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PORTFOLIO SELECTION WITH ROBUST ESTIMATORS CONSIDERING BEHAVIORAL BIASES IN A CAUSAL NETWORK

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Abstract. In this study, we develop a behavioral portfolio selection model that incorporates robust estimators for model inputs in order to reduce the need to change the portfolio over consecutive periods. It also includes Conditional Value at Risk as a sub-additive risk measure, which is preferable in behavioral portfolio selection. Finally, we model a varying risk attitude in a causal network in which investor behavioral biases and latest realized return are related to using a causation algorithm. We also provide a case study in Tehran Stock Exchange, where the results disclose that albeit our model is not mean-variance efficient, it selects portfolios that are robust, well diversified, and have less utility loss compared to a well-known behavioral portfolio model.

Mathematics Subject Classification. G11, G02, C02, C44, C61, C51.

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1. Introduction

In the area of behavioral finance, the first portfolio selection model was proposed in the beginning of the 21^{st} century by Shefrin and Statman [78], which is called Behavioral Portfolio Theory (BPT). This model is built based on the mental accounting approach [45] and the psychology of risk [54]. Furthermore, as conventional portfolio models cannot explain the asset allocation puzzle in Canner *et al.* [17], behavioral portfolio selection models were developed by Siebenmorgen and Weber [81] to consider more investor characteristics. Another outstanding work in this area is a Mental Accounting (MA) framework that is recommended by Das *et al.* [21]. Many recent studies have followed the MA approach because it is mean-variance efficient, which is also proven by Alexander and Baptista [4, 6]. For example, Alexander and Baptista [7] proposed a model that considers delegation to portfolio managers in MA, and Baptista [10] incorporated background risk to the MA. Moreover, a risk measure based on pure risk in MA is discussed in Momen *et al.* [59] and applied on Tehran Stock Exchange (TSE). Some other extensions of MA include the following: considering derivatives and non-normal returns by Das and Statman [22, 23], estimation risk by Alexander, Baptista, and Yan [8], and socially responsible investment by Bilbao-Terol *et al.* [12].

Other researches such as Nevins [66] recommended goal-based investing in which there is a sub-portfolio for each goal of the investor. This is expanded by Berkelaar and Kouwenberg [11] to include loss aversion,

Keywords. Behavioral portfolio selection, robust estimator, conditional value at risk, behavioral biases, causal relationship.

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TABLE 1. The model of this study in comparison with other behavioral portfolio selection models. In this table we summarize the comparison of our study (in the "Current Study" column) with other behavioral portfolio selection models in the literature (in the "Previous study" column) such as [7,8,10,11,15,19,21,53,66,78,81]. This table reveals the novelty of this paper.

Behavioral portfolio selection models	Previous studies	Current study
Robust estimators	No	Yes (Shrinkage)
Risk measure	Mostly VaR/ Pure risk/ Risk premium	Conditional value at Risk (Sub-additive)
Risk attitude	Constant	Varying (Based on behavioral biases and latest realized return)
Relationship among risk attitude, past performance, and behavioral biases	No	Yes (Causation method)
Real world data of behavioral biases and investor views on expected return	No	Yes (Tehran stock exchange)

and by Giorgi and Hens [33] to comprise several goals. Moreover, in a goal-based investing framework, an approach for elicitation of risks related to the goals of a family was developed by Brunel [15] that integrates Markowitz [56] with mental accounting. Furthermore, a continuous-time behavioral model was derived by Zhang *et al.* [93] that limits losses to an upper bound, and is similar to [42] which is based on prospect theory [45]. Also building on this theory, Shi *et al.* [80] devised multi-period utility maximization behavioral portfolio selection models. However, based on an alternative theory of disappointment with multiple reference points, Cillo and Delquié [19] formulated a mean-risk model. Finally, Liu *et al.* [53] developed a multi-period robust model using a meta-heuristic algorithm that considers behavioral factors.

In Sections 2, 3, and 4, we discuss three aspects of behavioral portfolio selection models, which our contribution is based on: model inputs, risk measure, and investor attitude toward risk (also identified as risk aversion and risk tolerance). Therefore, in each of these sections, we formally explain our problem statement in three steps: first we describe the current state of literature about each aspect of behavioral portfolio selection (as is). Second, we explain the shortcomings of the current state (the gap). Third, in order to improve these issues (to be), we propose a contribution, which is new in the context of behavioral portfolio selection. Finally, these contributions will be discussed in more details.

Based on the three steps mentioned above, we propose a behavioral portfolio selection model that has three main characteristics: first, it uses shrinkage robust estimators for model inputs in order to reduce the effect of outliers, and produces portfolios with less need to change over consecutive periods. Second, its risk measure is Conditional Value at Risk (CVaR), which is sub-additive, and hence more harmonious with the mental accounting structure of behavioral portfolio selection and third, it considers a varying risk attitude associated with investor behavioral biases and latest realized return. Therefore, as explained in Sections 2, 3 and 4, our contribution is threefold: 1- we use robust shrinkage estimators in a behavioral portfolio model, 2- we use CVaR risk measure that is linked with varying attitude in a behavioral portfolio model, 3- we model a varying risk attitude in relation with behavioral biases and investors latest realized return using a causation method. To the best of our knowledge, considering these three contributions in a single model makes it new in the context of behavioral portfolio selection. Moreover, as an interesting contribution for practitioners, we apply a behavioral portfolio model in a real market using real investors' behavioral biases, past performance, and risk profile. In order to further clarify the novelty of our study, we compare aspects of our work with available literature in Table 1.

