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ON THE M/G/1 RETRIAL QUEUE SUBJECTED TO BREAKDOWNS

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Abstract. Retrial queueing systems are characterized by the requirement that customers finding the service area busy must join the retrial group and reapply for service at random intervals. This paper deals with the M/G/1 retrial queue subjected to breakdowns. We use its stochastic decomposition property to approximate the model performance in the case of general retrial times.

Keywords. Retrial queue, breakdown, stochastic decomposition, approximation.

Mathematics Subject Classification. 60K25, 90B22, 68M20.

1. Introduction: model description

Retrial queueing systems are characterized by the requirement that customers finding the service area busy must join the retrial group and reapply for service at random intervals. These models arize in the analysis of different communication systems. For surveys on retrial queues see Falin [7], Artalejo [5], Templeton [14] and also monograph by Falin and Templeton [8].

Retrial queues subjected to breakdowns are of great importance because they occur in many practical situations. General issues related to servers with breakdowns are discussed in Gelenbe [9]. The author considered a transaction processing computer system subjected to failures, where the checkpoint-rollback-recovery

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technique is used to ensure reliability of the data. Assuming the instants of failure to be a Poisson process with constant failure rate, and using the means of a theoretical analysis of a queueing system representing system behavior, he derived the optimum checkpoint interval, which maximizes system availability, as a deterministic quantity and as a function of the system load. In Gelenbe and Hernandez [10], this result was extended to the case where the failure process is a time dependent Poisson process. Furthermore, the authors considered the Weibull failure rate model, and derived a method for obtaining the optimum checkpoint interval. A single server retrial queue with server subject to breakdowns and repairs was studied by Kulkarni and Choi [13]. With the help of the theory of regenerative processes they obtained the generating functions of the limiting distribution and other characteristics of the queue length process for two different models. In Aissani [1], a version of the unreliable M/G/1/1 retrial queue with variable service was considered. This approach permitted to consider the redundancy problem. Artalejo [4] obtained sufficient conditions for ergodicity of Markovian multiserver queues with retrials and breakdowns. He also presented a recursive algorithm to compute the steady-state probabilities for the M/G/1 retrial queue with breakdowns. Recent contributions on this topic include the papers of Artalejo and Gomez–Corral [6], Anisimov [3], Krishnamoorthy and Ushakumari [12].

This paper presents the analysis of the M/G/1 retrial queue subjected to breakdowns. We consider a single server queueing system at which primary customers arrive according to a Poisson stream with rate $\lambda > 0$. An arriving customer receives immediate service if the server is idle and functional, otherwise he leaves the service area temporarily to join the retrial group (orbit). Any orbiting customer will repeatedly retry until the time at which he finds the server idle-up and starts his service. The retrial times are arbitrarily distributed with distribution function $T(x)$ having finite mean $1/\theta$.

The service times follow a general distribution with distribution function $B(x)$ having finite mean $1/\gamma$ and Laplace–Stieltjes transform $\tilde{B}(s)$. We assume that the server is subject to Poisson active (when he is busy) and passive (when he is idle) breakdowns with rates μ and η, respectively. The time duration of active and passive interruptions follows random variables D_b and D_i with distribution functions $H(x)$ and $G(x)$ and Laplace–Stieltjes transforms $\tilde{H}(s)$ and $\tilde{G}(s)$, respectively. We assume that it follows the same type of distribution as the corresponding service time. No breakdowns may occur when the server is down. The customers whose service is interrupted by an active breakdown are obliged either to leave the system with probability $(1-c)$, or to join the orbit with probability c.

Finally, we admit the hypothesis of mutual independence between all random variables defined above.

The state of the system at time t can be described by means of the process $\{C(t), N_0(t), t \geq 0\}$ where $N_0(t)$ is the number of customers in the orbit, $C(t)$

is the state of the server as defined below:

$$C(t) = \begin{cases} 0 & \text{server is idle and functional} \\ 1 & \text{server is busy and functional} \\ 2 & \text{server is down due to a breakdown.} \end{cases}$$

One approach to study this model is a stochastic decomposition. Aissani and Artalejo [2] obtained some stochastic decomposition results for exponential retrial times, in particular the stochastic decomposition for the embedded Markov chain at idle-up epochs. Assuming this result as valid for general retrial times, we apply an approximation method for the computation of the steady-state queue size distribution. We study the effects of the retrial intensity and those of the breakdowns on the model performance. The paper is organized as follows. The next section contains some notations. We review the stochastic decomposition in Section 3. Section 4 deals with the approximation method. In Section 5, we present some numerical results.

