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Dynamical Systems

Local rigidity of restrictions of Weyl chamber flows

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Abstract

In this Note we consider examples of partially hyperbolic actions: restrictions of Weyl chamber flows on $\mathrm{SL}(n,\mathbb{R})/\Gamma$ ($n \geqslant 4$). We show that generic restrictions of rank at least two are locally rigid. Our approach combines the geometry of the invariant foliations for the action and the algebraic properties of the group $\mathrm{SL}(n,\mathbb{R})$. The method is applicable to restrictions of Weyl chamber flows on other homogeneous spaces. ***To cite this article: D. Damjanović, A. Katok, C. R. Acad. Sci. Paris, Ser. I 344 (2007).***

Résumé

Rigidite locale des restrictions de flots des chambres de Weyl. Dans cette Note on étudie des exemples d'actions partiellement hyperboliques : restrictions de flots des chambres de Weyl sur la variété $\mathrm{SL}(n,\mathbb{R})/\Gamma$ ($n \geqslant 4$). Nous démontrons que, génériquement, les restrictions de rang $\geqslant 2$ sont localement rigides. Notre approche combine la géométrie des feuilletages invariants et les propriétés algébriques du groupe $\mathrm{SL}(n,\mathbb{R})$. Cette approche est aussi applicable dans la démonstration de la rigidité locale des restrictions des autres flots des chambres de Weyl. ***Pour citer cet article : D. Damjanović, A. Katok, C. R. Acad. Sci. Paris, Ser. I 344 (2007).***

Version française abrégée

On considére des actions algébriques abeliénnes sur la variété quotient $X_n = \mathrm{SL}(n,\mathbb{R})/\Gamma$ avec $n \geqslant 4$. Ici Γ est un réseau uniforme de $\mathrm{SL}(n,\mathbb{R})$.

Si $\mathbb{D}_+ = \{(a_1,\dots,a_n) \in \mathbb{R}^n,\ \sum_{i=1}^{i=n} a_i = 0\}$ et $D_+ = \mathrm{diag}(\exp \mathbb{D}_+)$ désigne le groupe de matrices diagonales, l'action α_{0,D_+} de $\mathbb{D}_+$ sur X est définie par $\alpha_{0,D_+}(a,x) = \mathrm{diag}(\exp a)\cdot x,\ a \in \mathbb{D}_+,\ x \in X$. L'action α_{0,D_+} est une action d'Anosov : le feuilletage neutre est donné par les orbites de α_{0,D_+}.

Il est bien connu que l'action α_{0,D_+} (flot des chambres de Weyl sur X_n, notée par FCW) est $C^{\infty,1,\infty}$-localement rigide, voir [11]. C'est-à-dire que toute action de $\mathbb{D}_+$ de classe C^∞ sur X_n suffisamment C^1-proche est conjuguée à α_{0,D_+} par un difféomorphisme de classe C^∞.

Dans cette Note, on considére quelques actions partiellement hyperboliques : les restrictions $\alpha_{0,G}$ de FCW à des sous-groupes abéliens $\mathbb{G}$ du groupe $\mathbb{D}_+$ de rank $\mathbb{G} \geqslant 2$.

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Soient $H_{ij} := \{a = (a_1, \dots, a_n) \in \mathbb{D}_+ \colon a_i = a_j\}$, $0 \leqslant i \neq j \leqslant n$ les hyperespaces de Weyl. Un plan $\mathbb{P}$ de $\mathbb{D}_+$ est dit en *position générale* si les intersections de $\mathbb{P}$ avec des hyperspaces H_{ij} distincts sont distinctes.

Soit $\mathbb{G}$ un sous-espace de $\mathbb{D}_+$ et $\alpha_{0,G}$ la restriction de FCW à $G := \mathrm{diag}(\exp \mathbb{G})$. On dit que $\alpha_{0,G}$ est une *restriction générique* de FCW, si $\mathbb{G}$ posséde un réseau d'un plan $\mathbb{P}$ en position generale.

Le théorème suivant est notre résultat principal :

Théorème 0.1. *Soit $\alpha_{0,G}$ une restriction générique de FCW sur X_n ($n \geqslant 4$). Alors $\alpha_{0,G}$ est $C^{\infty,2,\infty}$-localement rigide.*

C'est une conséquence des résultats généraux de stabilité des actions normalement hyperboliques [8] et de la méthode plus générale des formes normales [11] pour obtenir la régularité de la conjugaison, que pour démontrer le Théorème 0.1 il suffit de démontrer la rigidité des cocycles pour de petites perturbations de $\alpha_{0,G}$. La démonstration du Théorème 0.1 utilise une méthode geometrique pour démontrer la rigidité des cocycles pour les restrictions génériques de FCW, voir [5].

