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Complex analysis

On the localization of the minimum integral related to the weighted Bergman kernel and its application

Sur la localisation de l'intégrale minimum liée au noyau de Bergman à poids et son application

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A B S T R A C T

In this note, under an additional condition, we present an alternative proof of a stability theorem for the boundary asymptotics of the Bergman kernel due to T. Ohsawa. Our method relies on the localization of the minimum integral related to the weighted Bergman kernel.

R É S U M É

Dans cette note, nous présentons, sous une certaine condition additionnelle, une preuve alternative d'un théorème de stabilité pour le comportement asymptotique à la frontière du noyau de Bergman, démontré antérieurement par T. Ohsawa. Notre méthode s'appuie sur la localisation de l'intégrale minimale liée au noyau de Bergman à poids.

1. Introduction and main results

In complex differential geometry, the study of holomorphic invariants has been motivated by a desire to be able to comprehend intrinsic values of sub-domains of complex manifolds. For this reason, the Bergman kernel and its boundary behavior have been extensively studied on various types of domains in $\mathbb{C}^n$, including pseudoconvex domains and those of D'Angelo finite type (see [2,7] and the references therein). In particular, the boundary behavior of the Bergman kernel of weakly pseudoconvex domains in $\mathbb{C}^n$ motivated the celebrated Ohsawa–Takegoshi L^2 extension theorem [8] on Stein manifolds. Recently, Błocki [1] and Guan, Zhou, and Zhu [3] obtained independently sharp estimates on this L^2 extension theorem. These results have rekindled the interest in the L^2 extension theorem and its application. For instance, Ohsawa posed the following problem.

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Problem. *Let $D = \{z : \rho(z) < 0\}$ be a domain in $\mathbb{C}^n$ with a C^2-smooth boundary, and let D_0 be a sub-domain of D defined by*

$$D_0 = \{z = (z', z_n) \in \mathbb{C}^{n-1} \times \mathbb{C} : \psi(z) + \log|z_n|^2 < 0\},$$

where $\psi(w) = -\log(-\rho(w))$ for all $w \in D$. Then, find a condition so that, up to constant multiples,

$$\lim_{(z',0)\to\partial D_0\cap\{z_n=0\}} \frac{K_{D_0}((z',0),(z',0))}{K_{D',\psi}(z',z')} = 1,$$

where $K_{D',\psi}(z', w')$ is the weighted Bergman kernel of $D' = D \cap \{z_n = 0\}$ with respect to the weight ψ.

T. Ohsawa recently gave an affirmative answer to this problem together with a refinement of his extension result with negligible weights (see [7] and the references therein). In this note, we present yet another proof of his answer to the above problem with a further condition that $\partial D_0 \cap \{z_n = 0\}$ are local holomorphic peak points. Our method essentially relies on a localizing argument of the minimum integral related to the weighted Bergman kernel.

We now briefly review some basics on a weighted version of the minimum integral below.

Definition 1. *Let Ω be a domain in $\mathbb{C}^n$ and φ any continuous function in Ω. For a fixed $\zeta \in \Omega$, we define a minimum integral $I_0^{\Omega,\varphi}(\zeta)$ of Ω with the weight φ by setting*

$$I_0^{\Omega,\varphi}(\zeta) = \inf\left\{\int_\Omega |f(z)|^2 e^{-\varphi(z)} d\lambda(z) : f \in A^2_\varphi(\Omega) \text{ and } f(\zeta) = 1\right\},$$

where $d\lambda$ is the Lebesgue measure on Ω and $A^2_\varphi(\Omega) := \mathcal{O}(\Omega) \cap L^2_\varphi(\Omega)$.

In a similar fashion to the unweighted case, the weighted Bergman kernel $K_{\Omega,\varphi}$ on the diagonal can be represented as follows.