The proofs and some results that are available in the literature are omitted, and interested readers are referred back to the original papers.

2. Notation

Let ξ_n be the time when the server enters the idle-up state for the n-th time; ϱ_n be the time at which the n-th fresh customer arrives at the server. We further consider the random process $\{C(t),\ N_0(t),\ t \geq 0\}$ as $t \to \infty$.

Let

$$\begin{aligned} \pi_k &= \lim_{n\to\infty} P\left(N_0\left(\xi_n^+\right) = k\right) \qquad k = 0,1,2,\ldots; \\ p_{i,j} &= \lim_{t\to\infty} P(C(t) = i; N_0(t) = j); \;\; r_{i,j} = \lim_{n\to\infty} P(C(\varrho_n^-) = i; N_0(\varrho_n^-) = j) \\ j &= 0,1,2,\ldots; \quad i = 0,1,2. \end{aligned}$$

Define $\quad \Phi(z) = \sum_{k=0}^{\infty} \pi_k z^k; \quad P_i(z) = \sum_{j=0}^{\infty} p_{i,j} z^j \quad i = 0,1,2.$

Let q_n be the number of customers in the orbit at instant ξ_n^+; X_i^n be the time elapsed since the last attempt made by the i-th customer in the orbit until instant ξ_n^+. Define $q = \lim_{n\to\infty} q_n$; $X_i = \lim_{n\to\infty} X_i^n$. When $q > 0$, we have a vector $X = (X_1, X_2, \ldots, X_q)$ of expended retrial times of the q orbiting customers present at an arbitrary time when the server enters the idle-up state. We denote by $f_q(x_1, x_2, \ldots, x_q)$ the joint density function of q and X.

3. Stochastic decomposition

Stochastic decomposition property of classical M/G/1 retrial queues with exponential and general retrial times states that the number of customers in the system is equal to the sum of two independent random variables: the number of customers in the ordinary M/G/1 queue with infinite waiting space and the number of customers in the M/G/1 retrial queue given that the server is idle [15, 16]. Aissani and Artalejo [2] introduced an auxiliary queue without retrials to establish the stochastic decomposition property of M/G/1 retrial queues with breakdowns and exponential retrial times. The decomposition result for queues with vacations, which obviously related to breakdowns, was first proved in a more general manner in Gelenbe and Iasnogorodski [11]. Assuming general interarrival times, the authors obtained an operational formula (together with sufficient conditions for ergodicity) relating the waiting time in stationary state of a queue with vacations to the waiting time of an equivalent GI/G/1 queue.

Consider a random process $\{C(t), N_0(t), t \geq 0\}$ as defined in Section 1. This is not a Markov process, but it has an embedded Markov chain at idle-up epochs $\{N_0(\xi_n^+)\}$. As in classical retrial queues, the ergodicity condition is independent of the retrial parameter θ. From [13], we have that the system is stable if

$$\rho = \lambda \frac{1-\tilde{B}(\mu)}{\mu}\left(1+\mu\left(E[D_b]+\frac{c}{\lambda}\right)\right) < 1. \tag{1}$$

In [2], the auxiliary queue is the M/G/1 queue with waiting line, breakdowns of the server and option for leaving the system after an interruption. The rules that govern this system are as follows. Customers queue up according to FCFS discipline. When an active breakdown occurs, the interrupted customer can leave the system or stay at the service facility until the repair is completed and then restart his service according to preemptive repeat different policy. In fact, this model is the main model given in Section 1 and simplified by neglecting the repeated attempts. The generating function $\bar{\Phi}(z)$ of the steady-state distribution $\{a_k, k \geq 0\}$ of the embedded Markov chain at idle-up epochs is defined in terms of the generating function $\Omega(z)$ for the number of customers that join the orbit during a fundamental server period (time from the epoch at which customer commences service until the epoch at which the server is able to start a new service time):

$$\bar{\Phi}(z) = K\frac{\eta z\tilde{G}(\lambda-\lambda z)-(\lambda-\lambda z+\eta)\,\Omega(z)}{z-\Omega(z)}, \tag{2}$$

