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Local and asymptotic properties of Linear Fractional Stable Sheets

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Abstract

In this Note we introduce representations of Linear Fractional Stable Sheets as wavelet random series. Using these representations, in the case where the paths are continuous, an anisotropic uniform and quasi-optimal modulus of continuity of these paths is obtained as well as an upper bound on their behavior at infinity. ***To cite this article: A. Ayache et al., C. R. Acad. Sci. Paris, Ser. I 344 (2007).***

Résumé

Comportement local et asymptotique du drap linéaire fractionnaire stable. Dans cette Note, nous introduisons une représentation du drap linéaire fractionnaire stable sous la forme d'une série aléatoire d'ondelettes. Au moyen de cette représentation, dans le cas où les trajectoires du processus sont continues, un module de continuité anisotropique uniforme quasi-optimal de ces trajectoires est obtenu ainsi qu'un contrôle de leur comportement à l'infini. ***Pour citer cet article : A. Ayache et al., C. R. Acad. Sci. Paris, Ser. I 344 (2007).***

Version française abrégée

Le drap linéaire fractionnaire stable (LFSS) de paramètres $\alpha \in]0,2[$ et $H = (H_1, \dots, H_N) \in]0,1[^N$ est le processus symétrique stable à valeurs réelles $X = \{X(t),\ t \in \mathbb{R}^N\}$ défini par (1), où $\{Z_\alpha(s),\ s \in \mathbb{R}^N\}$ est le drap de Lévy symétrique α-stable et $x_+ = \max(x, 0)$. Il convient de noter que, pour tout $n = 1, \dots, N$, la restriction de X à toute droite parallèle au n-ième axe est un mouvement linéaire fractionnaire stable (LFSM) sur $\mathbb{R}$ de paramètre de Hurst H_n (pour plus d'information concernant le LFSM voir par exemple [6,9,8,5]). Cette propriété d'anisotropie du LFSS peut avoir de l'importance du point de vue de la modélisation (voir par exemple [3]). Certaines propriétés locales et asymptotiques du drap brownien fractionnaire (FBS) on été obtenues dans [1] au moyen de méthodes d'ondelettes. L'objectif de cette note est de donner certaines extensions de ces résultats dans le cadre du LFSS. On se placera toujours dans le cas où $\underline{H} > 1/\alpha$ avec $\underline{H} = \min\{H_1, \dots, H_N\}$, c'est-à-dire dans le cas où les trajectoires sont continues sur $\mathbb{R}^N$ avec probabilité 1.

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Nous commençons par introduire une représentation du processus X sous la forme d'une série aléatoire d'ondelettes (Théorème 1.1). Cette décomposition permet d'obtenir un module de continuité anisotropique uniforme du LFSS sur les compacts (Théorème 1.2) ainsi qu'un contrôle du comportement de ses trajectoires à l'infini (Théorème 1.3). Enfin le Théorème 1.4 montre que ce module de continuité est quasiment optimal. Ces résultats sont résumés dans l'énoncé suivant :

Théorème 0.1. *Les trois propositions suivantes sont vraies presque sûrement* :

(i) *Pour tout* $\eta > 0$ *et tout compact* $\mathcal{K} \subset \mathbb{R}^N$, $\sup_{s,t\in\mathcal{K}} \frac{|X(s)-X(t)|}{\sum_{j=1}^N |s_j-t_j|^{H_j-1/\alpha-\eta}} < \infty$.

(ii) *Pour tout* $\eta > 0$, $\sup_{t\in\mathbb{R}^N} \frac{|X(t)|}{\prod_{j=1}^N (1+|t_j|)^{H_j} \log^{1/\alpha+\eta}(3+|t_j|)} < \infty$.

(iii) *Pour tout* $n = 1, \ldots, N$, *tout vecteur* $\hat{u}_n \in \mathbb{R}^{N-1}$ *dont les coordonnées sont toutes non nulles, tout* $\eta > 0$ *et tout intervalle* $]a, b[\subset \mathbb{R}$, $\sup_{s_n,t_n\in]a,b[} \frac{|X(s_n,\hat{u}_n)-X(t_n,\hat{u}_n)|}{|s_n-t_n|^{H_n-1/\alpha+\eta}} = \infty$, *où, pour tout* $x_n \in \mathbb{R}$, *nous notons* $(x_n, \hat{u}_n) = (u_1, \ldots, u_{n-1}, x_n, u_{n+1}, \ldots, u_N)$.

