the manufacturer is the leader, determines quality level when dealing with the supplier and participates in the retailer's advertising cost, resulted in the best value across almost all the parameters. The next best value was scenario of G3, in which the manufacturer had a higher decision making power at least over the retailer. Through a detailed sensitivity analysis, which varied key parameters of the games and investigated variations in main decision variables and profits, we demonstrated the validity of our findings at a much wider scale. It proved the importance of product quality and the role of local advertisement in influencing demand sensitivity. Effective implementation of such approach will enable supply chain managers to become sensitive to product quality and positively influence market demand through local advertisement on top of the traditional approach of speed and efficiency optimization. Future studies along this theme may investigate impacts of other demand structures on the results.

Appendix A.

Pertinent model and optimal solution for Game 1 (G1): Nash Equilibrium Approach

The decision problem of the manufacturer can formulate by differentiating Π'_M, with respect to q_M, a_M, and t as shown in equations (A.1)–(A.3).

$$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow -\theta'_M(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) + \frac{\lambda'_M}{q_M}(p'_M - p'_S - \theta'_M q_M)(\kappa'_M\sqrt{a_M} + \kappa'_R\sqrt{a_R}) - \eta_M q_M = 0 \quad (A.1)$$

$$\frac{\partial \Pi'_M}{\partial a_M} = \frac{1}{2}\kappa'_M a_M^{-\frac{1}{2}}(p'_M - p'_S - \theta'_M q_M)(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S})) - 1 = 0 \quad (A.2)$$

$$\frac{\partial \Pi'_M}{\partial t} = 0. \quad (A.3)$$

The decision problem of the retailer can be formulate by differentiating Π_R with respect to p'_R, a_R as shown in equations (A.4)–(A.5).

$$\frac{\partial \Pi'_R}{\partial p'_R} = 0 \Rightarrow (1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S})) - (p'_R - p'_M) = 0 \quad (A.4)$$

$$\frac{\partial \Pi'_R}{\partial a_R} = 0 \Rightarrow \frac{1}{2}\kappa_R a_R^{-\frac{1}{2}}(p'_R - p'_M)(1 - p'_R + \chi' \ln(q_M^{\lambda_M} q_S^{\lambda_S})) - (1 - t) = 0. \quad (A.5)$$

Finally, the decision problem of the supplier can be formulate by differentiating equation (3.22) with respect to q_S and p'_S as shown in equations (A.6) and (A.7).

$$\frac{\partial \Pi'_S}{\partial q_S} = 0 \Rightarrow -\theta'_S(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))(\kappa'_m\sqrt{a_M} + \kappa'_R\sqrt{a_R}) + \frac{\lambda_S}{q_S}(p'_S - \theta'_S q_S)(\kappa'_m\sqrt{a_m} + \kappa'_R\sqrt{a_R}) - \eta_S q_S = 0 \quad (A.6)$$

$$\frac{\partial \Pi'_S}{\partial p_S} = 0 \Rightarrow p'_S - \theta'_S q_S - (p'_M - p'_S - \theta'_m q_m) = 0. \quad (A.7)$$

Since t has a negative effect on the manufacturer's objective function, it is obvious that its optimal value should be zero. Furthermore, Π_M increases linearly with p'_M, which means the optimal value of p'_M is p'_R, resulting in zero profit for the retailer. For solving this problem, it is assumed that the retailer will not sell the product if he does not have a minimum marginal profit; thus, the manufacturer minimum marginal profit is considered as the retailer margin [7, 9, 24]. The approach is also used in this paper. Hence, the manufacturer incurs constraint (A.8).

$$p'_R - p'_M \geq p'_M - p'_S - \theta'_M q_M \Rightarrow p'_M \leq \frac{1}{2}(p'_R + p'_S + \theta'_M q_M). \quad (A.8)$$