Proposition 2. *Let Ω and φ be as above. Then the weighted Bergman kernel $K_{\Omega,\varphi}$ satisfies*

$$K_{\Omega,\varphi}(z,z) = \sup\left\{|f(z)|^2 : f \in A^2_\varphi(\Omega) \text{ and } \|f\|_{L^2_\varphi(\Omega)} \le 1\right\}.$$

Proof. For each $f \in A^2_\varphi(\Omega)$ such that $\|f\|_{L^2_\varphi(\Omega)} \le 1$, the reproducing property of $K_{\Omega,\varphi}$ and the Cauchy–Schwarz inequality imply

$$|f(z)|^2 = |\langle f, K_{\Omega,\varphi}(\cdot,z)\rangle_{L^2_\varphi(\Omega)}|^2 \le \|f\|^2_{L^2_\varphi(\Omega)} \|K_{\Omega,\varphi}(\cdot,z)\|^2_{L^2_\varphi(\Omega)} = \|f\|^2_{L^2_\varphi(\Omega)} K_{\Omega,\varphi}(z,z) \le K_{\Omega,\varphi}(z,z). \tag{1}$$

This observation ensures that, if $K_{\Omega,\varphi}(z,z) = 0$, then $f(z) \equiv 0$ for all $f \in A^2_\varphi(\Omega)$ such that $\|f\|^2_{L^2_\varphi(\Omega)} \le 1$. In the case when $K_{\Omega,\varphi}(z,z) > 0$, we define a function g by setting $g(p) = K_{\Omega,\varphi}(p,z)/\sqrt{K_{\Omega,\varphi}(z,z)}$ for each $p \in \Omega$. Then g satisfies the following:

$$|g(z)|^2 = K_{\Omega,\varphi}(z,z) \quad \text{and} \quad \|g\|^2_{L^2_\varphi(\Omega)} = 1.$$

This, in conjunction with (1), completes the proof. □

Moreover, the following proposition shows that the minimum integral $I_0^{\Omega,\varphi}$ can be viewed as the reciprocal of the weighted Bergman kernel on the diagonal, provided $K_{\Omega,\varphi}(p,p)$ does not vanish at $p \in \Omega$.

Proposition 3. *Let Ω and φ be as above. Then the minimum integral $I_0^{\Omega,\varphi}$ satisfies*

$$I_0^{\Omega,\varphi}(\zeta) = \frac{1}{K_{\Omega,\varphi}(\zeta,\zeta)},$$

whenever $K_{\Omega,\varphi}(\zeta,\zeta) \ne 0$ for all $\zeta \in \Omega$.

Proof. Let us first fix a point $\zeta \in \Omega$. Then we define a function f by setting $f(\xi) = K_{\Omega,\varphi}(\xi,\zeta)/K_{\Omega,\varphi}(\zeta,\zeta)$ for each $\xi \in \Omega$. This function f clearly satisfies $f(\zeta) = 1$. Since moreover

$$\int_\Omega |f(\xi)|^2 e^{-\varphi(\xi)} d\lambda(\xi) = \frac{1}{K_{\Omega,\varphi}(\zeta,\zeta)},$$

it follows that $I_0^{\Omega,\varphi}(\zeta) \le 1/K_{\Omega,\varphi}(\zeta,\zeta)$. For the opposite inequality, we note that for each $z \in \Omega$,

$$|f(z)|^2 = |\langle f, K_{\Omega,\varphi}(\cdot,z)\rangle_{L^2_\varphi(\Omega)}|^2 \le \|f\|^2_{L^2_\varphi(\Omega)} \|K_{\Omega,\varphi}(\cdot,z)\|^2_{L^2_\varphi(\Omega)} = \|f\|^2_{L^2_\varphi(\Omega)} K_{\Omega,\varphi}(z,z). \tag{2}$$

Then substitute $z = \zeta$ into the relation (2) to obtain

$$\frac{1}{K_{\Omega,\varphi}(\zeta,\zeta)} = \frac{|f(\zeta)|^2}{K_{\Omega,\varphi}(\zeta,\zeta)} \le \|f\|^2_{L^2_\varphi(\Omega)}. \tag{3}$$

Thus taking the infimum of the right-hand side of (3), we eventually reach that

$$\frac{1}{K_{\Omega,\varphi}(\zeta,\zeta)} \le I_0^{\Omega,\varphi}(\zeta),$$

as desired. □

We now state our first main result, which is a weighted version of the localization of the minimum integral in [5].

