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AN OVERVIEW OF REVENUE MANAGEMENT AND DYNAMIC PRICING MODELS IN HOTEL BUSINESS

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Abstract. Basic concepts and brief description of revenue management models and decision tools in the hotel business are presented. An overview of the relevant literature on dynamic pricing, forecasting methods and optimization models is provided. The main ideas of the authors' customized revenue management method for the hotel business are presented.

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1. Introduction

Price is one of the most effective variables of the business profit. By changing the price, hotel managers can encourage or restrict the demand in a short term, as well as regulate the on-hand inventories (free rooms). A rapid development of the information technologies, growth of the e-commerce and the universal deployment of the Internet have led to the situation that, in the first decade of the 21st century, the dynamic pricing tools have become an active component of the revenue management systems, see Feng and Gallego [39], Dasu and Tong [33], Anjos, Cheng and Currie [3] and Lin [80]. The main reasons for the increasing implementation of these tools are the following: (1) digital data processing allows efficient collection and use of valuable information about the demand and available inventory, prices of competitors, and processing this information in real time; (2) costs of retyping price tags and informing customers about the price changes have almost disappeared (Brynjolfsson and Smith [18]), (3) customers can easily follow the price changes. Abrate and Viglia [1] observed that Internet also has a feedback affect on dynamic pricing because online reputation becomes more important than the traditional star rating of the hotels.

Revenue management and dynamic pricing belong to the most popular intelligent decision tools to increase profitability of various businesses, see Palmer and McMahon-Beattie [96]. Through them, a hotel offers prices which correlate with the current level of the demand and occupancy, and respond to their changes. Dynamic pricing can be used as a tool to compete for the maximal profit with firms offering the same service (Rubel [102],

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Sybdari and Pyke [106]). Computer experiments conducted by Koenig and Meissner [67] and analytical results of Sato and Sawaki [103] revealed an advantage of dynamic pricing *versus* list pricing.

We will describe basic concepts of revenue management and dynamic pricing, their interrelation and applicability in the hotel business. A review of the relevant operational research literature will be given.

1.1. Definitions of revenue management

We will give several popular definitions of the revenue management in the hotel terminology. Note that the term of revenue management replaced the earlier concept of yield management, see Kimes [63]. El Haddad *et al.* [51] define revenue management as a tool that correlates supply of rooms with demand and maximizes income of a hotel by dividing its customers into different categories based on their booking choices and the current capacity of the hotel. Kimes and Wirtz [65] define the term as employment of the information systems and pricing strategies, which match orders with the corresponding free rooms over time. Jauncey *et al.* [58] consider revenue management as an integrated, continuous, systematic approach for maximizing the income coming from the sale of rooms with variable prices, based on the forecasted demand. Donaghy *et al.* [36] follow approximately the same concept, but also stress the importance of the market segmentation. They define revenue management as a method of maximizing the revenue, which increases the net income of a hotel through the correlation of the predicted number of available rooms with the predefined segments of the market at an optimal price. Jones and Hamilton [59] argue that the revenue management tries to maximize the room price when the demand exceeds the supply, and to maximize the hotel capacity when the supply exceeds the demand, without falling in price below the average cost. All the definitions point to the ability of the revenue management to increase the income of a company without a direct control of costs. The essence of all these definitions is that the *Hotel Revenue Management (HRM)* is a tool to increase the income of a hotel by making appropriate room prices and hotel capacity decisions. Elmaghraby and Keskinocak [37] stress that dynamic pricing models differ from the traditional optimization models of inventory management in which various given demand scenarios with fixed prices are considered.

1.2. Advantages

Dynamic pricing and revenue management techniques first appeared in the passenger air service in the late 1970's. Their advantages were fully revealed by American Airlines in 1985. There, the result of the first year of deployment of the revenue management and dynamic pricing systems led to the income increase by more than 14% and profit increase by 48%, see Nguyen [92]. In the 1990's, the hotel business has begun to adopt passenger air service experience of revenue management and dynamic pricing by adjusting their principles, models and tools for its own specificity. The implementation of the revenue management and dynamic pricing models in the hotel business turned possible because hotel, transportation and other service businesses have the following similar characteristics: (1) limited resources, such as rooms, passenger seats, rented cars, entertainment tickets; (2) the products or services with a limited period of sale, whose value deteriorates over time; (3) the ability to accept orders to be satisfied in the future; (4) low per product or service costs and high fixed costs; (5) fluctuating demand for products or services; (6) the ability to segment the market or customers, see Kimes [63] and Casado and Ferrer [22]. Many service companies possess these characteristics. That is why, in the recent past, such companies which offer renting of convention centers, golf courses, cars, traveling on cruise liners, as well as restaurants, shopping centers, etc., have begun to use revenue management and dynamic pricing in their operations, see Maddah [85]. Zhuang and Li [135] study a dynamic pricing problem with two revenue streams coming from the hotel rooms sales and a casino of this hotel.

1.3. Disadvantages and their handling

Kahneman *et al.* [60, 61], Kimes [62], Wirtz *et al.* [125] reason that, in spite of all obvious advantages of the revenue management and dynamic pricing for the user company, such as the increase of income and no risk in transition to dynamic pricing, these models can cause a sense of "unfair trade" to clients and can lead to the loss of their confidence. An explanation is that, due to the dynamic pricing, the room price for the same period of stay and the same room type can be different for different orders. Customers dissatisfaction from the fact that the same room was sold to another customer at a cheaper price can reduce the demand and lead to the hotel income reduction in the long run. El Haddad *et al.* [51] indicate that the high growth of income from using revenue management models should not be considered successful without rating a monetary equivalent of losing customers confidence to the hotel. Complementing this observations, Palmer and McMahon-Beattie [96] made a conclusion that not just the room prices, but the level of the individual customers awareness of the pricing rules affects clients confidence. Palmer and McMahon-Beattie also argued that certain types of clients are more compliant to the dynamic pricing and like its rules, while the other customers do not understand the rules, and this causes their dissatisfaction. Young, mobile and well-educated people tend to trust dynamic pricing. Moreover, some of them become rational buyers, who do not immediately agree to buy a service if the proposed price is lower than the price they perceive, but they start to analyze and "play" with the service offering companies, trying to forecast future price movement. In order to make revenue management models and decision tools able to account for the behavior of the rational customers, Besanko and Winston [12] suggested using game theory approaches. This suggestion was supported by Bitran and Caldentey [13], Elmaghraby and Keskinocak [37] and Lin and Sidbari [81]. Kwon *et al.* [70] employ evolutionary game theory to account for the demand learning. They describe the demand dynamics by means of a continuous time differential equation. However, the effectiveness of the game theory approaches for modeling the behavior of rational customers is still poorly investigated. Another way to address customer purchase behavior is to employ discrete choice models. Meissner and Strauss [88] suggest to correct bid prices based on any choice-based revenue management method that provides estimates of the marginal value of the service capacity. The basic idea is to start with an initial bid price, and then to raise bid prices in a greedy fashion to exclude service products that have a negative impact on the overall profit because of the buy-down effects.