The organization of the rest of this paper is as follows. In Sections 2, 3 and 4, we first review the literature of the individual aspects of our model, and then we bring up the need for improvement, and finally we present our contribution in line with the above introduction. In Section 5 we sum up all of our contributions in one straightforward model. In Section 6, the results and discussion are provided using market and survey data from Tehran Stock Exchange (TSE). Finally, in Section 7 the main remarks of the paper are concluded.

2. Model inputs

As Stein [87] argues, biased estimators usually result in better parameter estimation than their commonly preferred unbiased rivals. In particular, it can be shown that for estimating expected return and covariance in portfolio selection, regular unbiased estimators perform poorly compared to shrinkage estimators, in terms of utility loss function [30].

Many conventional portfolio selection models consider robustness, for example studies such as [25,34,36,58,69] investigate the superiority of robust estimators over conventional ones in portfolio selection models. Moreover, some studies conclude that robust estimators containing lower risk as assumptions of normal distribution usually do not hold in portfolio selection problems [49]. There are recent studies that address more specific models, such as Dong and Thiele [26] that propose a robust model for an investor who allocates his budget to several fund managers with a worst case risk. These works are in the area of conventional finance as they use mean-variance or minimum-variance portfolio models. Therefore, they neither contain a proper risk measure for a behavioral portfolio model [21, 77] nor consider behavioral biases in portfolio selection. On the other hand, most available behavioral portfolio models employ standard estimators for expected return and the covariance matrix (see for example [7, 10, 11, 15, 21, 53, 66, 78]). Although standard estimators benefit from simplicity and ease of implementation, they face two obstacles in the context of behavioral portfolio selection. First, portfolio weights are very variable from one period to the next, because standard estimators are very sensitive to model inputs, especially to extreme return outliers [29]. Second, models based on standard estimators, present poor out-of-sample risk-adjusted performance [25]. Fabozzi *et al.* [30] confirm the latter problem, and justify shrinkage estimators as a decent remedy for both of the above issues. In this study, we utilize two of the most well-known shrinkage estimators for expected return and covariance.

Among the most respected shrinkage estimators are those proposed by Jorion [43] for expected returns and Ledoit and Wolf [50] for the covariance matrix. Jorion [43] defines an estimator (r_s) for expected return as follows:

$$r_s = (1-\lambda)r + \lambda r_g e \tag{2.1}$$

Where r is the standard mean return, $r_g = \frac{e'\Sigma^{-1}r}{e'\Sigma^{-1}e}$, $\lambda = \frac{N+2}{N+2+T(r-r_gI)'\Sigma^{-1}(r-r_gI)}$, $e = [1, 1, \ldots, 1]'$, and N is the number of assets.

Ledoit and Wolf [50] express the covariance matrix estimator as:

$$\Sigma_s = \theta\Sigma_{CC} + (1-\theta)\Sigma \tag{2.2}$$

Where Σ is the standard covariance matrix, $\Sigma_{CC} = \Lambda C_{CC}\Lambda$, Λ is a diagonal matrix of the volatility of returns, and the element on the m-th row and n -th column $(m, n = 1, 2, \ldots, N)$ of C_{CC} equals ρ_{mn}.

$$\rho_{mn} = \begin{cases} 1 & , m = n \\ \dfrac{2}{(N-1)N}\sum_{i=1}^{N}\sum_{j=i+1}^{N}\rho_{ij} & , m \neq n \end{cases} \tag{2.3}$$

In equation (2.3), θ is called shrinkage intensity, which is calculated following Ledoit and Wolf [50]. As DeMiguel *et al.* [24] argue, the shrinkage intensity has a considerable effect on the performance of portfolio models.

3. Risk measure

As defined in Artzner *et al.* [9], a proper risk measure is called coherent and has four characteristics: Monotonicity, Translation Invariance, Homogeneity, and Subadditivity. Acerbi [1] discusses the convenience of coherent risk measures in more details. Subadditivity, which is of special interest in behavioral portfolio selection, means that the risk measure for several mental account portfolios after they have been combined should not be greater than the sum of their risk measures before they were combined. This is important in behavioral portfolios, which are built based on the assumption that people have different mental accounts.

The most popular risk measure in behavioral portfolio selection is Value-at-Risk (VaR), for example see [7, 10, 11, 15, 21, 66, 78]. VaR is defined as the maximum loss or minimum reward, which may occur in a defined confidence level [44]. This risk measure is popular because of its conceptual and calculation simplicity; however, it is not sub-additive. This means that if an investor has two mental accounts X and Y, $VaR(X+Y) \leq VaR(X) + VaR(Y)$ is not necessarily true. This is the first and most critical drawback for a behavioral risk measure. The next drawback of VaR originates from excluding extreme cases further than itself. This means that all of the weights are assigned to the VaR percentile of the distribution and none to all other percentiles. Hence, some even believe that VaR is not a measure of risk [88].

Another well-known risk measure is called Conditional Value at Risk (CVaR), which is first used in portfolio selection by Rockafellar and Uryasev [71]. CVaR is the expected loss conditional on the loss being more than the α-th percentile of the loss distribution. While in VaR, the question is "How bad can losses get?" the CVaR question is "If the situation gets bad, what is the expected loss?" [41].

While more studies such as Krokhmal *et al.* [48] also use CVaR in the portfolio selection models, others discuss advantages and disadvantages of CVaR *vs.* VaR [5, 6]. However, these models neither consider behavioral biases nor do they use robust estimators. Since CVaR provides better incentives for investors and is a better risk management tool in comparison with VaR [5], it is convenient to resolve the VaR issues which were mentioned above, by using CVaR as the risk measure for a behavioral portfolio model. Unlike VaR, CVaR considers the whole tail of loss or return distribution beyond VaR. Moreover, CVaR is sub-additive and coherent, which means that the following inequality is always true [1]:

$$CVaR(X+Y) \leq CVaR(X) + CVaR(Y) \tag{3.1}$$

CVaR, like VaR, is a function of α (the confidence level). In other words, the CVaR at the confidence level ? is the average of the worst $1-\alpha$ of losses:

$$CVaR_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_p dp \tag{3.2}$$

where q_p is the p loss percentile , and $p \in [0,1]$.

