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Partial differential equations

Null controllability for the semilinear heat equation in a non-smooth domain

Nulle contrôlabilité de l'équation de la chaleur semilinéaire dans un domaine non régulier

Tarik Ali-Ziane, Zahia Ferhoune, Ouahiba Zair

Laboratoire AMNEDP, Faculté de Mathématiques, USTHB, BP 32, El-Alia, 16111 Alger, Algeria

A R T I C L E I N F O

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A B S T R A C T

In this work we give a null-controllability result for the semi-linear heat equation in a **polygonal** or **cracked** bounded domain of $\mathbb{R}^2$. We suppose that the nonlinearity grows slower than $|s|\log^{3/2}(1+|s|)$ as $|s| \to \infty$ and then we prove our result by using Schauder's fixed point theorem.

R É S U M É

Dans ce travail, on donne un résultat de nulle contrôlabilité pour l'équation de la chaleur semi-linéaire dans un domaine borné de $\mathbb{R}^2$, **polygonal** ou **fissuré**. On suppose que la non-linearité croît moins vite que $|s|\log^{3/2}(1+|s|)$ quand $|s| \to \infty$, et on démontre le résultat par le théorème du point fixe de Schauder.

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1. Introduction

Let Ω be an open bounded subset of $\mathbb{R}^2$ with boundary $\Gamma = \partial\Omega$; we consider two cases:

1. Case 1: We suppose that Ω has a **polygonal** reentrant corner at point S ($S \in \Gamma$) with measure ϕ, $\pi < \phi < 2\pi$, and that $\Gamma \setminus \{S\}$ is regular.
2. Case 2: We suppose that Ω has one straight emerging **crack** σ, we denote by S its tip and by Γ_1 the part $\Gamma \setminus \sigma$, which is assumed to be smooth.

It is known that the geometry of Ω, described above, affects the domain of the Laplacian operator. In fact, it is not contained in $H^2(\Omega) \cap H_0^1(\Omega)$, but in the space $H^l(\Omega) \cap H_0^1(\Omega)$, $\frac{3}{2} < l < 2$ in case 1 and $1 < l < \frac{3}{2}$ in case 2; for more details see [6,7]. So this fact has consequences on the regularity of the solution to the problem we will consider.

E-mail address: ferzahia@yahoo.fr (Z. Ferhoune).

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For $T > 0$, we denote by Q_T and Σ_T the sets $(0,T)\times\Omega$ and $(0,T)\times\Gamma$, respectively, and we consider the semi-linear heat equation:

$$\begin{cases} \partial_t y - \Delta y + f(y) = v 1_\omega & \text{in } Q_T, \\ y = 0 & \text{on } \Sigma_T, \\ y(0,.) = y_0 & \text{in } \Omega, \end{cases} \tag{1}$$

where y is the state, $v \in L^r((0,T)\times\omega)$ with $r > 2$ is the control acting on the system through a non-empty subset ω of Ω, 1_ω is the characteristic function of ω, and $f:\mathbb{R}\to\mathbb{R}$ is supposed to be locally a Lipschitz-continuous function, $f(0)=0$ and checks that for each $\eta > 0$, there exists $C_\eta > 0$ such that

$$\left|\frac{f(s)}{s}\right|^{2/3} \le C_\eta + \eta \log\bigl(1+|s|\bigr), \quad \forall s \in \mathbb{R}. \tag{2}$$

The main goal of this paper is to study the null-controllability of (1) at time T, which means that for any initial condition $y_0 \in L^2(\Omega)$, we look at whether there exists a control $v \in L^r((0,T)\times\omega)$, $r > 2$, such that the corresponding boundary problem (1) admits a solution $y \in C^0([0,T]; L^2(\Omega))$ satisfying