1.4. Basic publications

Since the first practical success of the revenue management, an extensive research on this subject has been conducted, see, for example, Kimes [63], Bitran and Caldentey [13], Chiang *et al.* [29], Elmaghraby and Keskinocak [37], Weatherford and Bodily [119]. At present, theoretical knowledge, practical experience and application software are well developed in the revenue management for airlines (McGill and van Ryzin [87]). Papers on revenue management in other businesses often refer to the results from this business and employ its terminology. Similarity of the sale conditions between the hotel rooms and seats in the airplane explains that some authors describe only the transition conditions of a model from one area to another. For hospitality business, there exist several important studies considering systematic aspects of revenue management and dynamic pricing (Kimes [63], Jones and Hamilton [59], Chiang *et al.* [29], Ivanov and Zhechev [56, 57]), as well as the forecasting component alone (Burger *et al.* [19], Chen and Kachani [25], Phumchusri and Mongkolkul [98]) and an optimization component alone (Bitran and Monschein [14], Goldman *et al.* [49]). Academic publications rarely describe complex revenue management approaches for hotels. On the other hand, practical hotel revenue management systems for major hotel chains exist, see for example, Koushik *et al.* [69] and Pekgun *et al.* [97]. Based on data collected from almost 1000 European hotels, Abrate *et al.* [2] report that the large majority of hotels use dynamic pricing of some form. Our experience shows that there is a lack of revenue management models and methods, which are easy for use and include all necessary components, as well as inexpensive software tools for small and medium size hotels.

Our review considers studies of revenue management in the hotel business which have been carried out since the late 1990's mostly. We also touch research of revenue management in the other businesses which have direct implications for the hotel business. The rest of the paper is organized as follows. Section 2 describes general structure of an HRM system, surveys revenue management tools and classification schemes for dynamic pricing methods. In Section 3, processes of revenue management are considered. A special attention is paid to the processes of forecasting and optimization. Section 4 presents key ideas of our customized revenue management method for the hotel business. Section 5 summarizes this survey and points future research directions.

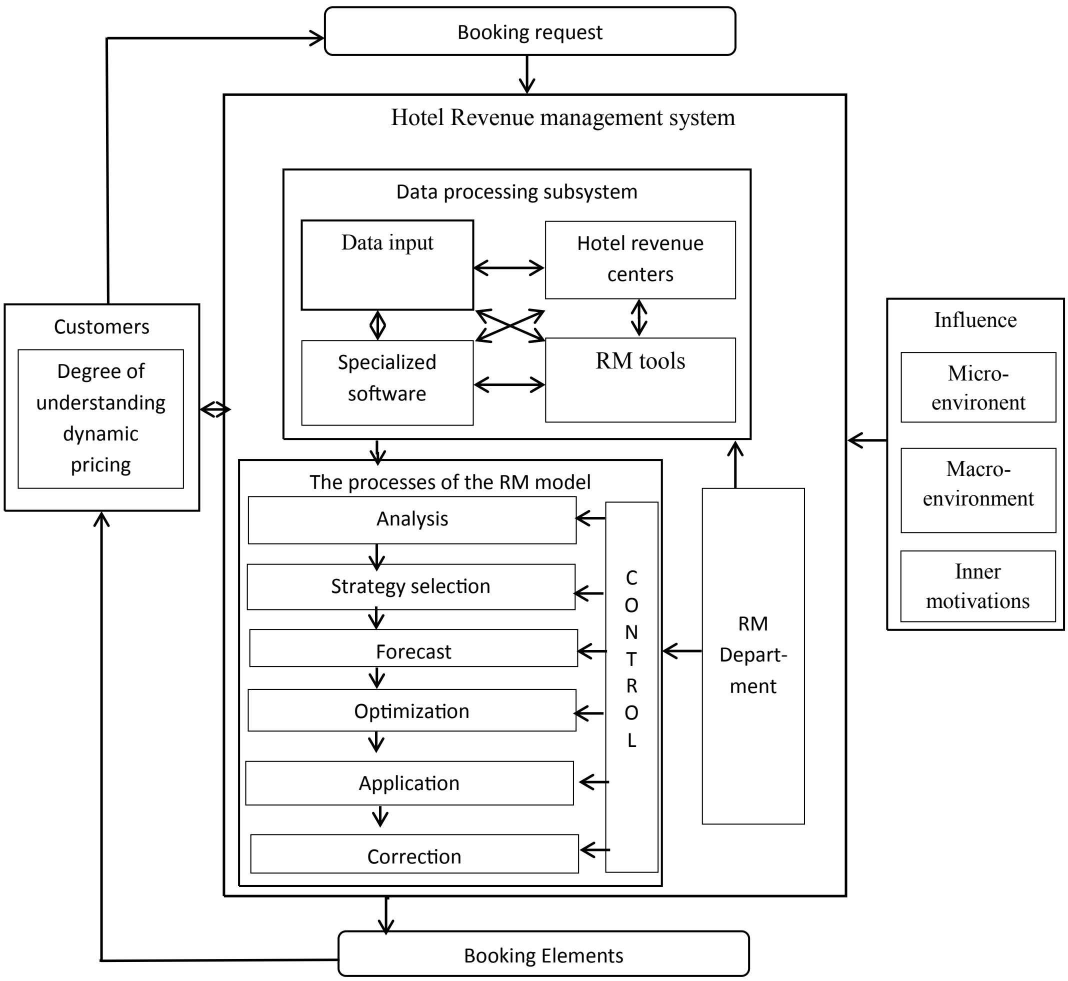

FIGURE 1. Revenue management system of a hotel.

2. Hotel revenue management system

2.1. System structure

Revenue management of a hotel can be represented as a system with interconnected elements. A general structure of such a system is given in Figure 1. It is a refined version of the structure suggested by Ivanov and Zhechev [57]. There, abbreviation RM stands for revenue management.