Cette méthode utilise la description des obstructions à la trivialisation des cocycles [9] pour les diffeomorphismes partiellement hyperboliques dont les feuilletages stable et instable forts sont localement transitifs. C'est-à-dire que pour $x, y \in X_n$ on peut trouver $x = x_1, x_2, \dots, x_k = y \in X_n$ tels que pour tout $i \in \{1, \dots, k\}$ le point x_{i+1} est dans une feuille passant par x_i du feuilletage stable ou instable (voir (ii) Section 3.3). La rigidité des cocycles est une conséquence de l'analyse de la structure des feuilletages de Lyapounov pour $\alpha_{0,G}$. On utilise la présentation de $\mathrm{SL}(n, \mathbb{R})$ [13] et la présentation (algébrique et topologique) de K_2, le noyau d'une extension centrale de $\mathrm{SL}(n, \mathbb{R})$ [12].

Les feuilletages de Lyapounov pour $\alpha_{0,G}$ sont localement transitifs et en plus (voir [1]) ils sont *robustement* localement transitifs en classe C^2. Alors, dans le même esprit que l'approche de [5], on obtient la rigidité des cocycles pour de petites perturbations de $\alpha_{0,G}$.

Remarque 1. Plus généralement, on applique la même méthode pour les actions de sous-groupes du groupe diagonal sur d'autres variétés quotients, voir Section 4.

Remarque 2. Avec la méthode geometrique pour démontrer la rigidité des cocycles pour les restrictions génériques de WCF et pour des petites perturbations, on peux démontrer la rigidité dons certains cas de petits cocycles *non-abéliens* (voir Section 4). En fait nous pensons que la rigidité locale reste vraie pour des classes particulieres d'actions partiellement hyperboliques dont la direction centrale est non-abelienne.

Les détails des démonstrations seront donnés dans [6] et [2].

1. Introduction

We consider abelian actions which are obtained as restrictions of the Weyl chamber flow (WCF). With the exception of the last section, notation WCF will be used for the action α_{0,D_+} of the group $\mathbb{D}_+$ on the space $\mathrm{SL}(n, \mathbb{R})/\Gamma$ (where Γ is a cocompact lattice in $\mathrm{SL}(n, \mathbb{R})$) defined as follows:

$$\alpha_{0,D_+}(a, x) = \mathrm{diag}(\exp a) \cdot x, \quad a \in \mathbb{D}_+,\ \mathbb{D}_+ = \left\{ (a_1, \dots, a_n) \in \mathbb{R}^n, \sum_{i=1}^{i=n} a_i = 0 \right\}.$$

Here $x \in \mathrm{SL}(n, \mathbb{R})/\Gamma$ and $\mathrm{diag}(\exp a) := \mathrm{diag}(\exp a_1, \dots, \exp a_n) \in D_+$, the group of diagonal elements in $\mathrm{SL}(n, \mathbb{R})$ with positive entries.

For $n = 2$, the action α_{0,D_+} is an Anosov flow. It can be identified with the geodesic flow on the surface $\mathrm{SO}(2)\backslash \mathrm{SL}(2, \mathbb{R})/\Gamma$ provided with the natural metric of constant negative curvature -1.[1]

For $n \geqslant 3$, the action α_{0,D_+} is an Anosov action, the orbit direction is isometric and is of dimension $n - 1 \geqslant 2$. It is proven in [11] that this action is $C^{\infty,1,\infty}$-*locally rigid* (see [3] for an exact definition). Let $\mathbb{P}_{ij} = \{(a_1, \dots, a_n) \in \mathbb{D}_+ \colon a_i = a_j\}$ for $1 \leqslant i \neq j \leqslant n$ denote the Weyl chamber walls; all the elements outside the walls are *regular*

[1] This surface has isolated conical singularities if Γ contains elliptic elements.

(i.e. normally hyperbolic with respect to the orbit foliation of α_{0,D_+}). For $n \geqslant 4$ we say that a 2-plane $\mathbb{P}$ in $\mathbb{D}_+$ is in *general position* if it intersects hyperplanes $\mathbb{P}_{ij}$ along distinct lines.