4. Risk attitude

Investor attitude toward risk can be defined through two different methods. First, eliciting risk aversion directly using questionnaires such as Pan and Statman [68], second asking the investor to state the maximum acceptable probability of failing to reach the aspiration level. The first type of risk attitude is very popular in portfolio models following Markowitz [56] (see for example [13, 31, 51, 52, 64, 94]), whereas the second type is very common in behavioral portfolio selection [7, 8, 10, 21, 78], because it allows the investor to have different risk attitudes in different mental accounts. Moreover, Das *et al.* [21] argue that investors are better at estimating their risk attitude through the second method. However, as Alexander and Baptista [4, 6] discuss, these two types of risk attitudes can be converted to each other mathematically.

As mentioned above, behavioral portfolios are one step ahead by using different risk attitudes in different accounts, however, behavioral portfolio studies such as Shefrin and Statman [78], Brunel [15], Berkelaar and Kouwenberg [11], Das *et al.* [21], Alexander and Baptista [7], Baptista [10], and Nevins [66] assume a constant risk attitude from one period to the next, which is against the hypothesis of changing risk aversion with investor wealth [38]. Other studies such as Morin and Suarez [63], Brunnermeier and Nagel [16], Guiso and Paiella [37], Shaw [76], and Sahm [73] also admitted this hypothesis in different ways. In addition, the economic sensibility of constant risk attitude is strictly challenged by Björk *et al.* [13]. While all these studies dispute the use of constant risk attitude, researches such as Brandt and Wang [14], and Holt and Laury [40] consider psychological factors to affect the varying risk attitude. Furthermore, Pompian [70], Pan and Statman [68], Roszkowski and Snelbecker [72], and Nordén [67] find behavioral biases to be among psychological factors that impact the risk attitude. On the other hand, behavioral biases are affected by past returns as stated by Vissing−Jorgensen [91], Chen and Kim [18], and Statman *et al.* [85]. Following these results and Grable *et al.* [35], we propose a varying risk attitude which depends on behavioral biases and investors latest realized return from the market.

In order to represent a relationship between risk attitude, behavioral biases and latest realized return of investor, an obvious option is to consider multivariate linear regression [75]. The problem is that regression does not infer or present causality between variables [74]. To infer causality based on data, one should follow causation methods such as the PC algorithm (named after its authors, Peter Spirtes and Clark Glymour), as implemented in Tetrad software [74], which results in a network of relations between desired variables. The PC algorithm looks for causal relationships in observational or experimental data while assuming that the true causal hypothesis is acyclic and no unknown common cause exists between any of two variables in the dataset. The algorithm decides on independence of pairs of variables (*e.g.* behavioral biases) by using conditional independence tests [83].

In order to use the output of the PC algorithm in modeling, we define $S = [b_1 b_2 \cdots b_M \alpha r^T]'$ and $s_i (i = 1, \ldots, M+2)$ as the ith element of S, where $b_k (k = 1, \ldots, M)$ is the k-th behavioral bias,r^T is the latest realized return (from the previous period), and M is the number of behavioral biases under consideration. Equation (4.1) summarizes the output of the PC algorithm:

$$A \times S = S_0 \tag{4.1}$$

where $S_0 = [b_{01} b_{02} \cdots b_{0M} \alpha_0 r_0^T]'$ is the vector of intercepts, A is an $(M+2) \times (M+2)$ matrix, the ij element (a_{ij}) of which is defined as follows.

$$a_{ij} = \begin{cases} 0 & , \text{No edge from } s_i \text{ to } s_j \\ 1 & , \quad i = j \\ -\text{edge coefficient from } s_i \text{ to } s_j & , \quad \text{Otherwise} \end{cases} \tag{4.2}$$

The above formulations are intended to draw a relationship between risk attitude and behavioral biases of the investor. The problem is that measuring behavioral biases in every update of the portfolio is not straightforward. This issue will be discussed in the next section and will be considered in modeling.

5. The proposed model

Krokhmal *et al.* [48] prove that there are three different equivalent ways to arrange risk measure and expected return in a portfolio selection model. First, including expected return as an objective function and risk measure as a constraint. Second, considering expected return as constraint and risk measure as objective function. Third, modeling both expected return and risk measure in one objective function. Although each of these ways have their advantages and disadvantages, the third one is easier to use, as it requires less input from the investor. For instance, it does not need the maximum acceptable risk level for calculating the optimum portfolio. Therefore, using this modeling approach and equations (2.1), (3.2), and (4.1), our behavioral portfolio selection model is written as:

$$\begin{aligned} Max \quad & w'.r_s - CVaR_\alpha \\ & e'.w = 1 \\ & A \times S = S_0 \end{aligned} \tag{5.1}$$

where w is asset weights (proportions) in portfolio, and $e' = [1, 1, \ldots, 1]$. The objective function includes both expected return and risk of portfolio to form the utility of the investor. The first constraint of the model guarantees the sum of weights to equal one. This model is constructed for an investor with one mental account.