1. Introduction and main results

The Linear Fractional Stable Sheet (LFSS) of parameters $\alpha \in (0, 2)$ and $H = (H_1, \ldots, H_N) \in (0, 1)^N$ is the real-valued symmetric α stable ($S\alpha S$) process $X = \{X(t),\ t \in \mathbb{R}^N\}$ defined as

$$X(t) = \int_{\mathbb{R}^N} \prod_{l=1}^N \left\{(t_l - s_l)_+^{H_l-1/\alpha} - (-s_l)_+^{H_l-1/\alpha}\right\} \mathrm{d}Z_\alpha(s), \tag{1}$$

where $\{Z_\alpha(s),\ s \in \mathbb{R}^N\}$ is the symmetric α-Stable Lévy Sheet and $x_+ = \max(x, 0)$. Observe that, for every $n = 1, \ldots, N$, X is a Linear Fractional Stable Motion (LFSM) in $\mathbb{R}$ of Hurst parameter H_n along the direction of the nth axis (see [6,9,8,5] for more information on LFSM). This anisotropic nature of X makes it a potential model for various spatial objects (see for example [3]). Some local and asymptotic properties of the ordinary Fractional Brownian Sheet (FBS), i.e. $\alpha = 2$, have been obtained in [1] by using the wavelet methods. Roughly speaking, the goal of our Note is to extend these results to LFSS. In all the sequel we set $\underline{H} = \min\{H_1, \ldots, H_N\}$ and always assume that the condition $\underline{H} > 1/\alpha$ holds. In fact the latter condition implies that the paths of X are almost surely continuous.

Let us introduce random series wavelet representations of LFSS. First a word about notations:

(i) For any $l = 1, \ldots, N$, the functions ψ^{H_l} and ψ^{-H_l} will respectively denote the left-sided fractional primitive of order $H_l + 1 - 1/\alpha$ and the right-sided fractional derivative of order $H_l + 1 - 1/\alpha$ of a compactly supported Daubechies wavelet ψ (see [4,7]), which are defined (up to a multiplicative constant) for all $x \in \mathbb{R}$ by

$$\psi^{H_l}(x) = \int_{\mathbb{R}} (x - y)_+^{H_l-1/\alpha} \psi(y)\,\mathrm{d}y \quad \text{and} \quad \psi^{-H_l}(x) = \frac{\mathrm{d}^2}{\mathrm{d}x^2} \int_{\mathbb{R}} (y - x)_+^{1/\alpha-H_l} \psi(y)\,\mathrm{d}y. \tag{2}$$

If the Daubechies wavelet ψ is smooth enough, then the functions ψ^{H_l} and ψ^{-H_l} are well-defined, continuously differentiable up to a given arbitrary order M and well-localized. By well-localized we mean that for $m = 0, 1, \ldots, M$,

$$\left|\frac{\mathrm{d}^m \psi^{H_l}}{\mathrm{d}x^m}(x)\right| \leqslant c_m\big(1 + |x|\big)^{-2} \quad \text{and} \quad \left|\frac{\mathrm{d}^m \psi^{-H_l}}{\mathrm{d}x^m}(x)\right| \leqslant c_m\big(1 + |x|\big)^{-2}. \tag{3}$$

(ii) $\{\epsilon_{j,k}\ (j, k) \in \mathbb{Z}^N \times \mathbb{Z}^N\}$ will denote the sequence of identically distributed $S\alpha S(\|\psi\|^N_{L^\alpha(\mathbb{R})})$ random variables defined as

$$\epsilon_{j,k} = \int_{\mathbb{R}^N} \prod_{l=1}^N \left\{2^{j_l/\alpha} \psi\big(2^{j_l} s_l - k_l\big)\right\} \mathrm{d}Z_\alpha(s). \tag{4}$$

Theorem 1.1. *LFSS can be expressed as*

$$X(t)=\sum_{(j,k)\in\mathbb{Z}^N\times\mathbb{Z}^N} 2^{-\langle j,H\rangle}\epsilon_{j,k}\prod_{l=1}^{N}\big\{\psi^{H_l}\big(2^{j_l}t_l-k_l\big)-\psi^{H_l}(-k_l)\big\},\quad t=(t_1,\dots,t_N)\in\mathbb{R}^N, \tag{5}$$

where, almost surely, the series (5) *is convergent* (*as a function of* t) *in any Hölder space* $C^\gamma(\mathcal{K})$ *of order* $\gamma\in[0,\underline{H}-1/\alpha)$ *for every compact set* $\mathcal{K}\subset\mathbb{R}^N$.