Given a subgroup $\mathbb{G}$ of $\mathbb{D}_+$, the action by left translations by elements

$$G := \exp \mathbb{G} := \{\operatorname{diag}(\exp a):\ a \in \mathbb{G}\}$$

on $\mathrm{SL}(n,\mathbb{R})/\Gamma$ is denoted by $\alpha_{0,G}$. If $\mathbb{G}$ contains a lattice in a 2-plane which is in general position then we refer to $\alpha_{0,G}$ as a *generic restriction* of the WCF. A generic restriction of the WCF is always generated by elements whose neutral foliation is the orbit foliation of the WCF. In this Note we announce a local rigidity result for a generic restriction of the WCF on $\mathrm{SL}(n,\mathbb{R})/\Gamma$. The detailed proofs will be given in [6] and [2].

2. The main result

Theorem 2.1. *Generic restriction $\alpha_{0,G}$ of the WCF on $X := \mathrm{SL}(n,\mathbb{R})/\Gamma$ is $C^{\infty,2,\infty}$-locally rigid: for every C^∞ action $\tilde{\alpha}_G$ of $\mathbb{G}$ which is sufficiently C^2-close to $\alpha_{0,G}$ there exists a C^∞ diffeomorphism h of $\mathrm{SL}(n,\mathbb{R})/\Gamma$ and there exists a homomorphism $i:\mathbb{G}\to\mathbb{D}_+$ such that*

$$\tilde{\alpha}_G\big(a, h(x)\big) = h\big(\alpha_{0,G}\big(i(a), x\big)\big),$$

for all $a \in \mathbb{G}$ and all $x \in \mathrm{SL}(n,\mathbb{R})/\Gamma$. The map h is close to the identity in the C^2 topology.

Remarks.

(i) Unlike both the hyperbolic case and the case of partially hyperbolic actions of $\mathbb{Z}^k$ by automorphisms of a torus [3,4] rigidity here does not imply that any small perturbation is smoothly conjugate to the original action up to an automorphism of the acting group. Rather it means that there is finite-dimensional family of standard algebraic perturbations such that any small perturbation is smoothly conjugate to one of those.

(ii) Another difference with both above-mentioned cases is in the assumption on the closeness of a perturbation, somewhat stronger than in the hyperbolic situation, but much weaker than for the partially hyperbolic actions on the torus where the number of derivatives required to carry out the KAM-type iteration scheme depends on the eigenvalues of the unperturbed algebraic action.

3. On the proof of Theorem 2.1

3.1. Reduction to Hölder perturbations in the neutral direction

Let $\tilde{\alpha}_G$ be a C^2-small perturbation of a generic restriction $\alpha_{0,G}$. Each regular element of the action $\alpha_{0,G}$ is normally hyperbolic to the orbit foliation $\mathcal{N}_0$ of the WCF. By the Hirsch–Pugh–Shub stability theory [8], the corresponding element of the perturbed action is normally hyperbolic to a Hölder foliation $\mathcal{N}$ with smooth leaves which is close to $\mathcal{N}_0$ and there exists a Hölder homeomorphism h' which takes the leaves of $\mathcal{N}$ to the leaves of $\mathcal{N}_0$; h' can be chosen smooth and close to identity along the leaves of $\mathcal{N}_0$.

Let α_G be the action obtained by conjugating $\tilde{\alpha}_G$ via h'. Therefore, α_G is a perturbation of $\alpha_{0,G}$ which preserves the leaves of the neutral foliation $\mathcal{N}_0$ and is C^2-small along the leaves of $\mathcal{N}_0$. However, a priori α_G is only Hölder transversally.

3.2. Reduction to the problem of trivialization of cocycles over perturbations

A perturbation α_G of $\alpha_{0,G}$ along the leaves of $\mathcal{N}_0$ is given by a map $\beta:\mathbb{G}\times X\to\mathbb{D}_+$:

$$\alpha_G(a,x) := \operatorname{diag}\big(\exp\beta(a,x)\big)\cdot x.$$

It is easy to see that β is a cocycle over α_G, i.e. for all $a, b \in \mathbb{G}$ and all $x \in X$ we have

$$\beta(a+b,x) = \beta\big(a,\alpha_G(b,x)\big) + \beta(b,x).$$

If β is cohomologous to a constant cocycle $i:\mathbb{G}\to\mathbb{D}_+$ i.e., $\beta(a,x)=H(\alpha_G(a,x))+i(a)-H(x)$, then the map $H:X\to\mathbb{D}_+$ induces a conjugacy $h(x):=H(x)^{-1}\cdot x$ between α_G and $\alpha_{0,G}$. Thus in order to obtain a Hölder conjugacy between $\tilde{\alpha}_G$ and $\alpha_{0,G}$ it suffices to show that every Hölder cocycle over a sufficiently small perturbation of $\alpha_{0,G}$ in the neutral direction, trivializes via a Hölder transfer map H to an injective homomorphism i.