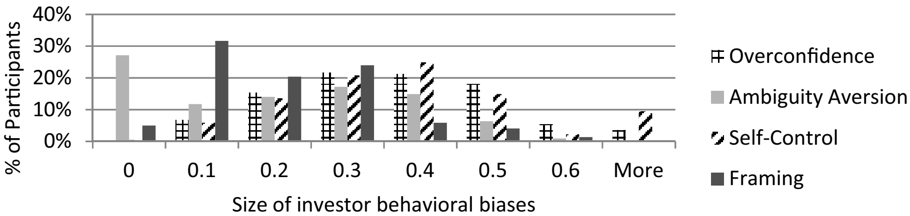


FIGURE 1. Survey participants were required to fill behavioral biases diagnostic tests in Pompian [70]. The results reveal that Tehran Stock Exchange (TSE) investors are behaviorally biased; therefore, behavioral biases cannot be ignored in portfolio selection.

In order to generalize it for a multi account investor, the model should be applied to each account separately. This makes more sense compared to available behavioral models such as [21], because the total risk of all accounts in our model is ensured to be less than or equal to the sum of risk for individual accounts (Eq. (3.1)) as the risk measure (CVaR) is sub-additive.

As it is defined in equation (4.1), S is a vector of behavioral biases, confidence level, and latest realized return. Anyway, due to the following two reasons measuring behavioral biases in every update of the portfolio is not easy. First, investors do not like to be questioned about their biases and α periodically, and second, even if investors respond to all interview questions for every portfolio update, the questionnaire would not be able to assess their true behavioral biases, because the questionnaire would be familiar to them. Hence, we measure investors' behavioral biases and α in the first selection of the portfolio, and then we run the model in equation (5.1) and wait for the real portfolio return to be realized. In the next updates of the portfolio, instead of measuring behavioral biases and α again, the last constraint of the model helps to estimate them using realized return from the last period. The resulting new α is used in calculating the risk measure in the objective function ($CVaR_\alpha$), hence it links behavioral biases to the decision variables (w). Therefore, in the first period, in-sample estimates are used in the model, while the model is run over out-of-sample data in the succeeding updates.

Hereafter, we call this model CVS (Conditional Value at Risk as risk measure and shrinkage estimators for estimation of expected return and covariance). Wherever standard estimators are used instead of shrinkage estimators, we present the model as CV (Conditional Value at Risk as risk measure and standard estimators for estimation of expected return and covariance).

6. Results and discussion

Two sets of data were selected to evaluate our model: first, historical return data of stocks in Tehran Stock Exchange (TSE). Second, similar to Momen *et al.* [60], questionnaires were distributed among investors. More details about the data set are presented in the appendix. The empirical part of the study is conducted as follows (for more studies in TSE see for example [3, 28, 59, 61, 65]):

(1) Investors were required to answer the Pompian [70] diagnostic tests for four behavioral biases: Overconfidence, Ambiguity Aversion, Self-Control, and Framing (for evidence on effects of behavioral biases on portfolio selection see for example [32]). Moreover, respondents were asked to provide preference scores for each alternative choice, which made measuring the size of their biases possible (Fig. 1). We have provided the results of reliability tests for the questionnaires in the appendix.

(2) Investors also determined the maximum probability of failing to reach the threshold, as a measure of attitude toward risk. Results are summarized in Figure 2.

(3) Respondents stated their latest realized return from the market with its related time period as shown in Figure 3, helped us in calculating the second constraint in equation (5.1).

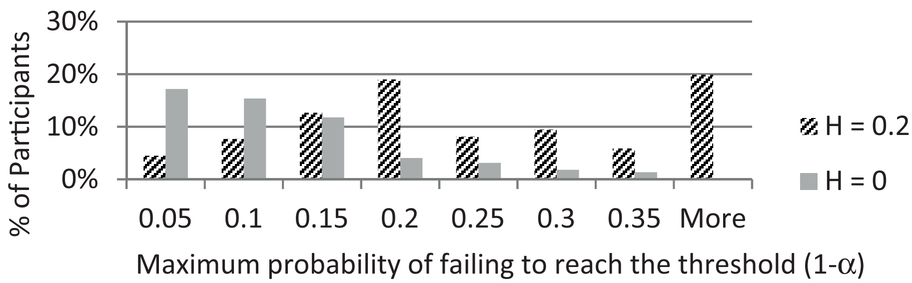


FIGURE 2. Survey participants answered this question: "If you hold a portfolio of assets in Tehran Stock Exchange, what is the most acceptable percentage of times that portfolio return is less than H?". Here we report the histogram of participant responses for two sets of aspiration levels (H).

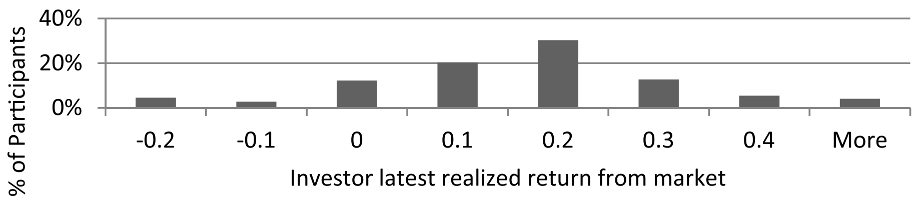


FIGURE 3. We have recorded participants latest realized return from TSE according to their self-declarations. This histogram presents their annualized returns.