The system operates as follows. A booking request comes from a client and the system registers it. The system includes the revenue management department, a subsystem of processes of the revenue management and a data processing subsystem. The latter subsystem has four closely related elements: (1) data input, (2) hotel revenue centers, (3) specialized software, and (4) revenue management tools. Input data contain all the information about the booking request and, possibly, information about the customer. Specialized software registers a booking request and begins its processing with a certain strategy. If the hotel has only one revenue center, then it is responsible only for the basic income from the room sales. If there are several revenue centers, then each of them is responsible for the corresponding service: spa and fitness area, restaurant and bar, game room, and others. The subsystem of processes treats a specific order and determines its status: the number and types of ordered rooms, period of stay and price. Revenue management department, directly or indirectly, approves the result of this treatment, and it goes back to the customer. The result and the decision approach of handling the orders affect the customers perception of the hotel pricing system and the hotel in general, and customers intention to deal with the hotel in the future. Revenue management system is constantly influenced by the external and internal environments.

2.2. Decision instruments

Choosing the right decision instrument, by which the revenue management system will maximize the hotel income, is very important. There exist many such instruments. Basically, they can be divided into price and non-price ones. Price based instruments include price discrimination, price barriers, dynamic pricing, guaranties of the lowest prices and other tools directly affecting the price. Non-price instruments do not change prices directly, but they are related to the resource management, control of overbooking, room availability and the duration of stay. Both types of instruments are often used in practice simultaneously.

2.3. Non-price instruments

Pullman and Rogers [100] examine resource management tasks from a general perspective. They divide them into short term and strategic ones. Strategic tasks are associated with a physical increase of the hotel capacity (number of rooms) depending on the demand. Short term tasks deal with planning everyday occupancy, check-in/check-out time and workforce timetabling. The process of the overbooking control is based on the assumption that, for some reason, a part of clients will not show to the hotel. Therefore, hotels may sell more rooms than they have, but it is important to plan the excess level. This topic was explored by Hadjinicola and Panayi [52], Ivanov [54, 55], Koide and Ishii [68], Netessine and Shumsky [90]. Less attention in the literature is paid to the control of the duration of stay. Kimes and Chase [64] and Vinod [116] investigated an opportunity for the hotel to fix the minimum number of nights to stay in the periods of high demand.

2.4. Price instruments

The core of price instruments is price discrimination, which is based on the price sensitivity of different groups of customers, such as tourists and business people, see Kimes and Wirtz [65], Hanks *et al.* [53], Ng [91]. Due to the price discrimination, the same room can be sold at different prices to the customers of different groups. To avoid the transition of customers from high prices to low, hotels set up price barriers, see Zhang and Bell [132]. Special conditions of room sale define these barriers. For example, a hotel may sell rooms at low prices only for certain days of the week or for a certain minimal duration of stay. It can keep a strict policy of cancellation or sell specific rooms only to certain types of customers. Sometimes hotels guarantee customers the lowest price, which is available on the market. This means that, if a client in 24 hours will find another hotel with a room at a lower price, they will equate the prices. This approach was explored by Carvell and Quan [21] and Demirciftci *et al.* [35]. Lanady [71] studied the problem of optimal market segmentation and suggested a model which assumes that the demand-price relations are non-linear. Guo *et al.* [50] developed a similar model and an optimal dynamic pricing strategy based on market segmentation for identical rooms in the online distribution channel of a hotel. Chen and Farias [24] studied a dynamic pricing model in which the demand is unpredictable. They suggested and analyzed a simple pricing policy which observes only sales.

2.5. Combining dynamic pricing with resource and inventory management

Many experts came to understanding that resource optimizing and inventory control decisions cannot be separated from the pricing decisions and that the dynamic pricing tools must be a part of the global revenue management system. An opportunity to handle the forecasted demand by the dynamic pricing tools as well as the optimization models of revenue management is the reason that the names of both methods have become interchangeable, see Boyd and Bilegan [16]. Van Ryzin and Gallego [113] indicate the natural affinity between pricing and resource management models. If price is treated as a variable, then it can be continuously monitored, and a decision to refuse an order can be effectuated by sufficiently raising the price. The revenue management problems through the prism of dynamic pricing were also studied by Ladany and Arbel [72], Gallego and Van Ryzin [47, 113], Feng and Gallego [39] and You [128]. Levi and Shi [77] and Chen *et al.* [28] suggested efficient and simple heuristic approaches to revenue management of reusable resources with advanced reservations within the scope of queuing theory. The results in [28] are applicable for fixed prices as well as for variable prices.

Integration of pricing and capacity allocation decisions have been carried out by Feng and Xiao [43, 44]. Their continuous-time models combine price and inventory decisions, and the pricing and capacity control policy is based on a sequence of precalculated threshold time points that take into consideration the inventory, price and the demand intensity. The set of thresholds are obtained by solving the Hamilton-Jacobi equation. This model applies to maximizing revenues for a single time period. A similar approach has been used by Shi *et al.* [105] for determining the production level and selling price of one type of a product in a make-to-stock manufacturing system. Cao *et al.* [20] extend studies of continuous-time models by incorporating a discounting revenue criterion into them.

McGill and Van Ryzin mentioned the works of Gaimon [46], Lau and Lau [74] and Weatherford [118], where the price determination and the resource management problems are combined. Gaimon attempted to consolidate price and capacity issues. Weatherford considered the average value of a normally distributed demand as a linear function of the price. Some researchers, for example, Boyd and Bilegan [16], tend to separate dynamic pricing models from the revenue management models. However, they still acknowledge their interrelation and similarity in certain cases such as the case of the one room type.

2.6. Classification of dynamic pricing models

There exist several classification schemes for the dynamic pricing models. Bitran and Caldentey [13] formulate a general problem of maximizing the income of a company, which owns a limited, deteriorating in value set of resources, and deals only with the price sensitive customers. For this problem, they suggest using various dynamic pricing models, dividing them into deterministic and stochastic ones. In each category, they study the cases of single and multiple types of products, and consider solutions with one static price for the whole season and with several dynamic prices. Elmaghraby and Keskinocak [37] divide dynamic pricing models into categories based on the following: (1) renewable or non-renewable resources; (2) dependent or independent demand; (3) myopic or rational consumers.

2.7. Price constraints

It should be mentioned that a search for an optimal pricing strategy often includes price constraints. Among the most common constraints are:

- choosing price from a given set, see Chatwin [23], Feng and Gallego [40], Feng and Xiao [41, 42];
- upper limit on the number of price changes, Feng and Gallego [39];
- a given shape of the price function: decreasing or increasing over time, special offers on certain days, see Bitran and Mondschein [15];
- price restrictions for a range of products;
- prices limited by costs.