Therefore, the optimal solution for p'_M is $\frac{1}{2}(p'_R+p'_S+\theta'_M q_M)$. By simultaneously solving equations (A.1)–(A.8), the following unique equilibrium is obtained:

$$\begin{array}{c} t=0, \quad a_M=\left(\frac{\kappa'_m Y^2}{32}\right)^2, \quad a_R=\left(\frac{\kappa'_r Y^2}{32}\right)^2 \\ p'_R=\frac{1}{4}(3Z+\theta'_M q_M+\theta'_S q_S), \quad p'_M=\frac{1}{2}(Z+\theta'_M q_M+\theta'_S q_S), \quad p'_S=\frac{1}{4}(Z-\theta'_M q_M+3\theta'_S q_S), \end{array} \tag{A.9}$$

where q_s and q_m are obtained from equations (A.10) and (A.11), which are nonlinear equations with two unknown variables:

$$\frac{\partial \Pi'_M}{\partial q_M}=0 \Rightarrow \frac{1}{128}(\kappa'^2_M+\kappa'^2_R)Y^3\left(\frac{\lambda'_M}{q_M}-\theta'_M\right)-\eta_M q_M=0 \tag{A.10}$$

$$\frac{\partial \Pi'_S}{\partial q_S}=0 \Rightarrow \frac{1}{128}(\kappa'^2_M+\kappa'^2_R)Y^3\left(\frac{\lambda'_S}{q_S}-\theta'_S\right)-\eta_S q_S=0, \tag{A.11}$$

where $Z=1+\ln(q^{\lambda'_M} q_S^{\lambda'_S}), Y=1+\ln(q^{\lambda'_M} q^{\lambda'_S})-\theta'_S q_S-\theta'_M q_M$.

Appendix B.

Pertinent model and optimal solution for Game 2 (G2): Backward Induction Scheme

Stage 2: In order to determine the equilibrium solution, the manufacturer's and the retailer's decision problems are initially solved in order to find the best responses of a_M, a_R, p'_M, p'_R and t to any given values of q_M, q_S and p'_S. Therefore, the decision problem of the manufacturer can be formulate by differentiating Π'_M, with respect to p'_M, a_M, and t as indicated in equations (B.1)–(B.3).

$$\frac{\partial \Pi'_M}{\partial p'_M}=0 \tag{B.1}$$

$$\frac{\partial \Pi'_M}{\partial a_M}=\frac{1}{2}\kappa'_M a_M^{-\frac{1}{2}}(p'_M-p'_S-\theta'_M q_M)(1-p'_R+\ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))-1=0 \tag{B.2}$$

$$\frac{\partial \Pi'_M}{\partial t}=0. \tag{B.3}$$

The decision problem of the retailer can be formulated by differentiating Π_R with respect to p'_R and a_R as indicated in equations (B.4) and (B.5).

$$\frac{\partial \Pi'_R}{\partial p'_R}=0 \Rightarrow (1-p'_R+\ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))-(p'_R-p'_M)=0 \tag{B.4}$$

$$\frac{\partial \Pi'_R}{\partial a_R}=0 \Rightarrow \frac{1}{2}\kappa_R a_R^{-\frac{1}{2}}(p'_R-p'_M)(1-p'_R+\ln(q_M^{\lambda'_M} q_S^{\lambda'_S}))-(1-t)=0. \tag{B.5}$$

Solving equations (B.1)–(B.5) is similar to G1, t is zero and $p'_M=\frac{1}{2}(p'_R+p'_S+\theta'_M q_M)$, the following unique solution is obtained for this stage:

$$\begin{array}{c} t=0, \quad p'_R=\frac{1}{3}(2Z+p'_S+\theta'_M q_M), \quad p'_M=\frac{1}{3}(Z+2p'_s+2\theta'_M q_M) \\ a_M=\left(\frac{\kappa'_m(Z-p'_S-\theta'_M q_M)^2}{18}\right)^2, \quad a_R=\left(\frac{\kappa'_r(Z-p'_S-\theta'_M q_M)^2}{18}\right)^2, \end{array} \tag{B.6}$$

where $Z=1+\ln(q_M^{\lambda'_M} q_S^{\lambda'_S})$.