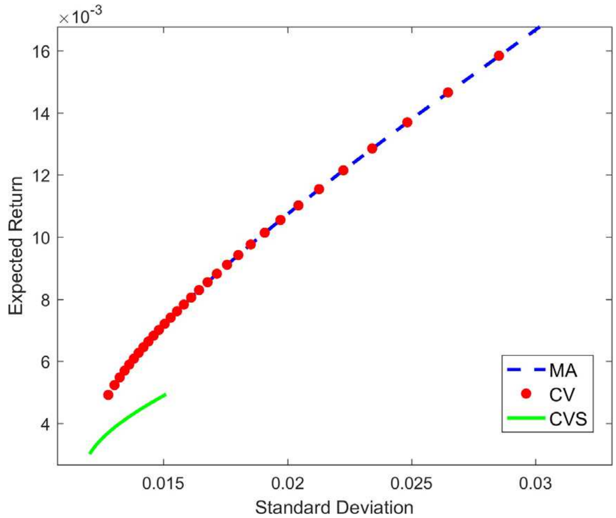


FIGURE 4. We calculated an efficient frontier for three models: our model (CVS), Mental Accounting (MA) of Das *et al.* [21] which is mean-variance efficient and a model like CVS in which shrinkage estimators are replaced by standard estimators (CV). This figure shows that our model is not mean- variance efficient.

Using the data gathered from the above survey, we draw a causal network using the PC algorithm [82]. Although there are other search algorithms in Tetrad that one can use, we preferred the PC algorithm because in our

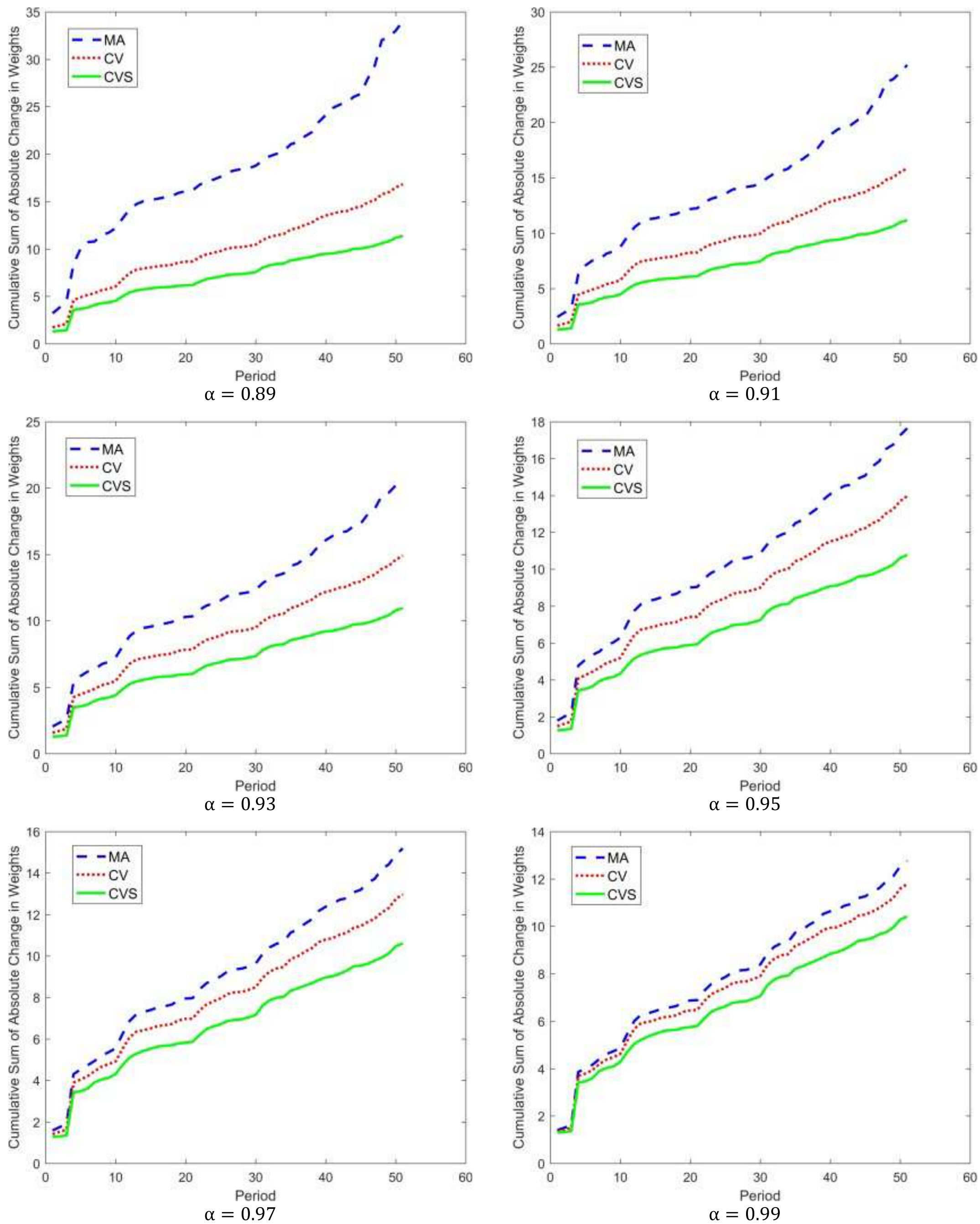
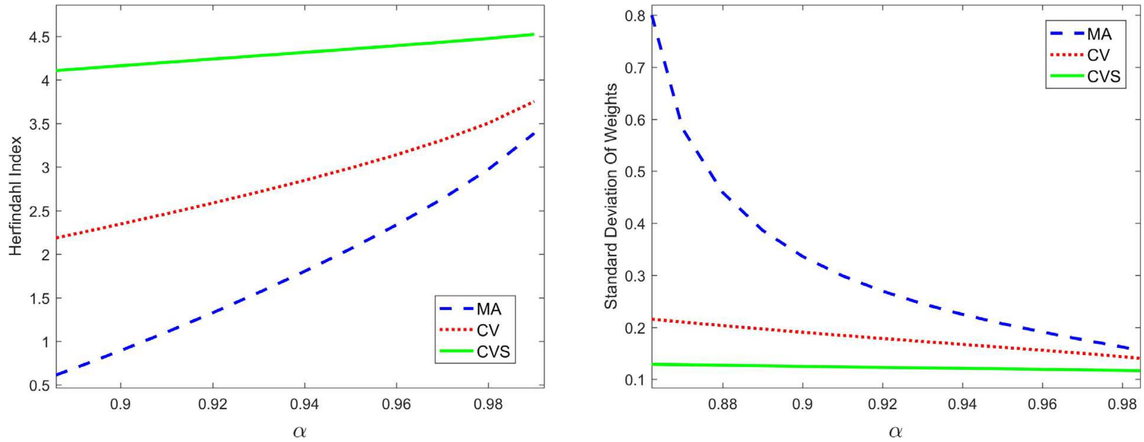