3.3. Geometric proof of cocycle rigidity

In [5] we proved rigidity of Hölder cocycles for generic restrictions of the WCF.

Theorem 3.1. *Real-valued Hölder (corr. C^∞) cocycles over generic restrictions $\alpha_{0,G}$ of the WCF, are cohomologous to constant cocycles. In particular, for every $\mathbb{D}_+$-valued Hölder (corr. C^∞) cocycle β over $\alpha_{0,G}$ there exists a Hölder (C^∞) map $H:\mathbb{G}\times X\to\mathbb{D}_+$ and a homomorphism $i:\mathbb{G}\to\mathbb{D}_+$ such that for all $a\in\mathbb{G}$ and all $x\in X$ we have $\beta(a,x)=H(\alpha_G(a,x))+i(a)-H(x)$.*

Unlike most earlier proofs of cocycle rigidity for hyperbolic or partially hyperbolic actions of higher rank Abelian groups our proof does not use harmonic analysis. Instead, the proof of Theorem 3.1 in [5] relies essentially on the following facts:

(i) The action $\alpha_{0,G}$ has *locally transitive* coarse Lyapunov foliations: there exists $N\in\mathbb{N}$ such that for every $\epsilon>0$ there exists $\delta>0$ such that every $x\in X$ and every y in δ-ball centered at x can be connected by a broken path whose pieces lie in the leaves of the coarse Lyapunov foliations (Lyapunov paths), and the length of every piece is less than ϵ.

(ii) For every partially hyperbolic diffeomorphism with the property that stable and unstable foliations are locally transitive one can describe the system of obstructions to cocycle trivialization in terms of the periodic cycle functionals (PCF) [9].

(iii) The obstructions vanish on contractible Lyapunov paths due to the invariance of the periodic cycle functional and the algebraic structure of $\mathrm{SL}(n,\mathbb{R})$ (more precisely, presentation of the group [13] and presentation of the kernel of its universal central extension i.e. Milnor's K_2 group [12]).

(iv) Due to Margulis' normal subgroup theorem every homomorphism from Γ into $\mathbb{D}_+$ vanishes; this implies vanishing of obstructions on all Lyapunov paths.
In the forthcoming paper [6] we show that the geometric argument for cocycle rigidity for $\alpha_{0,G}$ can be extended to sufficiently small perturbations.

Theorem 3.2. *Let $\tilde{\alpha}_G$ be a C^∞ action sufficiently close to $\alpha_{0,G}$ in C^2 topology and let β be a real-valued Hölder one-cocycle over $\tilde{\alpha}_G$. Then β is cohomologous to a constant cocycle via a continuous transfer function. If β is smooth, the transfer function is smooth along the leaves of the coarse Lyapunov foliations of $\tilde{\alpha}_G$.*

Remarks.

(i) Theorem 3.2 can be extended to cocycles which take values in some matrix groups, assuming that the cocycle is sufficiently small. (For the corresponding results for non-Abelian cocycles over the unperturbed action $\alpha_{0,G}$ see [5, Section 7].)

(ii) Clearly, the first statement of the theorem which concerns Hölder cocycles holds true for any action which is Hölder conjugate to $\tilde{\alpha}_G$, in particular the cocycle rigidity holds for the action α_G defined in Section 3.1.

We give here a brief outline of the main steps of the proof of Theorem 3.2.

(i) Persistence of local transitivity of strong stable and unstable foliations under C^2-small perturbations for partially hyperbolic diffeomorphisms is due to Brin and Pesin [1] and it is not difficult to produce a similar result for (coarse) Lyapunov foliations for partially hyperbolic actions.

(ii) This allows for a description of obstructions to cocycle trivialization in terms of the periodic cycle functional on closed Lyapunov paths for the perturbed action $\tilde{\alpha}_G$, and consequently for its conjugate α_G. Let $\mathcal{C}\bar{\mathcal{U}}_x$ be the collection of all closed Lyapunov paths for α_G starting at $x\in X$ and $\mathcal{C}\bar{\mathcal{U}}=\bigcup_{x\in X}\mathcal{C}\bar{\mathcal{U}}_x$. We drop $\mathcal{C}$ from the notation to denote corresponding collections of Lyapunov paths which are not necessarily closed. Denote by $\mathcal{CU}_x, \mathcal{CU}, \mathcal{U}_x, \mathcal{U}$ the corresponding collections of Lyapunov paths for the unperturbed action $\alpha_{0,G}$.