Stage 1: By substituting the solution of stage 2 in the profit functions of the manufacturer and the retailer (*i.e.* Eqs. (3.16) and (3.19)), equations (B.7) and (B.8) can be derived.

$$\text{Max} \quad \Pi'_M = \frac{1}{162}(\kappa'^2{}_M + \kappa'^2{}_R)(Z - p'_S - \theta'_M q_M)^4 - f_M - \frac{1}{2}\eta_M q_M^2 - \frac{1}{2}h\eta_S q_S^2 - \left(\frac{\kappa'_m(Z - p'_S - \theta'_M q_M)^2}{18}\right)^2 \tag{B.7}$$

$$\text{Max} \quad \Pi'_S = \frac{1}{54}(\kappa'^2{}_M + \kappa'^2{}_R)(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^3 - \frac{1}{2}\eta_S q_S^2 - fs. \tag{B.8}$$

Therefore, the decision problem of the manufacturer can be formulated by differentiating Π_M, with respect to q_M as indicated in equation (B.9).

$$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow \frac{4}{9\times 18}(\frac{\lambda_M}{q_M} - \theta'_M)(Z - p'_S - \theta'_M q_M)^3 - \eta_M q_M - 4\left(\frac{\kappa'_M}{18}\right)^2(\frac{\lambda_M}{q_M} - \theta'_M)(Z - p'_S - \theta'_M q_M)^3 = 0. \tag{B.9}$$

The decision problem of the supplier can be formulated by differentiating Π_S with respect to q_S, p'_S and as indicated in equations (B.10) and (B.11).

$$\frac{\partial \Pi_s}{\partial p'_S} = 0 \Rightarrow \frac{1}{3}(Z - p'_S - \theta'_M q_M) - (p'_S - \theta'_S q_S) = 0 \tag{B.10}$$

$$\frac{\partial \Pi'_S}{\partial q_S} = -\frac{1}{3\times 18}\theta'_S \kappa(Z - p'_S - \theta'_M q_M)^3 + \frac{1}{18}\frac{\lambda'_S}{q_S}\kappa(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^2 - \eta_S q_S = 0. \tag{B.11}$$

By simultaneously solving equations (B.9)–(B.11), the unique equilibrium indicated by equation (B.12) can be obtained.

$$\begin{array}{l} p'_S = \frac{1}{4}\left(Z - \theta'_M q_M + 3\theta'_S q_S\right), \quad p'_R = \frac{1}{4}(3Z + \theta'_M q_M + \theta'_S q_S), \quad p'_M = \frac{1}{2}(Z + \theta'_M q_M + \theta'_S q_S) \\ a_M = \left(\frac{\kappa'_M Y^2}{32}\right)^2, \quad a_R = \left(\frac{\kappa'_R Y^2}{32}\right)^2. \end{array} \tag{B.12}$$

Therefore, q_S and q_M are obtained from equations (B.13) and (B.14), which are nonlinear equations with two unknown variables:

$$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow \frac{1}{96}\left(1 - \frac{\kappa'^2{}_M}{2}\right)Y^3(\frac{\lambda'_M}{q_M} - \theta'_M) - \eta_M q_M = 0 \tag{B.13}$$

$$\frac{\partial \Pi'_S}{\partial q_S} = 0 \Rightarrow \frac{1}{512}(\kappa'^2{}_M + \kappa'^2{}_R)Y^3\left(\frac{\lambda'_S}{q_S} - \theta'_S\right) - \eta_S q_S = 0, \tag{B.14}$$

where $Y = 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}) - \theta'_S q_S - \theta'_M q_M$.

Appendix C.

Pertinent model and optimal solution for Game 3 (G3): Backward Induction Scheme

Stage 3: To determine the equilibrium solution, first the retailer's decision problem is solved in order to find the best responses of p'_R and a_R to any given values of a_M, p'_M, t, q_M, q_S and p_S. Therefore, the decision problem of the retailer can be formulated by differentiating Π'_R with respect to p'_R and a_R as indicated in equations (C.1) and (C.2).

$$\frac{\partial \Pi'_R}{\partial p'_R} = 0 \Rightarrow (1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S})) - (p'_R - p'_M) = 0 \tag{C.1}$$

$$\frac{\partial \Pi'_R}{\partial a_R} = 0 \Rightarrow \frac{1}{2}\kappa'_R a_R^{-\frac{1}{2}}(p'_R - p'_M)(1 - p'_R + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S})) - (1 - t) = 0. \tag{C.2}$$