3. Processes of revenue management

There exist different processes in revenue management. Tranter *et al.* [112] describe eight such processes: customer awareness, market segmentation, internal analysis, competitive analysis, demand forecasting, analysis of distribution channels, dynamic pricing and inventory control. Emeksiz *et al.* [38] suggest five processes to describe a revenue management system: preparation, supply and demand analysis, application of the revenue management system, its evaluation, and monitoring and making changes to the system. Based on the literature review and our experience in the hotel business, we suggest that five processes – analysis, forecasting, optimization, control and adjustment – can be used to adequately describe proper functioning of a HRM system. Analysis includes processing the input data, the most important of which are the demand and the information about the clients and the hotel resources. Forecasting and optimization are the two most important and necessary components of the whole system, see Cross [31]. At the transition from forecasting to optimization, there is a connection of the future demand with the hotel capacities. It is important to have a low forecast error, which makes the optimization model adequate. The choice of the forecasting method depends of the demand behavior, and the choice of the optimization tool depends on the truthfulness and accuracy of the input forecasted data and the computational complexity of the optimization problem. Control consists in monitoring the achievement of the main goal – maximization of income – and in identifying errors and omissions of the modeling approach. Adjustment aims at properly correcting the errors so that they do not appear in the future. Below we will describe in detail the two main components – forecasting and optimization.

3.1. Forecasting

Forecasting is an essential and necessary part of any HRM system. Its mission is to determine future demand for the hotel rooms. The quality of the revenue management system is highly dependent on the forecast accuracy. Pölt [99] calculated that, when using a revenue management system, reducing the forecast error by 20% leads to the 1% increase of the income. Before setting a forecasting model, the following questions have to be answered: (1) what to forecast; (2) which degree of aggregation of the forecasting objects to choose; (3) to restrict or not to restrict the demand; (4) which historical period, called *forecast base*, to use; (5) which *forecasting horizon* to choose; (6) which forecasting method to use; (7) which accuracy is reasonable.

3.1.1. Demand forecasting

The main forecasting object in the hotel business is the demand, each unit of which, called an order, a reservation or a booking, specifies the reservation date, the arrival date, the room type and the duration of stay. It can be also associated with a probability of cancellation. The reservations can be placed days, weeks or months before the arrival date. The nature of reservation cancellations is similar to the reservations, except for the two important features: one can only cancel a confirmed order, and an order can be canceled a given number of days before the arrival date. Sierag *et al.* [107] write that a model without cancellations can lead to a revenue loss of up to 20%. The difference between the number of real bookings and the number of cancellations is called net reservations.

The demand can be of different degree of aggregation – aggregated, partly aggregated and completely disaggregated demand – and this degree implies using the corresponding forecasting approach. The choice of the aggregation degree depends on the type of the available input data. The completely aggregated forecasting approach generates the overall future demand of the hotel, which is further divided between room categories based on the given ratios between them. The completely disaggregated approach generates future demand for each category, and then, if it is needed, the data is combined. Weatherford *et al.* [121] argue that the fully disaggregated forecast usually gives better results than partly aggregated or aggregated forecast.

The demand in a hotel business has a high degree of seasonality. If a small forecast base period is used, for example, eight–twelve weeks, then the seasonality cannot be properly addressed, and if the period is large, then the seasonality can be better addressed, but, in this case, a proper base period has to be chosen. A large forecast base period can make the forecast not responsive enough. The period for which the forecast is built is called forecasting horizon. Forecasting horizon can be long-term and short-term. The long-term horizon usually covers one year. The short-term horizon usually varies from one day to three months.

3.1.2. Forecasting methods

Lee [75] identifies three types of forecasting methods: historical, advanced and combined. Historical methods include exponential smoothing, moving average, copying demand from the same day of the previous year, linear regression and autoregressive methods of Box-Jenkins ARMA and ARIMA. ARMA method combines autoregressive method and the moving average method, and it applies only to the stationary time series. ARIMA methods extend ARMA methods for the non-stationary time series. The historical methods use data only from

TABLE 1. Forecasting methods.

Historical	Exponential smoothing	Burger *et al.* [19], Chen and Kachani [25], Rajopadhye *et al.* [101], Weatherford and Kimes [120], Yüksel [129], Phumchusri and Mongkolkul [98]
	Moving average	Burger *et al.* [19], Weatherford and Kimes [120], Yüksel [129]
	Autoregressive	*et al.* [19], Lim and Chan [78], Lim *et al.* [79], Yüksel [129]
Progressive	Additive	Chen and Kachani [25], Weatherford and Kimes [120]
	Multiplicative	Weatherford and Kimes [120]
Combined	Regressive	Burger et.al. [19], Chen and Kachani [25], Weatherford and Kimes [120]
	Weighted average	Chen and Kachani [25]

a certain period in the past, such as the total number of arrivals in a particular day. We observed that the early studies often concentrated on simple methods, while the later studies deal with the more sophisticated methods such as ARIMA. The results of the forecasting competition accomplished by Makridakis *et al.* [86] show that the sophisticated methods such as ARIMA do not perform statistically better than the simple methods in the computer experiments with real data.

Advanced methods, also called pickup methods, consider future as well as already committed reservations. There are additive and multiplicative versions of the advanced methods. The additive version assumes that the number of already committed reservations at a certain day before the arrival is independent of the final number of reservations for the arrival day. In contrast, the multiplicative version assumes that the number of already committed reservations influences future reservations. In the additive method, the number of reservations for a certain day T, forecasted at the current day $T-k$, is obtained as the sum of the number of already committed reservations for day T and the sum of k numbers c_t, $t=0,1,\dots,k$, where c_t is the number of reservations made for the day $T-t-i$, $i=0,1,\dots,L$, t days before the arrival and averaged over $i=1,\dots,L$, and $T-k-L$ is the first day of the historical period. In the multiplicative method, the forecasted number of reservations for day T forecasted at the current day $T-k$ is obtained as the product of the number of already committed reservations for day T and the product of k numbers p_t, $t=0,1,\dots,k$, where p_t is the average ratio of number of reservations made for day $T-t-u$ to the number of reservations at day $T-t-u+1$, $u=1,\dots,L$, and $T-k-L$ is the first day of the historical period.