Observe that the wavelet representations (5) of LFSS is a natural extension both of those of LFSM and FBS (see [1,2]). It will allow us to establish the following results: Theorem 1.2 provides a uniform modulus of continuity of LFSS, Theorem 1.3 gives an upper bound of its asymptotic behavior as $|t|\to\infty$ and Theorem 1.4 can be viewed as an inverse of Theorem 1.2.

Theorem 1.2. *Almost surely, for any compact set* $\mathcal{K}\subset\mathbb{R}^N$ *and any arbitrarily small* $\eta>0$, *one has*

$$\sup_{s,t\in\mathcal{K}}\frac{|X(s)-X(t)|}{\sum_{j=1}^{N}|s_j-t_j|^{H_j-1/\alpha-\eta}}<\infty. \tag{6}$$

Theorem 1.3. *Almost surely, for any* $\eta>0$, *one has*

$$\sup_{t\in\mathbb{R}^N}\frac{|X(t)|}{\prod_{j=1}^{N}(1+|t_j|)^{H_j}\log^{1/\alpha+\eta}(3+|t_j|)}<\infty. \tag{7}$$

Theorem 1.4. *Almost surely, one has that, for any interval* $(a,b)\subset\mathbb{R}$, *any* $\eta>0$, *any* $n=1,\dots,N$ *and any vector* $\hat{u}_n\in\mathbb{R}^{N-1}$ *with non-vanishing coordinates,*

$$\sup_{s_n,t_n\in(a,b)}\frac{|X(s_n,\hat{u}_n)-X(t_n,\hat{u}_n)|}{|s_n-t_n|^{H_n-1/\alpha+\eta}}=\infty, \tag{8}$$

where, for every real x_n, *we have set* $(x_n,\hat{u}_n)=(u_1,\dots,u_{n-1},x_n,u_{n+1},\dots,u_N)$.

Observe that Theorems 1.2 and 1.4 have already been obtained by Takashima (see [9]) in the particular case of LFSM (i.e. $N=1$); however, the method used by this author can hardly be adapted to LFSS. It is also worth noticing that the event of probability 1 on which (8) holds in Theorem 1.4 does not depend on $\hat{u}_n$. This is why the latter theorem cannot be obtained by simply using the fact that LFSS is a LFSM of Hurst parameter H_n along the direction of the nth axis.

Remark 1. Once the wavelet representation (5) is established, the scheme of proof for the above results is similar to those used in [1] for FBS, where a wavelet series representation similar to (5) is used, but with $\{\epsilon_{j,k}\}$ Gaussian. However the behavior of the wavelet series is quite different in the stable case and thus needs a specific treatment. The basic reason is that the supremum of iid stable variables behaves differently. This explains why the smoothness of LFSS is different of the one of FBS with the same parameter H.

2. Useful results and main ideas of the proofs

Let us first state a fundamental lemma:

Lemma 2.1. *Let* $\{\epsilon_\lambda,\ \lambda\in\mathbb{Z}^d\}$ *be an arbitrary sequence of identically distributed* $S\alpha S$ *random variables. Then, there exists an event* Ω_1^* *of probability* 1, *such that for any* $\eta>0$ *and any* $\omega\in\Omega_1^*$,

$$\big|\epsilon_\lambda(\omega)\big|\leqslant C_1(\omega)\prod_{l=1}^{d}\big(3+|\lambda_l|\big)^{1/\alpha+\eta}, \tag{9}$$

where $C_1>0$ *is an almost surely finite random variable, only depending on* η.