Theorem 4. *Let Ω be a smoothly bounded pseudoconvex domain in $\mathbb{C}^n$. Suppose that $p \in \partial\Omega$ is a local holomorphic peak point and φ is plurisubharmonic on Ω. Then for any neighborhood $\mathcal{U}$ of p in $\mathbb{C}^n$, we have*

$$\lim_{\zeta \to p} \frac{I_0^{\Omega\cap\mathcal{U},\varphi}(\zeta)}{I_0^{\Omega,\varphi}(\zeta)} = 1. \tag{4}$$

Exploiting this localization argument of the minimum integral related to the weighted Bergman kernel, we next come to the following second main result of this note.

Theorem 5. *Let Ω be a smoothly bounded pseudoconvex domain in $\mathbb{C}^n$ and ρ a defining function of Ω. Let H_Ω be a subset of a complex hypersurface defined by*

$$H_\Omega = \Omega \cap \left\{ z = (z', z_n) \in \mathbb{C}^{n-1} \times \mathbb{C} : H(z) = 0 \right\} \ne \emptyset$$

for a continuous function H. We denote by $\widetilde{\Omega}$ a sub-domain of Ω such that

$$\widetilde{\Omega} = \{ z \in \Omega : -\log(-\rho(z)) + 2\log|H(z)| < 0 \}$$

and $\psi(z) := -\log(-\rho(z))$ is plurisubharmonic on $\widetilde{\Omega}$. In particular, we have $H_\Omega = H_{\widetilde{\Omega}}$. Suppose that $p \in \partial\Omega \cap \overline{H_\Omega}$ is a local holomorphic peak point; there exist a neighborhood $\mathcal{U}$ of p and a biholomorphism Φ from $\widetilde{\Omega} \cap \mathcal{U}$ onto a bounded Hartogs domain

$$D := \left\{ (\tilde{z}', \tilde{z}_n) \in P_1 \Phi(H_{\widetilde{\Omega}} \cap \mathcal{U}) \times \mathbb{C} \subset \mathbb{C}^{n-1} \times \mathbb{C} : |\tilde{z}_n|^2 < \psi \circ \Phi^{-1}(\tilde{z}', 0) \text{ for a projection } P_1(\tilde{z}', \tilde{z}_n) := \tilde{z}' \right\}$$

satisfying that $\tilde{\psi}\Big|_{D \cap \{\tilde{z}_n = 0\}} := \psi|_{H_\Omega \cap \mathcal{U}} \circ \Phi^{-1}$ and $\Phi(p) \in \partial D \cap \overline{\Phi(H_\Omega \cap \mathcal{U})}$. Then we have

$$\lim_{\zeta \to p} \frac{K_{\widetilde{\Omega}}(\zeta,\zeta)}{K_{H_\Omega,\psi}(\zeta,\zeta)} = \frac{1}{\pi}.$$

We remark that in the case when $\widetilde{\Omega} = D$ and $H(z', z_n) = z_n$, the conclusion of Theorem 5 reduces to the conclusion of Theorem 0.2 of T. Ohsawa [7] with a further assumption that p is a local holomorphic peak point which is necessary for the localization arguments in this note.