Combined methods use the best features of the historical and advanced methods and combine them, either by weighted averaging or regression methods. The method of using neural networks also belongs to this group. Fildes and Ord [45] and Ben-Akiva [9] believe that the combined methods provide the most accurate forecast results. A short overview of the forecasting methods is given in Table 1.

3.1.3. Forecast accuracy

Making a proper choice of the forecast method is very important. Most often, accuracy is the main criterion for this choice. There are several measures to assess the accuracy of the forecast. An assessment based on the *Mean Absolute Error (MAE)* is the most simple and applicable method. Absolute deviations of the forecasted past values from the real past values can be calculated for each day of a historical period. The average of these deviations is the MAE. The smaller is the MAE value the better is the forecast. The *Mean Percentage Error (MPE)*, the *Mean Absolute Percentage Error (MAPE)*, the *Root Mean Square Deviation (RMSD)* and other measures indicators are also popular, see Phumchusri and Mongkolkul [98]. Armstrong and Collopy [4] carried out a fairly complete evaluation of error measures with respect to the reliability, construct validity, sensitivity to small changes, protection against outliers and relationship to decision making.

The effectiveness of the forecasting methods can be evaluated in different ways. Weatherford and Kimes [120] used real historical data of Choice Hotels and Marriott Hotels to compare the effectiveness of the forecasting methods. They deduced that the exponential smoothing, the moving average and the method of selecting already committed orders provide the most accurate forecasts. Based on the results reported in the literature, Fildes and Ord [45] concluded that combined methods give better accuracy compared to historical and progressive methods. Zakhary *et al.* [130] observed in their computer experiments with simulated data that the additive version of advanced method gives more accurate results than the multiplicative version. Schnaars [104] noted that, when the input data is highly variable, the method of transferring the demand from the same day in the past is superior to other popular methods. Despite some differences in the appraisals, all researchers agree that different methods should be applied to different data types, determined by the season, type of the customers and other characteristics.

Some researchers propose to incorporate experience and knowledge of experts into the forecasting methods, and combine them with the mathematical instruments. This direction of research is popular nowadays. Several authors state that the hotel managers are able to give a very accurate forecast for the two or three week period, see, for example, Rajopadhye *et al.* [101]. The human assessment is particularly useful in the presence of the external events that can affect the future demand.

3.2. Optimization

Optimization part of the revenue management system is extremely important. It is intended to solve the problem of maximizing the hotel revenue *via* identifying best prices or optimal allocation of limited resources (seats in airplanes, rooms in hotels) or both of them. Taking into account different types of rooms, price fares and durations of stay, this problem is not as simple as it seems. Details of the optimization methods in the revenue management are given by Weatherford [117], McGill and Van Ryzin [87], Boyd and Bilegan [16], Park and Piersma [95].

The first optimization models were developed for the area of passenger air transportation. Then, because of the similarity of mathematical models and the scope, they moved into the hotel business. We will review the existing revenue optimization models by using the air transportation terminology. Occasionally, we will provide hotel interpretation of the results. Optimization techniques of air transportation revenue management are most often published under the name of *seat inventory control*. Seat inventory control (optimization) techniques can be partitioned into two major groups – *class control* and *network seat inventory control* methods. Class control methods are based on stochastic principles which incorporate demand distributions and reservation and cancellation probabilities. They can be divided into *static* and *dynamic* solution methods. Static methods determine the best allocation of seats once, before sales start, based on the demand forecast and capacity information available at this moment. It is common way to use static methods repeatedly over the booking period. Dynamic methods allocate seats in each class over time, depending on the real-time information about reservations and seats availability. Every time the dynamic system gets a request, it decides on the acceptance or rejection of the reservation and the price. Network seat inventory control methods include deterministic and stochastic mathematical programming models, virtual nesting and bid price methods and simulation and dynamic systems approaches. Below we will review these techniques in more detail.

3.2.1. Seat inventory control

Seat inventory control models form the core of the optimization models in the air passenger transportation, see Chiang *et al.* [29]. They aim at maximizing the revenue through the right allocation of the limited number of seats to each of the fare classes. The seat requests occur over time before the flight departure. The seat request specifies a route and a specific fare class. Once an optimal allocation of the seats to the fare classes is computed based on the forecasted demand, it is used to develop a booking control policy, which specifies the rules of accepting or rejecting incoming seat requests. The nature of the customer requests is stochastic, and the customers can pay different prices. Prices for each class in each route segment are given and the airline offers them to the customers. Naturally, at a certain point in time it is more profitable to reject a low fare request for a seat in order to be able to accept a higher one later for the same seat.

The main methods of seat inventory control are: (1) single leg seat inventory control (class control), which optimizes the number of seats sold for each flight leg separately, and (2) *Origin-Destination and Fare (ODF)* class control, also called network seat inventory control, which optimizes the number of seats sold for the entire network of flight legs at all fare classes. ODF control operates with triples (origin, destination, fare class). The flight leg is the direct flight between two points without a stop. Each route in the network consists of one or more flight legs. If a flight is going from Minsk to Istanbul and then to Ankara, then Minsk-Istanbul and Istanbul-Ankara are the legs and Minsk-Ankara is the route. The network refers to the complete network of the flight legs offered by the airline.

3.2.2. Fare classes

Airlines create a set of services known as classes which vary not only because of the separate location of seats in the airplane. For example, assume that an airline sells seats in four classes – A, B, C and D. Each class is associated with its price. Class A is associated with the highest price and deluxe meal, and it has no restriction on the ticket exchange or refund. Class D price is the lowest, no meal is included, and the tickets cannot be exchanged or refunded. Classes B and C have reasonable prices, regular meal is included, and there are some restrictions on the ticket exchange and refund. Different customer segments prefer different fare classes.

3.2.3. Class control

For each leg the class control method determines a certain amount of seats that can be sold in each class. The amount of seats in each class can be different for each leg. For the entire route which comprises several legs, the seats of the same passenger must be of the same class for all legs. For example, a passenger can book tickets of class A on a route comprising leg 1 and leg 2 only if A class tickets are available on both legs. Let us consider the case of two legs POINT1-POINT2 and POINT2-POINT3 and assume that each of them has only one empty seat. There are only two customers willing to buy tickets. One passenger is willing to pay 70$ for class A in the leg POINT1-POINT2 and the other passenger is willing to pay 210$ for class A in the route of two legs POINT1-POINT2 and POINT2-POINT3. In the class control method, seats are available only if the leg and the class are both available at the same time. It is also impossible to block the 70$ request for the class A seat while the 210$ class A seat is still open for sale. The class control method does not control such cases and therefore loses opportunities to increase income.