Equations (C.1) and (C.2) lead to the unique equilibrium for this stage as indicated in equation (C.3).

$$p'_R = \frac{1}{2}(Z + p'_M), \quad a_R = \frac{\kappa'^2{}_R(Z - p'_M)^4}{64(1-t)^2}. \tag{C.3}$$

Stage 2: Substituting the solution of stage 3 in the profit function of the manufacturer (*i.e.* Eq. (3.16)), equations (C.4)–(C.5) can be obtained.

$$\text{Max} \quad \Pi'_M = (p'_M - p'_S - \theta'_M q_M)\frac{1}{2}(Z - p'_m)\left(\kappa'_M\sqrt{a_M} + \kappa'_R\frac{(Z - p'_M)^2}{8(1-t)}\right) - \frac{1}{2}(2f_M + \eta_M q_M^2)$$
$$- \left(a_M + t\frac{\kappa'^2{}_R(Z - p'_M)^4}{64(1-t)^2}\right) \tag{C.4}$$
$$\text{s.t.} \quad \theta'_S q_S + \theta'_M q_M \le p'_S + \theta'_M q_M \le p'_M \le p'_R \le 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}), \quad a_M \ge 0 \quad \text{and} \quad q_M \ge 0. \tag{C.5}$$

Therefore, the decision problem of the manufacturer can be formulate by differentiating Π'_M, with respect to p'_M, a_M, and t as indicated in equations (C.6)–(C.8).

$$\frac{\partial \Pi'_M}{\partial a_M} = 0 \Rightarrow \frac{1}{4}\kappa'_M a_M^{-\frac{1}{2}}(p'_M - p'_S - \theta'_M q_M)(Z - p'_M) - 1 = 0 \tag{C.6}$$
$$\frac{\partial \Pi'_M}{\partial t} = 0 \Rightarrow \frac{(p'_M - p'_S - \theta'_M q_M)}{16(1-t)^2} - \frac{(Z - p'_M)}{64(1-t)^2} - 2t\frac{(Z - p'_M)}{64(1-t)^3} = 0 \tag{C.7}$$
$$\frac{\partial \Pi'_M}{\partial p'_M} = 0 \Rightarrow \frac{1}{2}(Z - p'_M)\left(\kappa'_M\sqrt{a_M} + \kappa'^2{}_R\frac{(Z - p'_M)^2}{8(1-t)}\right)$$
$$- \frac{1}{2}(p'_M - p'_S - \theta'_M q_M)\left(\kappa'_M\sqrt{a_M} + \kappa'^2{}_R\frac{(Z - p'_M)^2}{8(1-t)}\right)$$
$$- \kappa'^2{}_R\frac{(p'_M - p'_S - \theta'_M q_M)(Z - p'_M)^2}{8(1-t)} + t\frac{\kappa'^2{}_R(Z - p'_M)^3}{16(1-t)^2} = 0. \tag{C.8}$$

Equations (C.6)–(C.8) lead to the unique equilibrium for this stage as given by equation (C.9).

$$a_M = \frac{1}{16}\kappa'^2{}_M(c-1)^2c^2(Z - p'_S - \theta'_M q_M)^4, \quad t = \frac{4-5c}{4-3c}, \quad p'_M = Z - c(Z - p'_S - \theta'_M q_M), Z, \tag{C.9}$$

where $c = \frac{8(1+\kappa)}{12+9\kappa+\sqrt{16+16\kappa^2+9\times\kappa^4}}$, $\kappa = \frac{\kappa'^2{}_R}{\kappa'^2{}_M}$ and $Y = 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}) - \theta'_S q_S - \theta'_M q_M$. The solution $p'_M = Z$ is not acceptable and only $p'_M = Z - c(Z - p'_S - \theta'_M q_M)$ is acceptable.