2. Proofs

2.1. Proof of Theorem 4

Throughout what follows, we use the same notation as in Theorem 4. Let h be a local holomorphic peak function at $p \in \partial\Omega$ and $\mathcal{U}$ the associated neighborhood of the point p as above. Now we choose another open neighborhood $\mathcal{V}$ of p such that $\mathcal{V} \Subset \mathcal{U}$ and $h \ne 0$ on $\mathcal{V}$. Then there is a constant $s \in (0,1)$ such that $|h| \le s$ on the closure $\overline{\Omega \cap (\mathcal{U} \setminus \mathcal{V})}$. Let us next choose a cut-off function $\chi \in C_0^\infty(\mathcal{U})$ such that $\chi = 1$ on $\mathcal{V}$ and $0 \le \chi \le 1$ on $\mathcal{U}$.

Now we shall utilize a L. Hörmander's estimate in [4] to get an upper bound of the ratio of the minimum integrals with a weight function in (4). In order to take this end, we first fix a point $\zeta \in \mathcal{V}$. Let us define a plurisubharmonic function $\tilde{\varphi}$ on Ω by setting $\tilde{\varphi}(z) = (2n+2)\log|z - \zeta|$ for the previously fixed point $\zeta \in \mathcal{V}$. Since φ is chosen as a plurisubharmonic function on Ω in our assumption, we note that $\varphi+\tilde{\varphi}$ is also plurisubharmonic on Ω. Given any function $f \in A^2_\varphi(\Omega \cap \mathcal{U})$, we define a smooth closed $(0,1)$-form $\alpha \in L^2_\varphi(\Omega) \cap L^2_{\varphi+\tilde{\varphi}}(\Omega)$ by setting $\alpha = \bar{\partial}(\chi f h^k)$ for each $k \geq 1$. Since α is indeed equal to $(\bar{\partial}\chi) f h^k$ from its definition, it follows that

$$\operatorname{supp}(\alpha) \subset \Omega \cap (\mathcal{U} \backslash \mathcal{V}). \tag{5}$$

By adopting Theorem 4.4.2 of L. Hörmander [4], we obtain a solution u to the equation $\bar{\partial} u = \alpha$ on Ω such that

$$\int_\Omega |u(z)|^2 e^{-\varphi(z)-\tilde{\varphi}(z)} (1+|z|^2)^{-2} d\lambda(z) \leq \int_\Omega |\alpha(z)|^2 e^{-\varphi(z)-\tilde{\varphi}(z)} d\lambda(z) < \infty. \tag{6}$$

This relation also holds if we replace $\varphi+\tilde{\varphi}$ by φ. The choice of $f \in A^2_\varphi(\Omega \cap \mathcal{U})$ and the relation (5) ensure the finiteness of the integral in (6). Taking the infimum of $1/(1+|z|^2)^2$ on Ω in (6), one can show that

$$\int_\Omega |u(z)|^2 e^{-\varphi(z)-\tilde{\varphi}(z)} d\lambda(z) \leq \sup_{z\in\Omega} (1+|z|^2)^2 \int_\Omega |\alpha(z)|^2 e^{-\varphi(z)-\tilde{\varphi}(z)} d\lambda(z).$$

We denote by C and $\widetilde{C}$ the values of $\sup_{z\in\Omega}(1+|z|^2)^2$ and $\sup_{z\in\Omega\cap(\mathcal{U}\backslash\mathcal{V})} 1/|z-\zeta|^{2n+2}$, respectively. Since $\alpha \in L^2_\varphi(\Omega) \cap L^2_{\varphi+\tilde{\varphi}}(\Omega)$, we see that

$$\int_\Omega |u(z)|^2 e^{-\varphi(z)-\tilde{\varphi}(z)} d\lambda(z) \leq C \int_\Omega |\alpha(z)|^2 e^{-\varphi(z)-\tilde{\alpha}(z)} d\lambda(z) \leq C\widetilde{C} \int_{\Omega\cap(\mathcal{U}\backslash\mathcal{V})} |\alpha(z)|^2 e^{-\varphi(z)} d\lambda(z) < \infty. \tag{7}$$