where

$$\Omega(z) = \tilde{B}(\lambda-\lambda z+\mu)+(1-c+cz)\,\mu\tilde{H}(\lambda-\lambda z)\,\frac{1-\tilde{B}(\lambda-\lambda z+\mu)}{\lambda-\lambda z+\mu}$$

and

$$K = \frac{1-\rho}{\lambda(1+\eta E[D_i])+\eta(1-\rho)}.$$

We have the following result about stochastic decomposition for $\{N_0(\xi_n^+)\}$ as $n \to \infty$:

$$\Phi(z) = \bar{\Phi}(z)\,\frac{P_0(z)}{P_0(1)}, \tag{3}$$

where

$$P_0(z) = K\left[K(1+\eta E[D_i]) + \left(1-\tilde{B}(\mu)\right)(1-\eta K)\left(E[D_b]+\frac{1}{\mu}\right)\right]^{-1} \times \exp\left\{\frac{\lambda}{\theta}\int_1^z \frac{1-\Omega(u)}{\Omega(u)-u}\mathrm{d}u + \frac{\eta}{\theta}\int_1^z \frac{1-\tilde{G}(\lambda-\lambda u)}{\Omega(u)-u}\mathrm{d}u\right\}$$

and $\dfrac{P_0(z)}{P_0(1)}$ is the generating function for the number of customers in the orbit given that the server is idle and functional.

We assume that the decomposition result (3) for exponential retrial times is also valid for general retrial times, in the same way as the result for M/G/1 retrial queues without breakdowns established in [15, 16].

4. Approximation method

From the decomposition result (3), we can see that the steady-state distribution $\{\pi_k, k \geq 0\}$ of the embedded Markov chain at idle-up epochs is a convolution of two distributions: the steady-state queue size distribution $\{a_k, k \geq 0\}$) for the auxiliary queue without retrials and $\{p_{0,k}, k \geq 0\}$.

Obviously $\pi_k = P(q = k)$ for $k \geq 0$. Since Poisson arrivals see time averages, we have $p_{0,k} = r_{0,k}$ for $k \geq 0$. Suppose that there are $k > 0$ customers in the orbit at an arbitrary time when the server enters the idle-up state. In such a case, we have

$$\pi_k = \int_0^\infty\int_0^\infty \cdots \int_0^\infty f_k(x_1, x_2, \ldots, x_k)\,\mathrm{d}x_1\mathrm{d}x_2\ldots\mathrm{d}x_k;$$

$$r_{0,k} = \int_0^\infty\int_0^\infty \cdots \int_0^\infty P(\delta(k; x_1, x_2, \ldots, x_k) = 0)\, f_k(x_1, x_2, \ldots, x_k)\,\mathrm{d}x_1\mathrm{d}x_2\ldots\mathrm{d}x_k,$$

where $\delta(k; x_1, x_2, \ldots, x_k) = 0$ if the next served customer is not one of the k orbiting customers (otherwise $\delta(k; x_1, x_2, \ldots, x_k) = 1$). Expended retrial times $X_1, X_2, \ldots, X_k$ of the k orbiting customers depend on each other in a very complicated way. This dependence implies that a derivation of an explicit formula for the joint density function $f_k(x_1, x_2, \ldots, x_k)$ is difficult, if not impossible.

An approximation to $f_k(x_1, x_2, \ldots, x_k)$ was proposed in [15]. By using this approximation and applying the decomposition property, the authors developed a method for the computation of the steady-state queue size distribution for classical retrial queues with general retrial times. We adapte this approximation method for the M/G/1 retrial queue with breakdowns. The approximation is based on the intuitive consideration (justified in many applications) that the average retrial time is very small relative to the average service time. Thus during a fundamental server period, orbiting customers make many retrials. Consequently the dependency among $X_1, X_2, \ldots, X_k$ is weak. Further, the more retrials an orbiting customer makes, the less the distribution of the expended retrial time depends on the time of the observation. According to renewal theory, the expended retrial time distribution observed at a time point sufficiently far from the origin is $m(x) = \int_0^x \theta (1 - T(u))\, du$. Therefore the joint density function $f_k(x_1, x_2, \ldots, x_k)$ can be approximated by

$$f_k(x_1, x_2, \ldots, x_k) \approx \pi_k \theta^k \prod_{i=1}^{k} (1 - T(x_i)).$$

Using the above approximation and following the renewal arguments given in [15], we can write that $r_{0k} \approx \pi_k b_k$ where $b_k = \int_0^\infty (1 - m(t))^k \lambda e^{-\lambda t} dt$. Since $1 - m(t) < 1$, it follows that $1 > b_k > b_{k+1}$ for $k = 1, 2, \ldots$ and $\lim_{k \to \infty} b_k = 0$.