$$y(x,T) = 0 \quad \text{in } \Omega. \tag{3}$$

When the domain Ω is regular, at least C^2, a lot of research is done to study the controllability of different problems, among them the semi-linear heat equations. In this case, we can firstly cite [4], where the approximate controllability is established. In [9], the author has studied approximate controllability with globally Lipschitz nonlinearities using the fixed-point method. We also cite [5], in which the authors proved exact controllability to trajectories where the non-linearity checks condition (2). They have used Carleman's estimates and Kakutani's fixed point theorem. In [3], the authors were interested by the exact controllability to trajectories with discontinuous diffusion coefficients. In [2], the authors have established a null-controllability result for the linear heat equation in polygonal or cracked domains of $\mathbb{R}^2$. They were able to justify Carleman's estimate by building a suitable weight function. Our work is a continuation of theirs. It consists in proving a similar result in the semi-linear case. We prove a global Carleman inequality for the linear problem with a potential. As in [2], the loss of regularity due to the geometry of the domain prevents us from doing some integrations by parts, so we use a density result. In case 2, we have circumvented the tip of the crack and worked in a sub-domain with a Lipschitzian boundary.

The main result of this paper is stated in the following theorem.

Theorem 1.1. *Let $T > 0$, $f:\mathbb{R}\to\mathbb{R}$ locally Lipschitz-continuous function such that*

$$\forall \eta > 0,\ \exists C_\eta > 0 : \left|\frac{f(s)}{s}\right|^{2/3} \le C_\eta + \eta \log\bigl(1+|s|\bigr), \quad \forall s \in \mathbb{R} \tag{4}$$

and $f(0)=0$. Then for each $y_0 \in L^2(\Omega)$, Problem (1) is null controllable at time T.

The proof we will provide is similar to the one given in [3], and can be reduced into three steps. In the first one, using a refined observability inequality resulting from Carleman's estimate, we prove an approximate controllability to the zero state for the linearized problem. In the second step, we will use uniform estimates deduced in the first one and from the fixed-point method, then we will get the approximate controllability result for the semi-linear case at a time lower than T. Finally, we pass to the limit and obtain the main result.

2. Proof of Theorem 1.1

2.1. Approximate controllability to zero for the linear problem

For $b \in L^\infty(Q_T)$, $y_0, q_T \in L^2(\Omega)$ we consider the following linear problem

$$\begin{cases} \partial_t y - \Delta y + by = v 1_\omega & \text{in } Q_T, \\ y = 0 & \text{on } \Sigma_T, \\ y(0,.) = y_0 & \text{in } \Omega, \end{cases} \tag{5}$$

and its adjoint problem

$$\begin{cases} -\partial_t q - \Delta q + bq = 0 & \text{in } Q_T, \\ q = 0 & \text{on } \Sigma_T, \\ q(T,.) = q_T & \text{in } \Omega. \end{cases} \tag{6}$$

Theorem 2.1. *For each $\varepsilon > 0$, there exists a control $v_\varepsilon \in L^r((0,T)\times\omega)$ such that the corresponding solution y_ε to problem (5) verifies*

$$\|y_\varepsilon(T,.)\|_{L^2(\Omega)} \le \varepsilon. \tag{7}$$

Moreover, v_ε can be chosen unique and satisfying the estimate

$$\|v_\varepsilon\|_{L^r((0,T)\times\omega)} \le C\|y_0\|_{L^2(\Omega)}, \quad C = e^{C(\Omega,\omega)(1+\frac{1}{T}+T+(T+T^{\frac{1}{2}})\|b\|_\infty+\|b\|_\infty^{\frac{2}{3}})}. \tag{8}$$

Proof. Let $\varepsilon > 0$ and consider the functional $J_\varepsilon : L^2(\Omega) \longmapsto \mathbb{R}$, defined by:

$$J_\varepsilon(q_T) = \frac{1}{2}\left(\int\limits_{(0,T)\times\omega} |q|^{r'}\,dt\,dx\right)^{2/r'} + \varepsilon\|q_T\|_{L^2(\Omega)} + \int\limits_\Omega q(0,x)y_0(x)dx. \tag{9}$$