In particular, (7) forces u to satisfy $u(\zeta)=0$ for the fixed point $\zeta \in \mathcal{V}$. Moreover, the boundedness of Ω implies that there exists a constant $C_1 > 0$ such that

$$\int_\Omega |u(z)|^2 e^{-\varphi(z)} (1+|z|^2)^{-2} d\lambda(z) \geq C_1 \int_\Omega |u|^2 e^{-\varphi(z)} d\lambda(z). \tag{8}$$

From (5) and the fact that $|h| \leq s \in (0,1)$, it follows that there exists a constant $C_2 > 0$ such that

$$\begin{aligned}
\int_\Omega |\alpha(z)|^2 e^{-\varphi(z)} d\lambda(z) &= \int_{\Omega\cap(\mathcal{U}\backslash\mathcal{V})} |\bar{\partial}\chi(z)|^2 |f(z)|^2 |h(z)|^{2k} e^{-\varphi(z)} d\lambda(z) \\
&\leq C_2 \int_{\Omega\cap(\mathcal{U}\backslash\mathcal{V})} |f(z)|^2 |h(z)|^{2k} e^{-\varphi(z)} d\lambda(z) \\
&\leq C_2 s^{2k} \|f\|^2_{L^2_\varphi(\Omega\cap\mathcal{U})}.
\end{aligned} \tag{9}$$

Combining (8) with (9), we deduce that

$$\begin{aligned}
\int_\Omega |u(z)|^2 e^{-\varphi(z)} d\lambda(z) &\leq \frac{1}{C_1} \int_\Omega |u(z)|^2 e^{-\varphi(z)} (1+|z|^2)^{-2} d\lambda(z) \\
&\leq \frac{1}{C_1} \int_\Omega |\alpha(z)|^2 e^{-\varphi(z)} d\lambda(z) \\
&\leq \frac{C_2}{C_1} s^{2k} \|f\|^2_{L^2_\varphi(\Omega\cap\mathcal{U})}.
\end{aligned} \tag{10}$$

Now we shall define a function F_k on Ω by setting $F_k = \chi f h^k - u$ for each $k \geq 1$. Then the linearity of $A^2_\varphi(\Omega)$ yields $F_k \in A^2_\varphi(\Omega)$. More precisely, using (10), we have

$$\begin{aligned}
\|F_k\|_{L^2_\varphi(\Omega)} &\leq \|\chi f h^k\|_{L^2_\varphi(\Omega)} + \|u\|_{L^2_\varphi(\Omega)} \\
&\leq \|f h^k\|_{L^2_\varphi(\Omega\cap\mathcal{U})} + \|u\|_{L^2_\varphi(\Omega)} \\
&\leq \|f\|_{L^2_\varphi(\Omega\cap\mathcal{U})} + \|u\|_{L^2_\varphi(\Omega)} \\
&\leq \left(1 + \sqrt{\frac{C_2}{C_1} s^{2k}}\right) \|f\|_{L^2_\varphi(\Omega\cap\mathcal{U})}.
\end{aligned} \tag{11}$$

We shall choose a function f so that f is the minimizing function for $I_0^{\Omega\cap\mathcal{U}}(\zeta)$, that is, $\|f\|^2_{L^2_\varphi(\Omega\cap\mathcal{U})} = I_0^{\Omega\cap\mathcal{U},\varphi}(\zeta)$ and $f(\zeta)=1$. Let us define a function g on Ω by setting $g = \frac{F_k}{h^k(\zeta)}$. Then this function g satisfies that $g \in A^2_\varphi(\Omega)$ and $g(\zeta)=1$. From the property of the minimum integral in Definition 1, it follows that

$$I_0^{\Omega,\varphi}(\zeta) \le \|g\|^2_{L^2_\varphi(\Omega)} \le \frac{\left(1+\sqrt{\frac{C_2}{C_1}s^{2k}}\right)^2}{|h(\zeta)|^{2k}}\|f\|^2_{L^2_\varphi(\Omega\cap\mathcal{U})} = \frac{\left(1+\sqrt{\frac{C_2}{C_1}s^{2k}}\right)^2}{|h(\zeta)|^{2k}} I_0^{\Omega\cap\mathcal{U},\varphi}(\zeta);$$

hence,

$$\frac{I_0^{\Omega,\varphi}(\zeta)}{I_0^{\Omega\cap\mathcal{U},\varphi}(\zeta)} \le \frac{\left(1+\sqrt{\frac{C_2}{C_1}s^{2k}}\right)^2}{|h(\zeta)|^{2k}}. \tag{12}$$