FIGURE 5. This figure presents different charts for different values of confidence level (α) showing the cumulative sum of absolute change in weights for three models: our model (CVS), Mental Accounting (MA) of Das *et al.* [21] which is mean-variance efficient and a model like CVS in which shrinkage estimators are replaced by standard estimators (CV). Results show that with different confidence levels in successive periods CVS portfolios need less change in portfolio weights. We used different αs to cover a wide range of possible investors, which shows the superiority of out-of-sample performance of our model.



a. Herfindahl Index b. Standard deviation of weights

Figure 6. Herfindahl index, which is a diversification ratio, is presented for three models: our model (CVS), Mental Accounting (MA) of Das *et al.* [21] which is mean-variance efficient, and a model like CVS in which shrinkage estimators are replaced by standard estimators (CV). Charts show that our model provides more diversified portfolios for different levels of α, compared to the other two alternatives. In order to maximize diversification, standard deviation of weights should be minimized. This figure shows our model (CVS), provides more diversified portfolios compared with Mental Accounting (MA) of Das *et al.* [21] which is mean-variance efficient and a model like CVS in which shrinkage estimators are replaced by standard estimators (CV).

data it produces a model with both higher p-value and lower degrees of freedom. The resulted network and its related equations are available in the appendix.

For the purpose of assessing the efficiency, we computed an efficient frontier of our model using both shrinkage and standard estimators. Figure 4 presents these two efficient frontiers along with Mental Accounting [21], which is a well-known behavioral portfolio selection that produces mean-variance efficient portfolios (for proof and discussion see [4, 6]). As Canner *et al.* [17] argue, these behaviorally selected portfolios are close to the efficient frontier. However, real world investors fancy portfolios that work well under different scenarios. This justifies inefficient CVS portfolios, as in Fabozzi *et al.* [30], where the investor must accept to give up some of the benefits to achieve protection from the model and estimation risks. For such an investor, portfolios based on robust estimators such as CVS are favorable because they are insured against worst-case misspecification of the model.

Figure 5 shows that for a wide range of confidence levels, in consecutive periods, our model is more robust than MA, meaning that the investor has to change his portfolio less intensively. This can save investor successive transaction costs. This figure reveals that different out-of-sample investors with different levels of confidence can obtain improved performance from our model, which amplifies the benefit of using the model of this study for different investors that have different risk attitudes.

In order to investigate the diversification effect of our model, we calculated two measures: first, the Herfindahl Index, which is defined as $HI(w) = (\sum_{i=1}^{N} w_i^2)^{-1}$ [2] (Fig. 6a), second, the diversification index which is defined as $DI(w) = StandardDeviation(w_1, w_2, \ldots, w_n)$ as in [81] (Fig. 6b). The results of both figures are very similar and show that our model provides a satisfying diversification.

With the purpose of assessing the performance of different portfolio selection models in the real world, we used the utility loss function of keeping $\hat{w}$, which is defined as:

$$L(w^*, \hat{w}) = u(w^*) - u(\hat{w}) \tag{6.1}$$

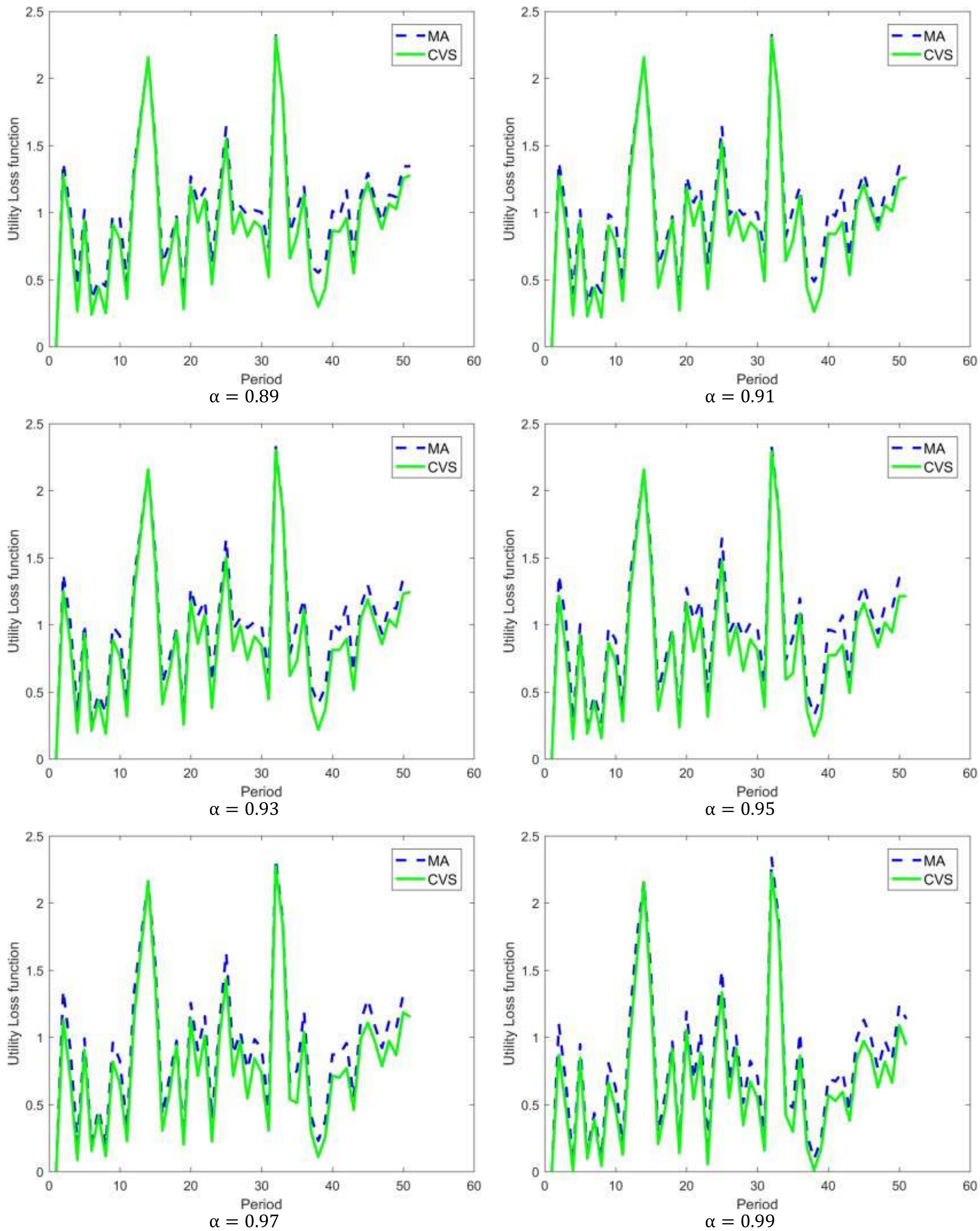


FIGURE 7. As utility loss shows the amount of loss in utility of an investor in the real world, less utility loss indicates a better model. Therefore, this figure shows the outperformance of our model (CVS) in comparison with Mental Accounting (MA) in Das *et al.* [21]. We used different αs to cover a wide range of possible investors, which shows the superiority of out-of-sample performance of our model. The mean absolute percentage deviation between MA and CVS equals 30.19% on average.

Where $u(x)$ is the utility function of keeping a portfolio with a weight vector of x, $w = (w_1, w_2, \ldots, w_N)$ is a vector of asset proportions in a portfolio, N equals the number of available assets, and any estimated portfolio weight vector can be defined as $w = f(r^1, r^2, \ldots, r^T)$, in which $r^1, r^2, \ldots, r^T$ are historical returns, and T is the number of past returns. This function highly depends on true values of expected return (r) and covariance matrix (Σ); however, in practice r and Σ are unknow $\hat{w}$, which is computed using expected return and covariance.

Obviously, an investor who holds w^* is more satisfied than anyone who keeps $\hat{w}$. This means that utility loss function is strictly positive, and its lower values show the superior performance of a model. Since this figure uses true values of expected return and covariance matrix after the period has finished, it is a proper out-of-sample performance measure. In Figure 7 the utility loss function for a wide range of confidence levels are presented, which shows superior performance for different out-of-sample investors with different levels of confidence. This means different investors with various risk attitudes can benefit from using the model of this study. More precisely, investors with α from 0.89 to 0.99, who use CVS have an average of 30.19% less utility loss in comparison with investors who use MA.

7. Concluding remarks

We pursue three objectives in a behavioral portfolio selection model in this study. First, reducing the need for extensive changes in portfolio weights from one period to the next. Second, incorporating a risk measure that is more compatible with the mental accounting concept and third, taking into account a varying investor attitude toward risk which depends on investor behavioral biases and the latest realized return from the market.

For the purpose of satisfying the above goals, we consider three specific parts of a model: model inputs, risk measure, and risk attitude. For approximating model inputs, we use shrinkage robust estimators, namely Jorion estimator [43] for expected return and Ledoit and Wolf estimator [50] for covariance matrix. For measuring risk, we take advantage of Conditional Value at Risk (CVaR), which is specifically appealing for a behavioral portfolio model as it is sub-additive. Finally, in order to estimate an updatable varying risk attitude, we recommend a causation network among risk attitude, behavioral biases, and investor realized return from the previous period.

With the intention of assessing our model and making discussion possible, we collected data from Tehran Stock Exchange (TSE) along with a survey data. Outcomes reveal that even if our model is not mean variance efficient similar to an available state of the art model, an investor who uses our model loses less utility in consecutive periods of the real world. Moreover, our model is more robust, and does not need as much change in asset proportions from one period to the next. Furthermore, our model produces well- diversified portfolios compared to a state of the art model in the literature.

The model of this study is constructed based on the relationship between behavioral biases and an individual investor attitude toward risk. Although this showed to be beneficial, it does not consider the effect of behavioral biases on market prices in the portfolio selection model, which we think has an enormous value in individual investor decision-making and remains open for future researches.

Appendix A.

The first part of our data set includes historical return data of Tehran Stock Exchange (TSE) for ten stocks from 2006/10/01 to 2015/10/01, which we retrieved from www.en.tsetmc.com (Tab. A.1). The second part of the data set is the questionnaire output from 228 TSE investors, aged 24 to 68 of both genders. It should be stated that investors with basic investment knowledge and willingness to participate in academic surveys are not widespread in TSE; therefore, we tried to keep as much respondents as possible in the survey by holding meetings in person with at most three of them in each session. Each meeting took about 45 minutes to 1 hour, which resulted in about 115 hours of meetings from January to August of 2015. Our participants were working in financial institutions such as banks, insurance companies, stock brokerages and investment consulting companies. However, we omitted the incomplete and careless responses; therefore, we only considered 201 questionnaires, which were more consistent.