Stage 1: Substituting the solution of stages 2 and 3 in the profit function of the manufacturer and the supplier, equations (3.22) and (C.4) are converted into equations (C.10) and (C.11).

$$\Pi'_M = \left(\frac{1}{2}c(1-c)d - e\right)(Z - p'_S - \theta'_M q_M)^4 - (f_M + \frac{1}{2}\eta_M q_M^2) \tag{C.10}$$
$$\Pi'_S = \frac{1}{2}c(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^3 d - (fs + \frac{1}{2}\eta_S q_S^2), \tag{C.11}$$

where $c = \frac{8(1+\kappa)}{12+9\kappa+\sqrt{16+16\kappa^2+9\times\kappa^4}}$, $d = \kappa'^2_M\frac{|c(1-c)|}{4} + \kappa'^2{}_R\frac{c^2}{8(1-t)}$ and $e = \frac{\kappa'^2{}_M c^2(1-c)^2}{16} + t\frac{\kappa'^2{}_R c^4}{64(1-t)^2}$.

By differentiating Π'_M with respect to q_M and Π'_S with respect to q_S, p'_S, equations (C.12)–(C.14) can be obtained:

$$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow (2c(1-c)d - 4e)\,(Z - p'_S - \theta'_M q_M)^3 (\frac{\lambda'_M}{q_M} - \theta'_M) - \eta_M q_M = 0 \tag{C.12}$$

$$\frac{\partial \Pi'_S}{\partial q_S} = 0 \Rightarrow -\frac{1}{2}\theta'_S cd(Z - p'_S - \theta'_M q_M)^3 + \frac{3}{2}cd\frac{\lambda'_S}{q_S}(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^2 - \eta_S q_S = 0 \tag{C.13}$$

$$\frac{\partial \Pi'_S}{\partial p'_S} = 0 \Rightarrow \frac{1}{2}c(Z - p'_S - \theta'_M q_M) - \frac{3}{2}c(p'_S - \theta'_S q_S) = 0. \tag{C.14}$$

By simultaneously solving equations (C.12)–(C.14), the unique equilibrium given in equation (C.15) is obtained for this game model:

$$\begin{aligned} t &= \frac{4-5c}{4-3c} \\ p'_M &= Z - c\frac{3}{4}Y, p'_S = \frac{3\theta'_S q_S - \theta'_M q_M + Z}{4}, p'_R = Z - c\frac{3}{8}Y \\ a_M &= \frac{1}{16}\kappa'^2{}_M (c-1)^2 c^2 \left(\frac{3}{4}\right)^4 Y^4, \quad a_R = \frac{1}{256}\kappa'^2{}_R (4-3c)^2 c^2 \left(\frac{3}{4}\right)^4 Y^4, \end{aligned} \tag{C.15}$$

where $Z = 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S})$ and $Y = 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}) - \theta'_S q_S - \theta'_M q_M$; q_s and q_M are obtained from equations (C.16) and (C.17), which are nonlinear equations with two unknown variables:

$$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow (2c(1-c)d - 4e)\left(\frac{3}{4}\right)^3 Y^3 (\frac{\lambda'_M}{q_M} - \theta'_M) - \eta_M q_M = 0 \tag{C.16}$$

$$\frac{\partial \Pi'_S}{\partial q_S} = 0 \Rightarrow \frac{1}{2}cd\left(\frac{3}{4}\right)^3 Y^3 (\frac{\lambda'_S}{q'_S} - \theta'_S) - \eta_S q_S = 0. \tag{C.17}$$

Appendix D.

Pertinent model and optimal solution for Game 4 (G4): Backward Induction Scheme

Stages 4 and 3: Stages 4 and 3 are the same as stages 3 and 2 of G3, respectively; therefore, at the end of Stage 3, we have the same solution as indicated in equations (C.3) and (C.9).