Here, q is the solution to (6) associated with the datum q_T, r' is the dual exponent to r. It is not difficult to see that J_ε is a continuous and strictly convex function, and arguing as in [9], we deduce that it is coercive. Therefore it achieves its minimum at a unique point $\widehat{q}_{T,\varepsilon} \in L^2(\Omega)$. Let $\widehat{q}_\varepsilon$ be the solution to (6) associated with $\widehat{q}_{T,\varepsilon}$. We take in (5) $v = v_\varepsilon$, where

$$v_\varepsilon = \operatorname{sgn}(\widehat{q}_\varepsilon)|\widehat{q}_\varepsilon|^{r'-1}\|\widehat{q}_\varepsilon\|^{2-r'}_{L^{r'}((0,T)\times\omega)}1_\omega. \tag{10}$$

Inequality (7) is proved by the classical method, while estimate (8) is derived from a refined observability inequality stated in the following proposition. □

Proposition 2.1. *There exists a constant $C > 0$ such that for each $q_T \in L^2(\Omega)$ and any r' sufficiently small, the associated solution to (6) satisfies:*

$$\|q(0)\|^2_{L^2(\Omega)} \le C\left(\int\limits_{\omega_T} |q|^{r'}\,dt\,dx\right)^{2/r'}, \quad C = e^{C(\Omega,\omega)(1+\frac{1}{T}+T+\|b\|_\infty^{\frac{2}{3}}+(T+T^{\frac{1}{2}})\|b\|_\infty)}. \tag{11}$$

Remark that if we make $r' = 1$ in estimate (11), we will find that given in [5].

Proof. Eq. (11) is a consequence of the following Carleman estimate and it is proved in the same way as in [3]. □

Before stating the Carleman estimate, we introduce some functions, among them the weight function.

$$\text{For } (t,x) \in Q_T, \text{ we set } \quad \alpha(t,x) = \frac{e^{2\lambda m\|\beta\|_\infty} - e^{\lambda(m\|\beta\|_\infty+\beta(x))}}{t(T-t)}, \qquad \xi(t,x) = \frac{e^{\lambda(m\|\beta\|_\infty+\beta(x))}}{t(T-t)},$$

where $\beta \in C^1(\overline{\Omega}) \cap W^{2,\infty}(\Omega)$ is the weight function constructed in [2], satisfying

1. $\beta > 0$ in Ω, $|\nabla\beta| > 0$ in $\overline{\Omega\setminus\omega}$ and $\frac{\partial\beta}{\partial\nu} < 0$ on $\Gamma\setminus\{S\}$ in case 1.
2. $\beta > 0$ in Ω, $|\nabla\beta| > 0$ in $\overline{\Omega\setminus\omega}$, $\frac{\partial\beta}{\partial\nu} < 0$ on Γ_1 and $\frac{\partial\beta}{\partial\nu_\pm} = 0$ on $\sigma\setminus\{S\}$ in case 2.

ν_+ and ν_- denote the (opposite) outward unit normal of the crack σ.

Proposition 2.2. *There exist three positive constants $\lambda_1 = \lambda_1(\Omega,\omega) \ge 1$, $s_1 = s_1(\Omega,\omega)(T+T^2)$ and $C(\Omega,\omega)$, such that for any $\lambda \ge \lambda_1$ and $s \ge s_1$, we have:*

$$\begin{aligned} &s^3\lambda^4\int\limits_{Q_T} e^{-2s\alpha}\xi^3|q|^2\,dt\,dx + s\lambda^2\int\limits_{Q_T} e^{-2s\alpha}\xi|\nabla q|^2\,dt\,dx \\ &\le C\left(s^3\lambda^4\int\limits_{(0,T)\times\omega} e^{-2s\alpha}\xi^3|q|^2\,dt\,dx, + \int\limits_{Q_T} e^{-2s\alpha}|\partial_t q + \Delta q|^2\right) \end{aligned} \tag{12}$$

for every function $q \in V = C^0([0,T]; L^2(\Omega)) \cap C^0([0,T[; D(-\Delta)) \cap C^1([0,T[; L^2(\Omega))$.