Assume that the steady-state distribution $\{a_k, k \geq 0\}$ is known (it is obtained by inversion of the generating function $\bar{\Phi}(z)$ given by (2)). Under this assumption, the result (3) can be expressed in the following form:

$$\pi_k = \frac{1}{1 - \rho} \sum_{i=0}^{k} a_i r_{0,k-i} \tag{4}$$

with

$$r_{0,k} \approx \pi_k b_k \tag{5}$$

and

$$\sum_{k=0}^{\infty} \pi_k = 1. \tag{6}$$

The set of equations (4–6) gives an approximate solution to $\{\pi_k, k \geq 0\}$. Let $\{\hat{\pi}_k, k \geq 0\}$ be the approximation to $\{\pi_k, k \geq 0\}$. With the help of (5), from (4) one can obtain

$$\hat{\pi}_k = c_k \hat{\pi}_0 \qquad k = 0, 1, 2, \ldots \tag{7}$$

where

$$c_0 = 1; \qquad c_k = \frac{1}{(1 - \rho)(1 - b_k)} \sum_{i=1}^{k} a_i b_{k-i} c_{k-i} \qquad k = 1, 2, \ldots$$

Using (7), from normalizing equation (6) one can find that the approximation to π_0 is

$$\hat{\pi}_0 = \frac{1}{\sum_{k=0}^{\infty} c_k}. \tag{8}$$

The computational procedure (7, 8) gives the unique positive solution if the series $\sum_{k=0}^{\infty} c_k$ converges. Since $1 > b_k > b_{k+1}$ for $k = 1, 2, \ldots$, the above condition is satisfied [15].

Suppose that the approximate solution $\{\hat{\pi}_k, k \geq 0\}$ is evaluated. Let N be the number of customers in the system at an arbitrary time when the server enters the idle-up state. Then, we can calculate

$$E[N] \approx \sum_{k=0}^{\infty} k\hat{\pi}_k; \qquad \mathrm{Var}[N] \approx \sum_{k=0}^{\infty} k^2\hat{\pi}_k - (E[N])^2 .$$

5. Numerical results

This section deals with numerical results made available by the computational procedure (7, 8). We consider the following service time distributions:

exponential (E) $\quad F(x) = 1 - \mathrm{e}^{-\gamma x} \quad x \geq 0;$

two-stage Erlang (E_2) $\quad F(x) = 1 - \mathrm{e}^{-2\gamma x} - 2\gamma x\mathrm{e}^{-2\gamma x} \quad x \geq 0;$

two-stage hyperexponential (H_2) $\quad F(x) = 1 - \zeta_1\mathrm{e}^{-\gamma_1 x} - \zeta_2\mathrm{e}^{-\gamma_2 x};$

$$\zeta_1 + \zeta_2 = 1; \quad \frac{\zeta_1}{\gamma_1} + \frac{\zeta_2}{\gamma_2} = \frac{1}{\gamma}; \quad x \geq 0.$$

For retrial times, we choose

exponential (E) $\quad T(x) = 1 - \mathrm{e}^{-\theta x} \quad x \geq 0;$

two-stage Erlang (E_2) $\quad T(x) = 1 - \mathrm{e}^{-2\theta x} - 2\theta x\mathrm{e}^{-2\theta x} \quad x \geq 0;$

two-stage hyperexponential (H_2) $\quad T(x) = 1 - \varsigma_1\mathrm{e}^{-\theta_1 x} - \varsigma_2\mathrm{e}^{-\theta_2 x} \quad x \geq 0;$

$$\frac{\varsigma_1}{\theta_1} + \frac{\varsigma_2}{\theta_2} = \frac{1}{\theta}; \; \varsigma_1 + \varsigma_2 = 1; \; \theta_1 = 2\varsigma_1\theta; \; \theta_2 = 2\varsigma_2\theta.$$

These distributions are the most representative. Furthermore from [15], we have that the approximation discussed in Section 4 works well as long as the retrial time distribution is relatively close to the exponential distribution in the sense that its coefficient of variation, cv, is close to that of the latter ($cv < 4$). Throughout this section, we let the mean service time, $1/\gamma$, be a unit time and the probability c be 0.9.