Stage 2: Substituting the solutions of stages3 and 4 in the profit function of the supplier, Π_S can be obtained from equations (D.1) and (D.2).

$$\text{Max} \quad \Pi'_S = \frac{1}{2}(p'_S - \theta'_S q_S)c(Z - p'_S - \theta'_M q_M)^3 d - \frac{1}{2}\eta_S q_S^2 - fs \tag{D.1}$$

$$\text{s.t.} \quad \theta'_S q_S + \theta'_M q_M \le p'_S + \theta'_M q_M \le p'_M \le p'_R \le 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}), \quad q_S \ge 0 \quad \text{and} \quad p'_S \ge \theta'_S q_S, \tag{D.2}$$

where $d = \left(\frac{1}{4}\kappa'^2{}_M \left|(c-1)c\right| + \frac{\kappa'^2{}_R c^2}{8(1-t)}\right)$.

Therefore the decision problem of the supplier can be formulated by differentiating Π'_S with respect to q_S and p'_S as indicated in equations (D.3) and (D.4),

$$\frac{\partial \Pi'_S}{\partial p'_S} = \frac{1}{2}cd(Z - p'_S - \theta'_M q_M)^3 - \frac{3}{2}cd(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^2 = 0 \tag{D.3}$$

$$\frac{\partial \Pi'_S}{\partial q_S} = -\frac{1}{2}\theta'_S cd(Z - p'_S - \theta'_M q_M)^3 + \frac{3}{2}cd\frac{\lambda'_S}{q_S}(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^2 - \eta_S q_S = 0. \tag{D.4}$$

Which lead to the following unique equilibrium for this stage as indicated in equation (D.5):

$$p'_S = \frac{1}{4}(3\theta'_S q_S - \theta'_M q_M + Z)$$
$$\frac{\partial \Pi'_S}{\partial q_S} = 0 \Rightarrow \left(\frac{3}{4}\right)^3 Y^3 dc \left(\frac{\lambda'_S}{q_S} - \theta'_S\right) - 2\eta_S q_S = 0. \tag{D.5}$$

Furthermore, since q_S is a function of q_M, $q_S = f(q_M)$, differentiating q_S with respect to q_M leads to:

$$\begin{aligned} &\tfrac{\partial}{q_M}\left(\tfrac{\partial \Pi'_S}{\partial q_S}\right) = 0 \Rightarrow \tfrac{\partial}{q_M}\left(\left(\tfrac{3}{4}\right)^3 Y^3 dc \left(\tfrac{\lambda'_S}{q_S} - \theta'_S\right) - 2\eta_S q_S\right) = 0 \\ &\Rightarrow 3\left(\tfrac{3}{4}\right)^3 dc \left(\tfrac{\lambda'_M}{q_M} - \theta'_M + \left(\tfrac{\lambda'_S}{q_S} - \theta'_S\right)\tfrac{\partial q_S}{\partial q_M}\right)\left(\tfrac{\lambda'_S}{q_S} - \theta'_S\right) Y^2 - \left(\tfrac{3}{4}\right)^3 Y^3 dc \left(\tfrac{\lambda_s}{q_S^2}\right)\tfrac{\partial q_S}{\partial q_M} - 2\eta_S \tfrac{\partial q_S}{\partial q_M}. \end{aligned} \tag{D.6}$$

After simplification of the mathematical statements, equation (D.7) gives the final result at stage 2.

$$\frac{\partial q_S}{\partial q_m} = \frac{3q_S\left(\frac{\lambda'_M}{q_M} - \theta'_M\right)\left(\frac{\lambda'_S}{q_S} - \theta'_S\right)}{Y\left(-\theta'_S + 2\frac{\lambda'_S}{q_S}\right) - 3q_S\left(\frac{\lambda'_S}{q_S} - \theta'_S\right)^2}. \tag{D.7}$$

Stage 1: Substituting the solution of stages 2, 3 and 4 in the profit function of the manufacture, Π_M can be rewritten as the model given in (D.8):

$$\begin{aligned} &\text{Max} \quad \Pi_M = \left(\tfrac{1}{2}(1-c)cd - e\right)\left(\tfrac{3}{4}\right)^4 Y^4 - \tfrac{1}{2}(2f_M + \eta_M q_M^2) \\ &\text{s.t.} \quad \theta'_S q_S + \theta'_M q_M \le 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}), \quad 0 \le h \le 1, \end{aligned} \tag{D.8}$$

where $e = (\frac{1}{16}\kappa'^2{}_M (c-1)^2 c^2 + t\frac{\kappa'^2{}_R c^4}{64(1-t)^2})$ and $e = \left(\frac{1}{16}\kappa'^2{}_M (c-1)^2 c^2 + t\frac{\kappa'^2{}_R c^4}{64(1-t)^2}\right)$.