(iii) For every $x \in X$ and $y \in \mathcal{N}_0(x)$, we describe continuous projections $\mathcal{P}_{x,y}$ and $\bar{\mathcal{P}}_{x,y}$ along the leaves of $\mathcal{N}_0$ so that $\mathcal{P}: \mathcal{CU} \to \bar{\mathcal{U}}$ and $\bar{\mathcal{P}}_{x,y}: \mathcal{C}\bar{\mathcal{U}}_x$ to $\mathcal{U}$.

(iv) By using the structure of closed contractible Lyapunov paths in $\mathcal{CU}$ [13] and continuity of $\bar{\mathcal{P}}_{x,y}$ in x and y, and of 1-parameter unipotent foliations we show that $\bar{\mathcal{P}}_{x,y}$ takes $\mathcal{C}\bar{\mathcal{U}}_x$ to $\mathcal{CU}_y$.

(v) The reverse projection $\mathcal{P}_{x,y}$ is such that for $\mathfrak{p} \in \mathcal{C}\bar{\mathcal{U}}_x$, $\mathcal{P}_{x,y}(\mathfrak{p})$ has endpoints on the same leaf of $\mathcal{N}_0$. This defines a map from $\mathcal{C}\bar{\mathcal{U}}$ to D_+ which is invariant along the leaves of Lyapunov foliations for the perturbation and due to the local transitivity of those foliations, the image of $\mathcal{C}\bar{\mathcal{U}}_x$ in D_+ is a subgroup of D_+ which does not depend on x. This subgroup however must be discrete due to the smallness of the perturbation and due to the fact that $\bar{\mathcal{P}}_{x,y}$ takes closed paths to closed paths.

(vi) This allows us to use the same reasoning to show that the obstructions for cocycle trivialization (periodic cycle functionals criterion from [9]) vanish for the perturbed action as we did for the linear action $\alpha_{0,G}$ in [5, Section 6].

3.4. Smoothness of the conjugacy

Proposition 3.3. *A Hölder map conjugating the action $\alpha_{0,G}$ to its C^2-small C^∞ perturbation $\tilde{\alpha}_G$, is smooth.*

The transversal smoothness of the conjugacy follows from the non-stationary normal forms method and the rigidity of centralizers for extensions [7]. As in [10] global smoothness is a consequence of the general fact that a function smooth along several smooth foliations whose tangent distributions with their Lie brackets generate the tangent space, is necessarily smooth.

4. Extension to other split classical groups

The proof of Theorem 3.2, and hence of Theorem 2.1, relies on complete algebraic as well as topological description of the universal central extension of $\mathrm{SL}(n, \mathbb{R})$. In general, for a connected real analytic semisimple Lie group the usual Lie group theoretic simply connected cover is its universal *topological* central extension.

Algebraic structure of the universal central extension of $\mathrm{SL}(n, \mathbb{R})$ follows from a general result of Steinberg [13, Theorem 3.2] which gives a presentation for simple Lie groups over a field, which are obtained from classical Lie algebra systems A_n, B_n, C_n and D_n, with $n \geqslant 2$. Let G be such a group of non-symplectic type with Lie algebra $\mathfrak{g}$ and $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. Let A be a connected component of a split Cartan subgroup of G. Let $\mathfrak{a}$ be the Lie algebra of A and let Φ be the root system of $\mathfrak{g}$ with respect to $\mathfrak{a}$. Let $\mathfrak{p}$ be a subalgebra of $\mathfrak{a}$ such that for all non-trivial $p \in \mathfrak{p}$ and for all $s, r \in \Phi$, $s \neq -r$ at least one of the elements $[p, s]$ or $[p, r]$ is not trivial. Let Γ be a cocompact lattice in G. Denote by $\alpha_{0,P}$ the action of $P := \exp \mathfrak{p}$ on G/Γ by left translations. Then we can show [2]:

Theorem 4.1.

(a) *If H is an abelian group then every H valued Hölder* (*corr. C^∞*) *cocycle over $\alpha_{0,P}$ is cohomologous to a constant cocycle via a Hölder* (*corr. C^∞*) *transfer function.*
(b) *If H is a matrix group with no non-trivial homomorphisms: $\Gamma \to H$ which are close to identity, then every H valued sufficiently small Hölder* (*corr. C^∞*) *cocycle over $\alpha_{0,P}$ is cohomologous to a constant cocycle via a Hölder* (*corr. C^∞*) *transfer function.*
(c) *If $\mathbb{K} = \mathbb{R}$ then $\alpha_{0,P}$ is $C^{\infty,2,\infty}$-locally rigid* (*up to an injective homomorphism of P into $A :=$* $\exp \mathfrak{a}$).

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