In the first time, we examine the performance of the approximation. Tables 1a–c, 2a, b and 3a, b present the approximate numerical results against those from a simulation study for the M/M/1, M/E_2/1 and M/H_2/1 retrial queues with breakdowns, respectively. The coefficients of variation of retrial times are $cv_{E_2} \approx 0.7$, $cv_E = 1$ and $cv_{H_2} = 1.5$. In these tables we observe a good agreement between the approximate and simulation results when there are M/M/1 and M/E_2/1 retrial queues with breakdowns for which the coefficient of variation of service times $cs \leq 1$. In the case of M/H_2/1 retrial queue with $cs = 2$, the approximation performs well, when the traffic intensity ρ is relatively low (see $\lambda = 0.3$). On the other hand, it fails when ρ is high (see $\lambda = 0.6$): the difference between the two solutions is highly significant.

TABLE 1a. The M/M/1 retrial queue with breakdowns.

$\lambda = 0.3$

θ	μ	η	$E[D_b]$	$E[D_i]$	ρ	Retrial times E_2 appr $E[N]$ / sim $E[N]$	E appr $E[N]$ / sim $E[N]$	H_2 appr $E[N]$ / sim $E[N]$
1	0.02	0.02	0.2	0.2	0.3129	0.5410	0.5684	0.6126*
						0.5501	0.5710	0.6771
	0.02	0.02	1	1	0.3176	0.5743	0.6031	0.6492*
						0.5839	0.6107	0.7175
	0.02	0.02	2	1	0.3235	0.6020	0.6313	0.6780*
						0.6138	0.6406	0.7493
3.3	0.02	0.02	0.2	0.2	0.3129	0.4677	0.4765	0.4943
						0.4656	0.4761	0.5004
	0.02	0.02	1	1	0.3176	0.4967	0.5060	0.5246
						0.4878	0.4999	0.5336
	0.02	0.02	2	1	0.3235	0.5222	0.5316	0.5504
						0.5110	0.5237	0.5630

TABLE 1b. The M/M/1 retrial queue with breakdowns.

$\lambda = 0.6$

θ	μ	η	$E[D_b]$	$E[D_i]$	ρ	Retrial times E_2 appr $E[N]$ / sim $E[N]$	E appr $E[N]$ / sim $E[N]$	H_2 appr $E[N]$ / sim $E[N]$
1	0.02	0.02	0.2	0.2	0.6082	2.3006	2.4414	2.5707*
						2.3493	2.4725	3.2444
	0.04	0.02	0.2	0.2	0.6161	2.3550	2.4999	2.6311*
						2.4179	2.5437	3.3228
	0.02	0.02	1	1	0.6176	2.4367	2.5823	2.7093*
						2.5115	2.6510	3.4378
	0.02	0.02	2	1	0.6294	2.6287	2.7786	2.8965*
						2.7036	2.8479	3.6875
	0.02	0.02	2	2		2.6678	2.8187	2.9351*
						2.7547	2.8944	3.7680
	0.2	0.2	1	1	0.75	4.3763*	4.6014*	4.6060*
						4.6301	4.8944	6.6056
3.3	0.02	0.02	0.2	0.2	0.6082	1.7492	1.7988	1.8710*
						1.7376	1.7947	2.0144
	0.04	0.02	0.2	0.2	0.6161	1.7842	1.8354	1.9095*
						1.7685	1.8251	2.0584
	0.02	0.02	1	1	0.6176	1.8556	1.9070	1.9801*
						1.8367	1.8933	2.1457
	0.02	0.02	2	1	0.6294	2.0158	2.0687	2.1406*
						1.9915	2.0453	2.3135
	0.02	0.02	2	2		2.0476	2.1009	2.1726*
						2.0114	2.0674	2.3835
	0.2	0.2	1	1	0.75	3.2816	3.3632	3.4351*
						3.3347	3.4161	3.8533

TABLE 1c. The M/M/1 retrial queue with breakdowns.