Since the function h is chosen as a local holomorphic peak function at $p \in \partial\Omega$, (12) implies that

$$\limsup_{\zeta\to p} \frac{I_0^{\Omega,\varphi}(\zeta)}{I_0^{\Omega\cap\mathcal{U},\varphi}(\zeta)} \le \left(1+\sqrt{\frac{C_2}{C_1}s^{2k}}\right)^2. \tag{13}$$

Then, letting $k \to +\infty$ in (13), we get

$$\limsup_{\zeta\to p} \frac{I_0^{\Omega,\varphi}(\zeta)}{I_0^{\Omega\cap\mathcal{U},\varphi}(\zeta)} \le 1. \tag{14}$$

In addition, it is inferred from the monotone increasing property of the minimum integral that the opposite inequality

$$\frac{I_0^{\Omega,\varphi}(\zeta)}{I_0^{\Omega\cap\mathcal{U},\varphi}(\zeta)} \ge 1.$$

Thus, combining (14) with the previous relation, we complete the proof.

2.2. Proof of Theorem 5

Throughout what follows, we use the same notation as in Theorem 5. By employing the localization argument of the minimum integral related to the Bergman kernel in [5], we note that

$$K_{\widetilde{\Omega}}(\zeta,\zeta) = K_{\widetilde{\Omega}\cap\mathcal{U}}(\zeta,\zeta) \tag{15}$$

as ζ tends to p. Then, combining Theorem 4 with (15), it follows that

$$\lim_{\zeta\to p}\frac{K_{\widetilde{\Omega}}(\zeta,\zeta)}{K_{H_\Omega,\psi}(\zeta,\zeta)} = \lim_{\zeta\to p}\frac{K_{\widetilde{\Omega}\cap\mathcal{U}}(\zeta,\zeta)}{K_{H_\Omega\cap\mathcal{U},\psi}(\zeta,\zeta)}. \tag{16}$$

Applying the transformation formulas for the Bergman kernel and the weighted Bergman kernel to (16), one can deduce that

$$\lim_{\zeta\to p}\frac{K_{\widetilde{\Omega}\cap\mathcal{U}}(\zeta,\zeta)}{K_{H_\Omega\cap\mathcal{U},\psi}(\zeta,\zeta)} = \lim_{\zeta\to p}\frac{|\det \mathrm{Jac}(\Phi,\zeta)|^2 K_{\Phi(\widetilde{\Omega}\cap\mathcal{U})}(\Phi(\zeta),\Phi(\zeta))}{|\det \mathrm{Jac}(\Phi,\zeta)|^2 K_{\Phi(H_\Omega\cap\mathcal{U}),\tilde{\psi}}(\Phi(\zeta),\Phi(\zeta))} = \lim_{\zeta\to p}\frac{K_D(\Phi(\zeta),\Phi(\zeta))}{K_{D\cap\{|\tilde{z}_n|=0\},\tilde{\psi}}(\Phi(\zeta),\Phi(\zeta))}.$$

Thus we conclude that

$$\lim_{\zeta\to p}\frac{K_{\widetilde{\Omega}}(\zeta,\zeta)}{K_{H_\Omega,\psi}(\zeta,\zeta)} = \lim_{\zeta\to p}\frac{K_D(\Phi(\zeta),\Phi(\zeta))}{K_{D\cap\{|\tilde{z}_n|=0\},\tilde{\psi}}(\Phi(\zeta),\Phi(\zeta))} = \frac{1}{\pi}$$

by using the Forelli–Rudin construction in [6].

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