3.2.4. Static solutions

Littlewood [82] was the first to propose static solutions with two classes. He suggested closing the class of a low price and transfer remaining seats to the higher class when the expected income from the sale of the next seat in this class is lower than the expected income from the sale of the same seat in the higher class. Belobaba [8] offered a so-called nested approach for multiple classes, which is a modification of the approach of Littlewood [82]. The new approach has been termed the *Expected Marginal Seat Revenue approach (EMSRa)*. It produces so called nested protection levels. Such levels are defined as upper bounds on the number of seats allocated to the fare classes. Optimal policies of this approach were independently presented by Curry [32] and Wollmer [127]. Curry suggested that the distribution of the demand is continuous, while Wollmer supposed that it is discrete. Brumelle and McGill [17] suggested another approach, named EMSRb, which considers both continuous and discrete distribution of the demand. It is based on the idea of equating the marginal revenues in the various fare classes. The authors state that the EMSRb approach provides greater protection for higher valued fare classes than the EMSRa approach. The nested approach is commonly used to solve class control problems.

Multistage static stochastic programming model for airlines business was presented by Williams [123]. Since stochastic programming models have become nowadays a very popular decision tool in many applications, including hotel business, let us describe this model in detail. We will use the hotel terminology because the problem in Williams [123] admits an evident hotel business interpretation.

The hotel owns rooms of three types $i = 1, 2, 3$. Types 1 and 2, and 2 and 3 are called *adjacent* to one another. The booking horizon is divided into T time periods and the current time period is $t = 0$. In each time period $t = 0, 1, \dots, T-1$ room reservations are made for time period T. In time period T, there are n_i rooms of type i, and r_i percent of rooms of this type can be transformed into the rooms of the adjacent types. The price of a room of type i to be used in time period T, which is booked and paid in time period t, $0 \le t \le T-1$, can take one of the values $c_{t,i,1}, \dots, c_{t,i,O_t}$, where O_t is the number of price options in time period t. The model of Williams [123] decides the room prices and the numbers of rooms of each type for each time period in the planning horizon. The demand values are the numbers of rooms of each type which will be booked in the current time period and they will be used in time period T. It is assumed that the demand is uncertain and that its values depend on the price. Assume that the forecast gives S_t *demand scenarios* for time period t. While the demand values depend on the price, it is assumed that the demand scenarios do not depend on the price. They depend on the external economical, political and social factors. The demand scenarios in time period t are assumed to be independent events that form a full system of events in this time period. Let the probability of scenario s in time period t, $1 \le s \le S_t$, be $p_{t,s}$. We have $\sum_{s=1}^{S_t} p_{t,s} = 1$.

The model suggests the construction of a *scenario tree*. The tree has $T+1$ *levels* denoted $t = 0, 1, \dots, T$, each consisting of a number of *nodes*. Each node (t, s) of level t is associated with a demand scenario s in time period t, $t = 0, 1, \dots, T$, $s = 1, \dots, S_t$. Level 0 consists of the artificial node $(0, 0)$, where 0 is an artificial scenario that happens with probability 1 in time period 0. It is assumed that, for each node $(t+1, b)$, there is exactly one *arc* $((t, a), (t+1, b))$, which means that the scenario b in time period $t+1$ happens after the scenario a in time period t, $t = 0, 1, \dots, T-1$. This assumption makes precedence relations between the nodes to be tree-like. If there is an arc $((t, a), (t+1, b))$, then the node (t, a) is called a *parent* of the node $v = (t+1, b)$ and denoted as $prnt(v)$. Each node (t, s_t) of level t is associated with a unique *scenario path* $v = ((0, 0), (1, s_1), (2, s_2), \dots, (t, s_t))$ ending in this node, $s_\tau \in \{1, \dots, S_\tau\}$, $\tau = 1, \dots, t$. Since we are in time period 0, the probability that the scenario path $v = ((0, 0), (1, s_1), (2, s_2), \dots, (t, s_t))$ will lead to the demand scenario s_t in time period t is equal to $P_v = \prod_{\tau=1}^{t} p_{\tau, s_\tau}$. Let V_t denote the set of all scenario paths ending in the nodes of level t, $t = 1, \dots, T$. Due to the tree-like precedence relations, $|V_t| = S_t$. Assume that, for each scenario path $v \in V_t$, the demand in time period t for rooms of type i and price o to be used in time period T, denoted as $d_{v,i,o}$, is known or forecasted. There are the following decision variables:

1. $x_{v,i,o}$ – the number of rooms of type i for time period T to be sold in time period t at price o assuming that the scenario path $v \in V_t$ has realized, $0 \le t \le T-1$;
2. $y_{v,i,o}$ – auxiliary indicator variable; $y_{v,i,o} = 1$ if $x_{v,i,o} > 0$ and $y_{v,i,o} = 0$ if $x_{v,i,o} = 0$, $v \in V_t$, $0 \le t \le T-1$;
3. $z_{v,i}$ – auxiliary variable that expresses the total number of rooms of type i for time period T to be sold along the scenario path v, $v \in V_t$, $0 \le t \le T$.

The deterministic model of the problem can be formulated as follows.

$$\max \quad \sum_{t=0}^{T-1} \sum_{v \in V_t} \sum_{o=1}^{O_t} P_v c_{t,i,o} x_{v,i,o}, \tag{3.1}$$

$$\text{s.t.} \ \sum_{o=1}^{O_t} y_{v,i,o} = 1, \ v \in V_t; \ i = 1, 2, 3; \ t = 0, \dots, T-1, \tag{3.2}$$

$$x_{v,i,o} \le d_{v,i,o} y_{v,i,o}, \ v \in V_t; \ i = 1, 2, 3; \ o = 1, \dots, O_t, \ t = 0, \dots, T-1, \tag{3.3}$$

$$z_{v,i} = \sum_{o=1}^{O_1} x_{0,i,o},\ v \in V_1;\ i = 1,2,3, \tag{3.4}$$

$$z_{v,i} = z_{prnt(v),i,o} + \sum_{o=1}^{O_t} x_{prnt(v),i,o},\ v \in V_t; prnt(v) \in V_{t-1};\ i = 1,2,3;\ t = 2,\ldots,T, \tag{3.5}$$

$$z_{v,1} \le (n_1 + \lfloor \frac{r_2 n_2}{100} \rfloor),\ v \in V_T, \tag{3.6}$$