			$\lambda = 0.6$					
θ	μ	η	$E[D_b]$	$E[D_i]$	ρ	Retrial	times	
						E_2	E	H_2
						appr $E[N]$	appr $E[N]$	appr $E[N]$
						sim $E[N]$	sim $E[N]$	sim $E[N]$
10	0.02	0.02	0.2	0.2	0.6082	1.5945	1.6117	1.6414
						1.5843	1.6082	1.6587
	0.04	0.02	0.2	0.2	0.6161	1.6241	1.6419	1.6725
						1.6120	1.6352	1.6901
	0.02	0.02	1	1	0.6176	1.6925	1.7103	1.7405
						1.6792	1.7034	1.7595
	0.02	0.02	2	1	0.6294	1.8436	1.8619	1.8920
						1.8284	1.8515	1.9185
	0.02	0.02	2	2		1.8733	1.8918	1.9218
						1.8519	1.8754	1.9698
	0.2	0.2	1	1	0.75	2.9740	3.0025	3.0374
						2.9398	2.9767	3.1195

From numerical results shown in Tables 1a–c, 2a, b and 3a, b, we can see that increasing the rate of active (passive) breadowns μ (η) as well as increasing the mean time duration of active (passive) interruption $E[D_b]$ ($E[D_i]$) deteriorates the accuracy of the approximation. We also observe that the approximation fails when the mean retrial time $1/\theta$ is not sufficiently small relative to the mean service time $1/\gamma$ (the failure is denoted by *). On the other hand, as is expected, increasing the retrial rate θ results in a sensitive improvement of its performance.

Another observation is that the accuracy of the approximation deteriorates as the retrial time distribution departs from the exponential one in the sense that its coefficient of variation $cv > 1$.

Finally, one can see that the mean number of customers in the system at an arbitrary idle-up epoch $E[N]$ shares a similar property with that of the mean number of customers in the system observed in [15] for classical retrial queues: $E[N]$ is an increasing function of the second moment of the retrial time distribution. In Section 1, it was assumed that no breakdown occurs during interruption. The mean time between two consecutive active (passive) breakdowns is $1/\mu$ ($1/\eta$). If we choose the values of $E[D_b] > 1/\mu$ and (or) $E[D_i] > 1/\eta$, the above property does not hold (see Tab. 4).

Now we study the effects of the retrial rate and those of the breakdowns on the mean number of customers in the system at an arbitrary idle-up epoch $E[N]$. From numerical results shown in Tables 1a–c, 2a, b and 3a, b, we can see that $E[N]$ seems not to be affected very much when the rate of passive breakdowns η increases while, on the other hand, $E[N]$ is significantly affected by the increase of the mean time duration of passive interruption $E[D_i]$. The traffic intensity

Table 2a. The M/E_2/1 retrial queue with breakdowns.

$\lambda = 0.3$								
θ	μ	η	$E[D_b]$	$E[D_i]$	ρ	Retrial	times	
						E_2	E	H_2
						appr $E[N]$	appr $E[N]$	appr $E[N]$
						sim $E[N]$	sim $E[N]$	sim $E[N]$
1	0.02	0.02	0.2	0.2	0.31	0.5118	0.5399	0.5859*
						0.5211	0.5480	0.6528
	0.02	0.02	1	1	0.3192	0.5428	0.5724	0.6203*
						0.5540	0.5820	0.6967
	0.02	0.02	2	1	0.3251	0.5679	0.5980	0.6465*
						0.5801	0.6086	0.7288
3.3	0.02	0.02	0.2	0.2	0.31	0.4383	0.4474	0.4659
						0.4344	0.4454	0.4740
	0.02	0.02	1	1	0.3192	0.4651	0.4766	0.4940
						0.4568	0.4703	0.5023
	0.02	0.02	2	1	0.3251	0.4879	0.4976	0.5172
						0.4786	0.4900	0.5266

Table 2b. The M/E_2/1 retrial queue with breakdowns.