TABLE A.1. Names and Symbols of ten Tehran Stock Exchange (TSE) are shown here. They are from different activity sectors.

Symbol	Name	Activity Sector
TRNS	Iran Transfo	Electric Devices
PETR	Petro. Inv.	Chemical Products
GDIR	Ghadir Inv.	Multi-Disciplinary
MSMI	I. N. C. Ind.	Basic Metals
MKBT	Iran Tele. Co.	Telecommunication
SIPA	Saipa	Cars & Spare Parts
BAMA	Bama	Extraction of metallic ores
PKSH	Pakhsh Alborz	Pharmaceuticals
FOLD	Mobarakeh Steel	Basic Metals
NOLZ	Iranol	Oil Products

TABLE A.2. Results of Cronbach's alpha tests. This table shows that because $\alpha_{Cronbach} \geq 0.7$, all four questionnaires of Pompian [70] are reliable.

	Overconfidence	Ambiguity Aversion	Self-Control	Framing
Number of questions in the questionnaire	8	2	3	4
the standardized Cronbach's alpha	0.7782	0.7590	0.7612	0.8918
the unstandardized Cronbach's alpha	0.7626	0.7590	0.7479	0.8898

In the questionnaire, we asked about four behavioral biases: Overconfidence, Ambiguity Aversion, Self-Control, and Framing, which are defined as follows. Overconfidence is one's unreasonable belief in his intuitive reasoning, judgment and cognitive abilities [18, 62, 84, 86, 90, 92]. Ambiguity Aversion is hesitation of a decision maker when probability distributions of events are uncertain [27, 39, 47]. Self-Control is investor reluctance to save for the future instead of spending now [55, 79] . Framing means that investor reactions to similar situations vary due to differences in context [46, 57, 89].

We performed Cronbach's alpha test [20] to assess the reliability of our questionnaires, which are borrowed from [70]. The results show that all four questionnaires are quite reliable (Tab. A.2).

Based on our data, a causal network among behavioral biases, α , and latest realized return, is obtained from Tetrad software, which is shown in Figure A.1.

As discussed in Section 5., in the first period the gathered data are used directly in the model, while in the following periods, the model uses these data to draw a causal relationship between parameters and uses this relationship to estimate the updated value of parameters. In the present case, according to Figure A.1, there are the following equations:

$$\begin{cases} -0.0156SC - 0.0789OC + 0.1312r^T + \alpha = 0.7129 \\ 0.0297OC + SC = 0.4093 \\ -0.0629AM + 0.0522FR - 0.2266r^T + OC = 0.2978 \\ 0.1235AM - 0.1231r^T + FR = 0.2017 \\ 0.1717SC + 0.1669r^T + AM = 0.2812 \\ r^T = 0.1214 \end{cases} \tag{A.1}$$

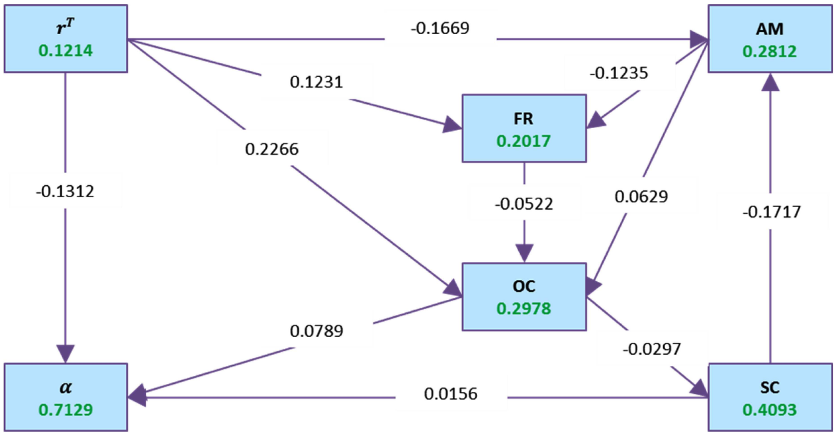


FIGURE A.1. This causal network is resulted from applying the PC algorithm of Tetrad software for data of our survey. The data as shown in the network includes: latest realized return from the market (r^T), α (the confidence level), Ambiguity Aversion (AM), Overconfidence (OC), Framing (FR), and Self-Control (SC). Numbers on the edges are edge coefficients and those inside the nodes are intercepts.

Where r^T equals the latest realized return (0.1214), and these equations result in the following matrix notation:

$$A = \begin{bmatrix} 1.0000 & 0.0000 & 0.1717 & 0.0000 & 0.0000 & 0.1669 \\ 0.1235 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & -0.1231 \\ 0.0000 & 0.0000 & 1.0000 & 0.0297 & 0.0000 & 0.0000 \\ -0.0629 & 0.0522 & 0.0000 & 1.0000 & 0.0000 & -0.2266 \\ 0.0000 & 0.0000 & -0.0156 & -0.0789 & 1.0000 & 0.1312 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}, \quad S = \begin{bmatrix} AM \\ FR \\ SC \\ OC \\ \alpha \\ r^T \end{bmatrix}, \quad S_0 = \begin{bmatrix} 0.2812 \\ 0.2017 \\ 0.4093 \\ 0.2978 \\ 0.7129 \\ 0.1214 \end{bmatrix} \tag{A.2}$$

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