Proof. The proof is similar to that given in [2]. □

2.2. The fixed-point method

We will suppose that $y_0 \in L^\infty(\Omega)$ and $f \in C^1$. When $y_0 \in L^2(\Omega)$ or f is locally Lipschitz continuous, we argue as in [3] and [5]. Let $R > 0$ be given positive constant, we will fixe it later. We put:

$$g(s) = \begin{cases} \frac{f(s)}{s} & \text{if } s \neq 0, \\ f'(0) & \text{if } s = 0, \end{cases} \tag{13}$$

$$T^R = \min\left\{T, \|g\|_{L^\infty(-R,R)}^{-2/3}, \|g\|_{L^\infty(-R,R)}^{-1/3}\right\} \tag{14}$$

and consider the truncation function $T_R : \mathbb{R} \longrightarrow \mathbb{R}$ such that

$$T_R(s) = \begin{cases} s & \text{if } |s| \leq R, \\ R\,\mathrm{sgn}(s) & \text{otherwise.} \end{cases} \tag{15}$$

For each $z \in L^2(Q_{T^R})$ we consider the linearized problem

$$\begin{cases} \partial_t y - \Delta y + g(T_R(z))y = v1_\omega & \text{in } Q_{T^R}, \\ y = 0 & \text{on } \Sigma_{T^R}, \\ y(0,.) = y_0 & \text{in } \Omega. \end{cases} \tag{16}$$

Remark that (16) is a linear problem on y with a potential $b = g(T_R(z)) \in L^\infty(Q_{T^R})$.

Now for each $\varepsilon > 0$, we build a non-linear mapping $\Lambda : L^2(Q_{T^R}) \longrightarrow L^2(Q_{T^R})$ defined as follows: for each $z \in L^2(Q_{T^R})$, there exists a unique control $v_{\varepsilon,z} \in L^r((0,T^R) \times \omega)$ given by (10) and that verifies:

$$\|v_{\varepsilon,z}\|_{L^r((0,T^R)\times\omega)} \leq C\|y_0\|_{L^2(\Omega)}, \quad C = \mathrm{e}^{C(\Omega,\omega)(1+\frac{1}{T^R}+T^R+(T^R+{T^R}^{\frac{1}{2}})\|g\|_{L^\infty(-R,R)}+\|g\|_{L^\infty(-R,R)}^{\frac{2}{3}})}. \tag{17}$$

Then set $\Lambda(z) = y_{\varepsilon,z}$, where $y_{\varepsilon,z}$ is the unique solution to (16), it checks (7) for $T = T^R$.

Proposition 2.3. *Λ is continuous and compact, with bounded range.*

Proof. To prove the continuity and the compactness of Λ we need to use the compact imbedding from $H_0^1(\Omega)$ into $L^2(\Omega)$, which occurs if $\partial\Omega$ is Lipschitz as in case 1 (see [7]). In case 2, once again, the geometry of the domain brings up a new difficulty, but a small geometric manipulation allows us to reduce it to the case 1; this is the subject of the following lemma.

Lemma 2.1. *Let Ω be a domain of $\mathbb{R}^2$ satisfying the description in case 2 above. Then the space $H_0^1(\Omega)$ is compactly imbedded in $L^2(\Omega)$.*

Proof. We extend the crack to the boundary Γ; it intersects at a point S' so that we obtain two Lipschitz open subsets Ω_1, Ω_2 such that $\Omega = \Omega_1 \cup \Omega_2 \cup SS'$, and the lemma is a consequence of the compact imbedding of $H^1(\Omega_i)$ in $L^2(\Omega_i)$. □

Continuity of Λ. Let z_n be a sequence of $L^2(Q_{T^R})$ such that $z_n \to z$ in $L^2(Q_{T^R})$. For each z_n, $n \geq 1$, there exist a control v_{ε,z_n} and an associated y_{ε,z_n} solution to problem (16). By construction, $v_{\varepsilon,z_n} = \mathrm{sgn}(\widehat{q}_{\varepsilon,z_n})|\widehat{q}_{\varepsilon,z_n}|^{r'-1}\|\widehat{q}_{\varepsilon,z_n}\|_{L^{r'}((0,T^R)\times\omega)}^{2-r'} 1_\omega$, where $\widehat{q}_{\varepsilon,z_n}$ solves the adjoint problem (6) with the datum $\widehat{q}_{\varepsilon,T^R,z_n}$ a minimizer of the associated functional $J_{\varepsilon,n}$. Here and in the sequel, $J_{\varepsilon,n}$ denotes the functional J_ε corresponding to the potential $g(T_R(z_n))$.