Lemma 2.1 follows from the fact that for any $\nu \in ((1/\alpha+\eta)^{-1}, \alpha)$,

$$\mathbb{E}\left(\sup_{\lambda\in\mathbb{Z}^d} \frac{|\epsilon_\lambda|^\nu}{\prod_{j=1}^d (3+|\lambda_j|)^{\nu(1/\alpha+\eta)}}\right) \leqslant c_2 \sum_{\lambda\in\mathbb{Z}^d} \prod_{j=1}^d (3+|\lambda_j|)^{-\nu(1/\alpha+\eta)} < \infty.$$

Sketch of the proof of Theorem 1.1. Let us first fix $t \in \mathbb{R}^N$. By expanding for every $l = 1, \dots, N$ the function $s_l \mapsto (t_l - s_l)_+^{H_l-1/\alpha} - (-s_l)_+^{H_l-1/\alpha}$ in the unconditional basis of $L^\alpha(\mathbb{R})$, $\{2^{j_l/\alpha}\psi(2^{j_l}s_l - k_l),\ j_l \in \mathbb{Z} \text{ and } k_l \in \mathbb{Z}\}$ and by using standard properties of stochastic integral with respect to dZ_α one can prove that for any fixed t, the series (5) converges in probability to $X(t)$. Let us now set for every $t \in \mathbb{R}^N$ and $m \in \mathbb{N}$,

$$U_m(t) = \sum_{(j,k)\in D_m^N} 2^{-\langle j,H\rangle}\epsilon_{j,k} \prod_{l=1}^N \left\{\psi^{H_l}(2^{j_l}t_l - k_l) - \psi^{H_l}(-k_l)\right\}, \tag{10}$$

where $D_m = \{(J,K) \in \mathbb{Z}^2;\ |J| \leqslant m \text{ and } |K| \leqslant 2^{m+1}\}$. It follows from (3), Lemma 2.1 and some technical computations that $(U_m(\cdot,\omega))_{m\in\mathbb{N}}$ is a Cauchy sequence in the Hölder space $C^\gamma(\mathcal{K})$ for any $\omega \in \Omega_1^*$. □

The proof of Theorem 1.2 relies on Lemma 2.1 and the following lemma:

Lemma 2.2. *For any $l = 1, \dots, N$ and any arbitrarily small $\eta > 0$, define the functions*

$$S_{H_l,\eta}(x,y) = \sum_{(J,K)\in\mathbb{Z}^2} 2^{-JH_l}\left|\psi^{H_l}(2^Jx - K) - \psi^{H_l}(2^Jy - K)\right|(3+|J|)^{1/\alpha+\eta/2}(3+|K|)^{1/\alpha+\eta/2} \tag{11}$$

and

$$T_{H_l,\eta}(x) = \sum_{(J,K)\in\mathbb{Z}^2} 2^{-JH_l}\left|\psi^{H_l}(2^Jx - K) - \psi^{H_l}(-K)\right|(3+|J|)^{1/\alpha+\eta/2}(3+|K|)^{1/\alpha+\eta/2}. \tag{12}$$

Then there is a constant $c_3 > 0$ such that for all $x, y \in [0,1]$ one has

$$S_{H_l,\eta}(x,y) \leqslant c_3|x-y|^{H_l-1/\alpha-\eta} \quad \text{and} \quad T_{H_l,\eta}(x) \leqslant c_3. \tag{13}$$

Sketch of the proof of Theorem 1.2. For the sake of simplicity we will assume that $\mathcal{K} = [0,1]^N$. It follows from (5), (9), (11), (12) and (13) that one has for every $\omega \in \Omega_1^*$ and every $s, t \in [0,1]^N$

$$\begin{aligned}
\left|X(s,\omega) - X(t,\omega)\right| &\leqslant \sum_{j=1}^N \left|X(t_1,\dots,t_{j-1},s_j,s_{j+1},\dots,s_N;\omega) - X(t_1,\dots,t_{j-1},t_j,s_{j+1},\dots,s_N;\omega)\right| \\
&\leqslant C_4(\omega)\sum_{j=1}^N \left(\prod_{l=1}^{j-1} T_{H_l,\eta}(t_l)\right) \times \left(\prod_{l=j+1}^{N} T_{H_l,\eta}(s_l)\right) \times S_{H_j,\eta}(t_j,s_j) \\
&\leqslant C_5(\omega)\sum_{j=1}^N |t_j - s_j|^{H_j-1/\alpha-\eta}. \qquad \square
\end{aligned}$$

Sketch of the proof of Theorem 1.3. Roughly speaking this proof follows the same lines as that of Theorem 2 in [1]. However we have to replace $\sqrt{\log(3+|J|+|K|)}$ by $(3+|J|)^{1/\alpha+\eta/2}(3+|K|)^{1/\alpha+\eta/2}$. □