$$z_{v,2} \le (n_2 + \lfloor \frac{r_1 n_1 + r_3 n_3}{100} \rfloor),\ v \in V_T, \tag{3.7}$$

$$z_{v,3} \le (n_3 + \lfloor \frac{r_2 n_2}{100} \rfloor),\ v \in V_T, \tag{3.8}$$

$$z_{v,1} + z_{v,3} \le (n_1 + n_3 + \lfloor \frac{r_2 n_2}{100} \rfloor),\ v \in V_T, \tag{3.9}$$

$$z_{v,1} + z_{v,2} + z_{v,3} \le n_1 + n_2 + n_3,\ v \in V_T, \tag{3.10}$$

$$x_{v,i,o} \in Z_+,\ v \in V_t;\ i = 1,2,3;\ o = 1,\ldots,O_t;\ t = 0,\ldots,T-1, \tag{3.11}$$

$$y_{v,i,o} \in \{0,1\},\ v \in V_t;\ i = 1,2,3;\ o = 1,\ldots,O_t;\ t = 0,\ldots,T-1, \tag{3.12}$$

$$z_{v,i} \in Z_+,\ v \in V_t;\ i = 1,2,3;\ o = 1,\ldots,O_t;\ t = 0,\ldots,T. \tag{3.13}$$

The objective function (3.1) is the total expected income from selling rooms in time periods $t = 0, 1, \ldots, T-1$ for time period T. The equations (3.2) guarantee that in any time period only one price option will be chosen for each room type. The relations (3.3) ensure that for any scenario path and any price option the number of rooms sold for each of the three room types does not exceed the corresponding demand. The equations (3.4) and (3.5) represent recursive calculation of values of variables z *via* values of variables x. The inequalities (3.6)–(3.10) exhibit upper bounds on the total number of rooms of each type available in time period T and their combinations.

3.2.5. Dynamic solutions

In the discrete time dynamic programming model of Lee and Hersh [76] demand for each class is modeled by an inhomogeneous Poisson process of a Markovian type in such a way that, at any given time t, the booking requests before time t do not affect the decision to be made at time t. The decision rule is that a booking request is accepted if its fare exceeds the opportunity costs of the seat. Authors define opportunity costs as the expected marginal value of the seat at time t. Kleywegt and Papastavrou [66] showed that the class control problem can be formulated as a dynamic stochastic knapsack problem. Subramanian *et al.* [109] added accounting for cancellations to the model proposed by Lee and Hersh.

3.2.6. Network seat inventory control

Comparing with the class control method, the network seat inventory control method is more efficient for reservations which include transfers, because it optimizes the entire network of flights in all fare classes offered by the airline. One of the techniques of this method is to distribute the expected income of the entire route in proportion to its legs and then to use the class control method for each leg. Glover *et al.* [48], Talluri and Van Ryzin [111] and many others formulate the problem of network seat inventory control as the following deterministic mathematical programming problem.

$$\max \quad \sum_{i \in I} r_i x_i, \tag{3.14}$$

$$\text{s.t.} \sum_{i \in I(l)} x_i \le c_l,\ l \in L, \tag{3.15}$$

$$x_i \le d_i,\ i \in I, \tag{3.16}$$

$$x_i \ge 0,\ i \in I. \tag{3.17}$$

where I is the set of all pairs (route, class), r_i is the price of one seat for the (route, class) pair i, variable x_i is the number of orders for the pair i, L is the set of legs in the network, $I(l)$, $I(l) \subset I$, is the set of pairs (route, class) for the leg l, c_l is the capacity of the leg l, and d_i is the expectation of the number of orders for the pair i. The problem is to determine numbers of orders which maximize the total income $\sum_{i \in I} r_i x_i$. Aziz *et al.* [5] suggested an adaptation of this model for hotel revenue maximization.

Let x^* denote an optimal solution of the problem (3.14)–(3.17). A booking control policy is generated by setting upper bound x_i^* on the number of orders for each pair i, $i \in I$. As it is mentioned by many authors, e.g., Pak and Piersma [95] and de Boer *et al.* [34], the optimal revenue value of (3.14)–(3.17) is an upper bound for the same stochastic problem. The problem (3.14)–(3.17) assumes that there is a single flight in a single time window for each route in the network. Multiple flights of the same route can be considered by making copies of this route. A stochastic version of the model (3.14)–(3.17) was suggested by Wollmer [126]. This model, called *Expected Marginal Revenue (EMR)* model, is the following.

$$\max \quad \sum_{i \in I} \sum_{k=1}^{c_i^0} r_i P_{D_i \ge k} X_{i,k}, \tag{3.18}$$

$$\text{s.t.} \sum_{i \in I(l)} \sum_{k=1}^{c_i^0} X_{i,k} \le c_l,\ l \in L, \tag{3.19}$$

$$X_{i,k} \in \{0,1\},\ i \in I,\ k = 1, 2, \ldots, c_i^0, \tag{3.20}$$

where D_i is the demand for the (route, class) pair i, $P_{D_i \ge k}$ is the probability that this demand will be at least k, and $c_i^0 = \max\{c_l \mid i \in I(l), l \in L\}$ is the largest number of seats available along all legs of the pair i. Decision variable $X_{i,k}$ is equal to 1 if at least k seats are reserved for the pair i, and it is equal to 0 otherwise. The value of $r_i P_{D_i \ge k}$ represents the expected marginal revenue of allocating an additional kth seat to the pair i. A more sophisticated model of similar type that addresses service product upgrades is suggested by Steinhardt and Gönsch [108].

General stochastic network models, which are based on Markov decision processes, dynamic programming decomposition and several types of approximations, are offered by Van Ryzin and Talluri [114], Zhang and Lu [133] and Zhang and Weatherford [134]. Meissner and Strauss [89] incorporated customer choice into these models, in which probability of selecting a certain product by the arriving customer is given. A customized application of Markov decision processes to the problem of determining rental rates in the apartment lease industry is suggested by Chen *et al.* [27]. Özkan *et al.* [93] formulate a Markov decision process for situations in which demand depends on the current external environment, representing economic, financial, social or other factors that affect customer behavior.

3.2.7. Virtual nesting and bid price methods

The most frequently used approaches in the network seat inventory control are the virtual nesting and the bid price method. The virtual nesting approach is similar to the class control method, but it eliminates the major inconvenience of the latter method by creating "virtual buckets" of seats based on the value rather than on the class. The approach creates value based virtual buckets on each leg, and then requests for each leg in each pair (route, class) are assigned to these virtual buckets.