Since $J_{\varepsilon,n}$ is uniformly coercive (see [9]), the sequences $\widehat{q}_{\varepsilon,T^R,z_n}$, $\widehat{q}_{\varepsilon,z_n}$ are uniformly bounded in $L^2(\Omega)$ and $L^2(0,T^R;H_0^1(\Omega))$, respectively. Then there exist $\widehat{q}_{\varepsilon,T^R} \in L^2(\Omega)$ and $\widehat{q}_\varepsilon \in L^2(0,T^R;H_0^1(\Omega))$ such that up subsequences

$$\widehat{q}_{\varepsilon,T^R,z_n} \rightharpoonup \widehat{q}_{\varepsilon,T^R} \quad \text{in } L^2(\Omega) \quad \text{and} \quad \widehat{q}_{\varepsilon,z_n} \rightharpoonup \widehat{q}_\varepsilon \quad \text{in } L^2\big(0,T;H_0^1(\Omega)\big). \tag{18}$$

We claim that $\widehat{q}_{\varepsilon,z_n}$ converges strongly to $\widehat{q}_{\varepsilon,z}$ in $L^2(Q_{T^R})$ and that $\widehat{q}_{\varepsilon,z}$ solves the adjoint problem of (16).

Indeed, $\partial_t\widehat{q}_{\varepsilon,z_n}$ is bounded in $L^2(0,T^R;H^{-1}(\Omega))$; thus

$$\widehat{q}_{\varepsilon,z_n} \in L^2\big(0,T^R;H_0^1(\Omega)\big) \cap H^1\big(0,T^R;H^{-1}(\Omega)\big), \quad \forall n \geq 1. \tag{19}$$

Combining (18), (19) and using compact embedding from $H_0^1(\Omega)$ into $L^2(Q_{T^R})$ given by Lemma 2.1 (in case 2) and thanks to the Aubin–Lions Lemma, we deduce that for a subsequence, still denoted by $n : \widehat{q}_{\varepsilon,z_n} \to \widehat{q}_{\varepsilon,z}$ strongly in $L^2(Q_{T^R})$ where $\widehat{q}_{\varepsilon,z}$ solves the adjoint problem of (16). Then one can conclude that, up to a subsequence, $v_{\varepsilon,z_n} \rightharpoonup v_{\varepsilon,z}$ weakly in $L^2((0,T^R) \times \omega)$ with $v_{\varepsilon,z}$ satisfies (17). Accordingly, at least for a subsequence $y_{\varepsilon,z_n} \to y_{\varepsilon,z}$ in $L^2(0,T^R;H_0^1(\Omega))$, where $y_{\varepsilon,z}$ is the solution to (16) with a control $v_{\varepsilon,z}$.

We argue as in [9] to check that the limit $\widehat{q}_{\varepsilon,T^R,z}$ in (18) is the minimizer of the functional J_ε associated with the limit control problem (16).

Boundedness of the range of Λ. Estimate (17) and classical energy estimate for the problem (16) show that there exists a positive constant C, depending only on T^R and $\|g\|_{L^\infty(-R,R)}$, such that

$$\|v_{\varepsilon,z}\|_{L^2((0,T^R)\times\omega)} \le C\|y_0\|_{L^2(\Omega)}, \qquad \|y_{\varepsilon,z}\|_{L^2(0,T^R;H_0^1(\Omega))} \le C\|y_0\|_{L^2(\Omega)}.$$

The last inequality means that the range of Λ is bounded in $L^2(Q_{T^R})$.