Remark 2. In view of the above proofs, Theorems 1.2 and 1.3 hold, more generally, for any process $Y = \{Y(t), t \in \mathbb{R}^N\}$ having a wavelet representation of the form

$$Y(t) = \sum_{(j,k)\in\mathbb{Z}^N\times\mathbb{Z}^N} c_{j,k}\epsilon_{j,k} \prod_{l=1}^N \left\{\Psi_l(2^{j_l}t_l - k_l) - \Psi_l(-k_l)\right\},$$

where the Ψ_l's are continuously differentiable and well-localized functions, $\{c_{j,k},\ j,k \in \mathbb{Z}^N\}$ is a sequence of complex-valued coefficients satisfying $|c_{j,k}| \leqslant c_6 2^{-\langle j,H\rangle}$ for every $j,k \in \mathbb{Z}^N$ ($c_6 > 0$ being a constant) and $\{\epsilon_{j,k},\ j,k \in \mathbb{Z}^N\}$ is an array of identically distributed complex-valued random variables satisfying $\sup_{j,k} \mathbb{E}[|\epsilon_{j,k}|^\nu] < \infty$ for all $\nu < \alpha$. In contrast Theorem 1.4 will rely on the precise definition of $\{\epsilon_{j,k},\ j,k \in \mathbb{Z}^N\}$ in (4).

From now on our goal will be to give the main lines of the proof of Theorem 1.4. For the sake of simplicity we will assume that $N = 2$, $n = 1$, $(a,b) = (0,1)$ and $\hat{u}_n = \hat{u} \in (\theta, 1)$, where θ is an arbitrarily small positive real number. For every $(j,k) \in \mathbb{N} \times \mathbb{Z}$, $u \in \mathbb{R}$ and $\omega \in \Omega_1^*$ (obtained from Lemma 2.1) let us set

$$G_{j,k}(u,\omega) = 2^{j(1+H_1)} \int_{\mathbb{R}} X(s,u,\omega)\psi^{-H_1}\big(2^j s - k\big)\,\mathrm{d}s. \tag{14}$$

In view of (3) and Theorem 1.3 the $S\alpha S$ field $\{G_{j,k}(u)\}_{u\in(\theta,1)}$ is well-defined and has continuous paths almost surely. This field can be viewed as a projection of LFSS on $\mathbb{R}$. The proof of Theorem 1.4 mainly relies on the following lemma:

Lemma 2.3. *One has almost surely, for any arbitrarily small $\eta > 0$,*

$$\liminf_{j\to\infty} \Big\{ 2^{-j(1/\alpha-\eta)} \inf_{u\in(\theta,1)} \sup_{0\leqslant k\leqslant 2^j} \big|G_{j,k}(u)\big| \Big\} > 0. \tag{15}$$

Observe that (15) means that $\inf_{u\in(\theta,1)} \max_{0\leqslant k\leqslant 2^j} |G_{j,k}(u)|$ increases faster than $2^{j(1/\alpha-\eta)}$ as $j \to \infty$. To prove Lemma 2.3 we need some preliminary results. Let us first give some useful properties of the random variables $G_{j,k}(u)$:

Proposition 2.4. *Let $u \in (\theta,1)^{N-1}$ be fixed, then the following results hold*:

(a) $\{G_{j,k}(u),\ (j,k) \in \mathbb{N} \times \mathbb{Z}\}$ *is a sequence of identically distributed $S\alpha S(\sigma(u))$ random variables, where*

$$\sigma^\alpha(u) = \|\psi\|^\alpha_{L^\alpha(\mathbb{R})} \int_{\mathbb{R}} \big|(u-s)_+^{H_2-1/\alpha} - (-s)_+^{H_2-1/\alpha}\big|^\alpha\,\mathrm{d}s. \tag{16}$$

(b) *Let $L > 0$ be a constant such that the support of ψ is included in $[-L, L]$. Then for any integers $p > 2L$ and $j \geqslant 0$, $\{G_{j,qp}(u);\ q \in \mathbb{Z}\}$ is a sequence of independent random variables.*

To obtain Proposition 2.4 we use that the random variable $G_{j,k}(u)$ has the following stochastic integral representation:

Proposition 2.5. *For every $(j,k) \in \mathbb{N} \times \mathbb{Z}$ and $u \in (\theta,1)$ one has almost surely*

$$G_{j,k}(u) = \int_{\mathbb{R}^2} \big[2^{j/\alpha}\psi\big(2^j s_1 - k\big)\big(\big\{(u-s_2)_+^{H_2-1/\alpha} - (-s_2)_+^{H_2-1/\alpha}\big\}\big)\big]\,\mathrm{d}Z_\alpha(s_1,s_2). \tag{17}$$

The first step for proving Lemma 2.3 consists of showing that (15) holds when inf over $u \in (\theta,1)$ is replaced by an inf over the numbers of the form $M^{-j}\hat{k}$ with $\hat{k} \in (M^j\theta, M^j) \cap \mathbb{Z}$. We denote by M an arbitrary number satisfying $\frac{\alpha\max\{H_1,H_2\}-1}{\alpha\min\{H_1,H_2\}-1} < \frac{\log M}{\log 2}$.

Lemma 2.6. *For any $j \in \mathbb{N}$ let*

$$\nu(j) = \inf\Big\{ \sup_{0\leqslant k\leqslant 2^j} \big|G_{j,k}\big(M^{-j}\hat{k}\big)\big|;\ M^{-j}\hat{k} \in (\theta,1) \text{ and } \hat{k} \in \mathbb{Z} \Big\}. \tag{18}$$

Then one has almost surely for any arbitrarily small $\eta > 0$,

$$\liminf_{j\to\infty} 2^{-j(1/\alpha-\eta)}\nu(j) > 0. \tag{19}$$

Lemma 2.6 can be obtained by using the Borel–Cantelli Lemma, Proposition 2.4 and the fact that any $S\alpha S(\sigma)$ random variable X is heavy-tailed and satisfies $\lim_{t\to+\infty} t^\alpha \mathbb{P}(|X| > t) = \sigma^\alpha c_\alpha$ where $c_\alpha > 0$ is a constant only depending on α.

The second step for proving Lemma 2.3 is to show that the increments of the field $\{G_{j,k}(u)\}_{u\in(\theta,1)}$ can be controlled uniformly in the indices j and k, namely the following proposition:

Proposition 2.7. *Let Ω_1^* be the event of probability* 1 *that was introduced in Lemma* 2.1*. Then for any $\eta > 0$, there is an almost surely finite random variable $C_7 > 0$ such that for every $j \in \mathbb{N}$, $k \in \{0, \ldots, 2^j\}$, $u, v \in (\theta, 1)$ and $\omega \in \Omega_1^*$, one has*

$$\left|G_{j,k}(u,\omega) - G_{j,k}(v,\omega)\right| \leqslant C_7(\omega)2^{jH_1}|u - v|^{H_2-1/\alpha-\eta}. \tag{20}$$

The proof of the latter proposition is similar to that of Lemma 3 in [1]. By putting together Proposition 2.7 and Lemma 2.6 one can get Lemma 2.3. We are in a position to sketch the proof of Theorem 1.4.

Sketch of the proof of Theorem 1.4. Let Ω_2^* be the event of probability 1 on which (15) holds and let Ω_1^* be the event of probability 1 introduced in Lemma 2.1. We denote $\Omega_3^* = \Omega_1^* \cap \Omega_2^*$. Suppose ad absurdum that there exists $\omega_0 \in \Omega_3^*$ such that (8) is not satisfied i.e. there exists $\hat{u} \in (\theta, 1)$ and some constants $C_8 > 0$ and $\eta > 0$ such that, for any $s, t \in (0, 1)$,

$$\left|X\big((s,\hat{u}),\omega_0\big) - X\big((t,\hat{u}),\omega_0\big)\right| \leqslant C_8|s - t|^{H_1-1/\alpha+\eta}. \tag{21}$$

Then using the main ideas of the proof of Lemma 4 in [1] one can show that there is a non-trivial interval $I \subset (0, 1)$ and $C_9 > 0$ such that for any $(j, k) \in \mathbb{N} \times \mathbb{Z}$ satisfying $2^{-j}k \in I$, one has $|G_{j,k}(\hat{u}, \omega_0)| \leqslant C_9 2^{j(1/\alpha-\eta)}$. This contradicts (15). □

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