Differentiating Π_M with respect to q_M, equation (D.9) is obtained:

$$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow (2(1-c)cd - 4e)\left(\frac{3}{4}\right)^4\left(\frac{\lambda'_M}{q_M} - \theta'_M + \left(\frac{\lambda'_S}{q_S} - \theta'_S\right)\frac{\partial q_S}{\partial q_m}\right) Y^3 - \eta_M q_M = 0. \tag{D.9}$$

Finally, the optimal solutions for this game model can be obtained through equation (D.10).

$$\begin{aligned} &p'_S = \tfrac{3\theta'_S q_S - \theta'_M q_M + Z}{4}, \quad p'_M = Z - \tfrac{3c}{4}Y, \quad p'_R = Z - \tfrac{3c}{8}Y \\ &a_R = \tfrac{\kappa'^2{}_R c^4}{64(1-t)^2}\left(\tfrac{3Y}{4}\right)^4, \quad a_M = \tfrac{1}{16}\kappa'^2{}_M (c-1)^2 c^2 \left(\tfrac{3Y}{4}\right)^4, \\ &t = \tfrac{4-5c}{4-3c}. \end{aligned} \tag{D.10}$$

Substituting equation (D.7) into equation (D.9) and simultaneously solving it together with equation (D.6), q_S and q_M are obtained.

Appendix E.

Pertinent model and optimal solution for Game 5 (G5): Backward Induction Scheme

Stage 3: This stage is the same as stage 2 of G2; therefore, the unique solution as indicated in equation (E.1) is obtained for this stage in a similar manner to that of equation (B.6) in G2:

$$\begin{aligned} &t = 0, \quad p'_R = \tfrac{1}{3}(2Z + p'_S + \theta'_M q_M), \quad p'_M = \tfrac{1}{3}(Z + 2p'_S + 2\theta'_M q_M) \\ &a_M = \left(\tfrac{\kappa'_M (Z - p'_S - \theta'_M q_M)^2}{18}\right)^2, \quad a_R = \left(\tfrac{\kappa'_R (Z - p'_S - \theta'_M q_M)^2}{18}\right)^2, \end{aligned} \tag{E.1}$$

where $Z = 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S})$.

Stage 2: Substituting the solution of stage 2 in the profit functions of the supplier, Π_S (*i.e.* Eq. (3.22)), equations (E.2) and (E.3) can be obtained.

$$\text{Max} \quad \Pi'_S = \frac{1}{3\times 18}\kappa(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^3 - \frac{1}{2}\eta_S q_S^2 \tag{E.2}$$

$$\text{s.t.} \quad \theta'_S q_S + \theta'_M q_M \le p'_S + \theta'_M q_M \le p'_M \le p'_R \le 1 + \ln(q_M^{\lambda'_M} q_S^{\lambda'_S}), \quad q_S > 0. \tag{E.3}$$

Differentiating Π'_S with respect to q_S, p'_s, equations (E.4) and (E.5) are obtained.

$$\frac{\partial \Pi'_S}{\partial p_S} = 0 \Rightarrow \frac{1}{3\times 18}\kappa(Z - p'_S - \theta'_M q_M)^3 - \frac{1}{18}\kappa(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^2 = 0 \tag{E.4}$$

$$\frac{\partial \Pi'_S}{\partial q_S} = -\frac{1}{3\times 18}\theta'_S\kappa(Z - p'_S - \theta'_M q_M)^3 + \frac{1}{18}\frac{\lambda'_S}{q_S}\kappa(p'_S - \theta'_S q_S)(Z - p'_S - \theta'_M q_M)^2 - \eta_S q_S = 0. \tag{E.5}$$

Equations (E.4) and (E.5) lead to equations (E.6) and (E.7).