$\lambda = 0.6$								
θ	μ	η	$E[D_b]$	$E[D_i]$	ρ	Retrial	times	
						E_2	E	H_2
						appr $E[N]$	appr $E[N]$	appr $E[N]$
						sim $E[N]$	sim $E[N]$	sim $E[N]$
1	0.02	0.02	0.2	0.2	0.61	2.0957*	2.2434*	2.3925*
						2.1656	2.3065	3.1457
3.3	0.02	0.02	0.2	0.2	0.61	1.5401	1.5927	1.6733*
						1.5303	1.5872	1.8225
	0.04	0.02	0.2	0.2	0.6220	1.5930	1.6477	1.7310*
						1.5731	1.6369	1.8958
	0.02	0.02	1	1	0.6206	1.6328	1.6873	1.7695*
						1.6162	1.6761	1.9283
	0.02	0.02	2	1	0.6324	1.7688	1.8250	1.9074*
						1.7449	1.8136	2.0799
	0.02	0.02	2	2		1.7939	1.8504	1.9329*
						1.7612	1.8275	2.1170
	0.2	0.2	1	1	0.7809	3.3719	3.4688	3.5553*
						3.4612	3.5399	4.0358
10	0.02	0.02	0.2	0.2	0.61	1.3849	1.4031	1.4364
						1.3781	1.3976	1.4475
	0.04	0.02	0.2	0.2	0.6220	1.4302	1.4492	1.4837
						1.4123	1.4382	1.5025
	0.02	0.02	1	1	0.6206	1.4691	1.4880	1.5220
						1.4548	1.4776	1.5379
	0.02	0.02	2	1	0.6324	1.5960	1.6155	1.6497
						1.5752	1.6044	1.6744
	0.02	0.02	2	2		1.6189	1.6386	1.6729
						1.5933	1.6220	1.7159
	0.2	0.2	1	1	0.7809	3.0060	3.0402	3.0833
						2.9702	3.0138	3.1588

TABLE 3a. The M/H_2/1 retrial queue with breakdowns.

			$\lambda = 0.3$					
θ	μ	η	$E[D_b]$	$E[D_i]$	ρ	Retrial	times	
						E_2	E	H_2
						appr $E[N]$	appr $E[N]$	appr $E[N]$
						sim $E[N]$	sim $E[N]$	sim $E[N]$
1	0.02	0.02	0.2	0.2	≈0.30	0.6574	0.6782	0.7178*
						0.6724	0.6929	0.7971
	0.02	0.02	1	1	0.3016	0.7087	0.7337	0.7716*
						0.6914	0.7210	0.8331
	0.02	0.02	2	1	0.3072	0.7466	0.7721	0.8100*
						0.7248	0.7566	0.8554
3.3	0.02	0.02	0.2	0.2	≈0.30	0.5884	0.5960	0.6106
						0.5938	0.5998	0.6227
	0.02	0.02	1	1	0.3016	0.6384	0.6434	0.6586
						0.6254	0.6361	0.6727
	0.02	0.02	2	1	0.3072	0.6713	0.6795	0.6946
						0.6573	0.6713	0.7112

TABLE 3b. The M/H_2/1 retrial queue with breakdowns.

			$\lambda = 0.6$					
θ	μ	η	$E[D_b]$	$E[D_i]$	ρ	Retrial	times	
						E_2	E	H_2
						appr $E[N]$	appr $E[N]$	appr $E[N]$
						sim $E[N]$	sim $E[N]$	sim $E[N]$
1	0.02	0.02	0.2	0.2	≈0.60	3.0530	3.1686	3.1990
						3.3042	3.4969	3.7919
3.3	0.02	0.02	0.2	0.2		2.5635	2.6031	2.6379
						2.8139	2.8536	2.9054
10	0.02	0.02	0.2	0.2		2.4248	2.4384	2.4542
						2.7035	2.7224	2.8039
50	0.02	0.02	0.2	0.2		2.3708	2.3735	2.3771
						2.6234	2.6733	2.7232

TABLE 4. The M/M/1 retrial queue with breakdowns.

	$\mu = \eta = 1$; $E[D_b] = E[D_i] = 2$.				
λ	ρ	θ	Retrial	times	
			E_2	E	H_2
			$cv \approx 0.7$	$cv = 1$	$cv = 1.5$
			appr $E[N]$	appr $E[N]$	appr $E[N]$
0.3	0.9	1	8.8876	9.2112	8.6167
		10	6.8784	6.9141	6.8948
		100	6.6792	6.6828	6.6815

ρ (given by (1)) as well as $E[N]$ are very sensitive in changes of the rate of active breakdowns μ and above all of the mean time duration of active interruption $E[D_b]$. These changes can increase drastically the value of ρ as well as of $E[N]$. Finally, the performance of the system deteriorates as the mean retrial time $1/\theta$ approachs the mean service time $1/\gamma$.

We conclude that the performance of the approximation discussed in Section 4 is affected very much by the type of service time distribution as well as by the type of retrial time distribution. In other words, increasing the coefficient of variation of service times and that of retrial times has significant adverse influence on the accuracy of this approximation. The approximation method works well as long as the mean retrial time is sufficiently inferior to the mean service time. Ultimately, the breakdowns have an adverse effect on the accuracy of the approximation.

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