Compactness of Λ. Let B be a bounded set of $L^2(Q_{T^R})$. We have to show that $\Lambda(B)$ is relatively compact in $L^2(Q_{T^R})$. For each $z \in B$, $y = y_{\varepsilon,z} = \Lambda(z)$, is bounded in $L^2(0,T^R;H_0^1(\Omega))$ and satisfies:

$$\begin{cases} \partial_t y - \Delta y = k & \text{in } Q_{T^R}, \\ y = 0 & \text{on } \Sigma_{T^R}, \\ y(0,.) = y_0 & \text{in } \Omega, \end{cases} \tag{20}$$

with

$$k = v 1_\omega - g\big(T_R(z)\big) y$$

which is uniformly bounded in $L^2(Q_{T^R})$.

We proceed as in [9]. The solution y is written as

$$y = y_1 + y_2$$

where y_1 and y_2 solve (20) respectively without a second member and with vanishing initial data. It is obvious that y_1 is a fixed element of $L^2(0,T^R;H_0^1(\Omega))$. Thanks to [8], we know that y_2 belongs to a bounded set of $L^2(0,T^R;H_0^1(\Omega)) \cap H^1(0,T^R;L^2(\Omega))$, which, as a consequence of a compact imbedding from $H_0^1(\Omega)$ into $L^2(\Omega))$ given by Lemma 2.1 and Aubin–Lions compactness Lemma, is a relatively compact set of $L^2(Q_{T^R})$. This concludes the proof of Proposition 2.3.

Hence, from Schauder's theorem, the mapping Λ has a fixed point, $y_\varepsilon = \Lambda(y_\varepsilon)$, which solves the problem (16) with a control v_ε and satisfies (7) for $T = T^R$.

2.3. End of the proof and conclusion

Since v_ε, y_ε and $\partial_t y_\varepsilon$ are bounded uniformly in ε, in $L^2((0,T^R)\times\omega)$, $L^2(0,T^R;H_0^1(\Omega))$ and $L^2(0,T^R;H^{-1}(\Omega))$, respectively, we deduce that, when $\varepsilon \to 0$ at least for subsequence, $y_\varepsilon \to y^R$ in $L^2(Q_{T^R})$, where y^R solves the problem:

$$\begin{cases} \partial_t y^R - \Delta y^R + g(T_R(y^R))y^R = v^R 1_\omega & \text{in } Q_{T^R}, \\ y^R = 0 & \text{on } \Sigma_{T^R}, \\ y^R(0,.) = y_0 & \text{in } \Omega. \end{cases} \tag{21}$$

Consequently $y_\varepsilon(T^R) \to y^R(T^R)$ in $L^2(\Omega)$ and $y^R(T^R) = 0$.

On the other hand, since $v^R \in L^r((0,T^R)\times\omega)$ with $r > 2$, we have (see [1]) the following estimate

$$\|y^R\|_\infty \le e^{T^R\|g(T_R(y^R))\|_\infty}\|y_0\|_\infty + T^R e^{T^R\|g(T_R(y^R))\|_\infty}\|v^R\|_{L^r((0,T^R)\times\omega)}. \tag{22}$$

Using the definition of T^R, estimate (17) and that $\|g\|_{L^\infty(-R,R)}^{2/3} \le C_\eta + \eta\log(1+|R|)$ $\forall \eta > 0$, we deduce from (22) that y^R satisfies:

$$\|y^R\|_\infty \le C_2(\Omega,\omega,T,\eta,y_0)(1+R)^{\eta C_3(\Omega,\omega,T)}. \tag{23}$$

Now let us extend by zero v^R and y^R to the whole cylinder Q_T, and for the sake of simplicity we still call them v^R and y^R. It is clear that (23) holds for y^R. To conclude the proof, we choose $\eta = 1/(2C_3)$ and $R > 0$ such that $C_2(\Omega,\omega,T,\eta,y_0)(1+R)^{\eta C_3(\Omega,\omega,T)} < R$, this leads to $\|y^R\|_\infty \le R$ and then $T_R(y^R) = y^R$. In conclusion, we have found a control $v \in L^r((0,T)\times\omega)$ such that the solution y to (1) verifies (3). This completes the proof of our result. □

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