$$p'_S = \frac{1}{4}\left(Z - \theta'_M q_M + 3\theta'_S q_S\right) \tag{E.6}$$

$$\frac{\partial \Pi'_S}{\partial q_S} = 0 \Rightarrow \kappa(\frac{3}{4}Y)^3\left(\frac{\lambda'_S}{q_S} - \theta'_S\right) - 48\eta_S q_S = 0. \tag{E.7}$$

Furthermore, since q_S is a function of q_M (*i.e.* $q_S = f(q_M)$), differentiating q_S with respect to q_M results in equation (E.8).

$$\begin{aligned} &\tfrac{\partial}{\partial q_M}\left(\tfrac{\partial \Pi'_S}{\partial q_S}\right) = 0 \Rightarrow \tfrac{\partial}{q_M}\left(\kappa(\tfrac{3}{4}Y)^3\left(\tfrac{\lambda'_S}{q_S} - \theta'_S\right) - 48\eta_S q_S\right) = 0 \\ &\Rightarrow 3\kappa\left(\tfrac{3}{4}\right)^3\left(\tfrac{\lambda'_M}{q_M} - \theta'_M + \left(\tfrac{\lambda'_S}{q_S} - \theta'_S\right)\tfrac{\partial q_S}{\partial q_M}\right)\left(\tfrac{\lambda'_S}{q_S} - \theta'_S\right)Y^2 - \kappa\left(\tfrac{3}{4}\right)^3 Y^3\left(\tfrac{\lambda'_S}{q_S^2}\right)\tfrac{\partial q_S}{\partial q_M} - 48\eta_S\tfrac{\partial q_S}{\partial q_M} = 0. \end{aligned} \tag{E.8}$$

Through mathematical simplification, equation (E.9) can then be obtained.

$$\frac{\partial q_S}{\partial q_M} = \frac{3q_S\left(\frac{\lambda'_M}{q_M} - \theta'_M\right)\left(\frac{\lambda'_S}{q_S} - \theta'_S\right)}{Y\left(2\frac{\lambda'_S}{q_S} - \theta'_S\right) - 3q_S\left(\frac{\lambda'_S}{q_S} - \theta'_S\right)^2}. \tag{E.9}$$

Stage 1: Substituting the solution of stages 2 and 3 in the profit functions of the Manufacturer, equation (E.10) can be derived.

$$\Pi'_M = \left(\frac{1}{9\times 18}(\kappa'^2{}_M + \kappa'^2{}_R) - \left(\frac{\kappa'_M}{18}\right)^2\right)(\frac{3}{4}Y)^4 - f_M - \frac{1}{2}\eta_M q_M^2. \tag{E.10}$$

Differentiating Π_M with respect to q_M, equation (E.11) is obtained.

$$\frac{\partial \Pi'_M}{\partial q_M} = 0 \Rightarrow (\frac{3}{4})^4\left(\frac{1}{9\times 18}(\kappa'^2{}_M + \kappa'^2{}_R) - \left(\frac{\kappa'{}_M}{18}\right)^2\right)\left(\frac{\lambda'_M}{q_M} - \theta'_M + (\frac{\lambda'_S}{q_S} - \theta'_S)\frac{\partial q_S}{\partial q_M}\right)Y^3 - \eta_M q_M = 0. \tag{E.11}$$

Finally, the optimal solutions for this game model are as follows:

$$\begin{aligned} &t = 0, \quad a_M = \left(\tfrac{\kappa'_m Y^2}{32}\right)^2, \quad a_R = \left(\tfrac{\kappa'_r Y^2}{32}\right)^2 \\ &p'_R = \tfrac{1}{4}(3Z + \theta'_M q_M + \theta'_S q_S), \quad p'_M = \tfrac{1}{2}(Z + \theta'_S q_S + \theta'_M q_M), \quad p'_S = \tfrac{1}{4}(Z - \theta'_M q_M + 3\theta'_S q_S). \end{aligned} \tag{E.12}$$

q_S and q_M are obtained by substituting equation (E.9) in equation (E.11) and simultaneously solving it with equation (E.7).

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