Table 2. Virtual nesting of seats.

Buckets/legs	POINT1-POINT2, number of seats	POINT2-POINT3, number of seats
Bucket 1 (high value)	1	1
Bucket 2 (low value)	0	0

Consider the example of two legs and two passengers in Section 3.2.3. Two virtual buckets are created in this case: Bucket 1 is for high value requests, and Bucket 2 is for low value requests, see Table 2. Seats are made available in Bucket 1 on both legs. To block a low value request and make a high value request eligible, the method will assign the 70$ Class A request on the leg POINT1-POINT2 to Bucket 2 and the 210$ Class A request on the legs POINT1-POINT2 and POINT1-POINT2 to Bucket 1. Note that, if there are multiple fare requests, the process of assigning them to the buckets is not trivial. There are several approaches to assign different fare requests to the buckets, see Williamson [124] and Boer [34].

The bid price method is similar to virtual nesting but it avoids complications with assigning requests for pairs (route, class). The bid price is associated with the shadow price and the displacement/opportunity cost of reducing the capacity of the leg by one seat, see Williamson [124]. A shadow price is linked to each leg in the network and it represents a marginal lost from reducing the capacity of this leg by one seat. The bid price (value, opportunity cost of selling one seat) of a pair (route, class) in the network is equal to the sum of the shadow prices over the legs comprising the route. A class for a route is opened for sale if the fare associated with this pair (route, class) exceeds its bid price. Otherwise, the class is closed. An advantage of the bid price method is that it takes into account the remaining capacity and open/closed pair (route, class) status only. Once a class is opened, there are no limits on the number of accepted requests. In order to control the selling process, the bid prices are refreshed periodically. Thus, some classes are closed and some classes are opened. The bid price method was explored by Williamson [124], Wei [122], Talluri and Van Ryzin [110].

3.2.8. Simulation and dynamic approaches

Bertsimas and de Boer [10] presented a simulation based approach for the network seat inventory control problem. Their approach is a combination of the deterministic linear programming and approximate dynamic programming. It considers expected revenue function as a function of the booking limits. The linear programming model finds initial optimal values of those booking limits. Then the approach improves solutions by considering the stochastic nature of the demand and employing virtual nesting. The booking period is divided into small time periods, and the booking process is simulated for the current time period. The booking control policy is formed only for the current period. Revenue is calculated as the sum of the current period revenue and the estimated revenue of the future periods, which depends on the remaining capacity.

A full dynamic solution of the network seat inventory control problem was first obtained by Chen *et al.* [26]. They formulated the problem as a Markov decision problem and used a linear programming model for the calculation of the objective function. The objective function depends on the time until departure and the remaining capacity of the flights. The customer requests are assumed to be independent of each other and Markovian. In order to accept or reject a request, it is decided whether its fare exceeds opportunity costs or not. The method does an off-line approximation of the objective function but the booking policy is implemented on-line. A similar approach is also suggested by Cooper and Homem-de-Mello [30].

3.2.9. Similarity of air transportation and hotel businesses

Optimization models and methods are almost the same for airlines and hotels. For an example, consider a situation that the hotel can transform any room to be of any type, the number of rooms can change over time, no client can change the room type during the entire stay but changing the rooms is possible. For this situation, the equivalent notions in both businesses are given in Table 3. Because of these relations, the linear

Table 3. Equivalent notions.

Air transportation	Hotel business
Leg	Night
Route	Period of stay
Class	Room type
Capacity of leg l	Number of rooms for night l
Expected number of orders for the pair (route, class)	Expected number of bookings for the pair (period of stay, room type)
Price of one seat for the pair (route, class)	Price of one night for the pair (period of stay, room type)

programming model (3.14)–(3.17) can be used to maximize the total income of the hotel.

The class control method can be interpreted and used for the hotel business too. In the hotel terminology it can be called "room type control" method. The method establishes availability of room types for each date in the planning horizon. A guest can order a room of a certain type if it is available for sale for each date of the stay period of the order. Similar to the class control method, "room type control" method cannot examine reservations by the length of stay. Therefore, reservations for a night or two occupy rooms and do not let the system to accept the reservations with longer length of stay, which leads to the ineffective usage of the resources. An analogue of the virtual nesting method associates different combinations of the triple "arrival date – length of stay – room type" with the different buckets of room requests for each night. The buckets differ by prices of room types. A room is sold if the corresponding room type is present in the same bucket during the entire period of stay. An analogue of the bid price method determines the bid price for every night. A room is sold if the total payment for the corresponding stay exceeds the sum of the bid prices of the nights in the entire stay period.

In a recent review, Ivanov and Zhechev [57] observed that stochastic programming (Goldman *et al.* [49], Lai and Ng [73], Liu *et al.* [83, 84]) and simulation methods (Baker and Collier [7], Rajopadhye *et al.* [101], Zakhary *et al.* [131]) prevail among the optimization tools. Deterministic linear programming methods (Goldman *et al.* [49], Liu *et al.* [84]), integer programming methods (Bertsimas and Shioda [11]), dynamic programming methods (Badinelli [6], Bertsimas and Shioda [11]) and fuzzy goal programming methods (Padhi and Aggarwal [94]) received less attention, but there is a growing interest in them. The bid price method (Baker and Collier [7]) is poorly used in the hotel business.

4. Our revenue management approach

Our revenue management approach for the hotel business employs a rolling horizon decision strategy. It combines adaptive methods of demand forecasting, dynamic pricing and resource management methods for profit maximization. The main stages of the approach are: determination of input parameters and decision variables, data input, forecasting, determination of demand-price relations, optimization and output of the results. Profit maximization is gained by solving a mathematical programming problem with a concave quadratic objective function and linear constraints. Our method allows transforming a room of one type into a room of an adjacent type, it takes into account non-linearity of the objective function and it is efficient in terms of computational time and computer memory. By using the existing terminology, our method can be classified as that of the network seat inventory control type and it can be employed to manage a single property hotel or a hotel chain, provided that the season periods of all hotels are the same. Our method is expected to be competitive to the existing optimization models of stochastic programming, dynamic programming and fuzzy goal programming, which require much more computing resources, what makes them inappropriate for large scale problems or too expensive in order to be run on a high-performance computing hardware. A brief description of our revenue management method is given below.

4.1. Input parameters and decision variables

We conjecture that disaggregation of the demand into several categories will increase the accuracy of the forecast method. We suggest that the demand categories are to be determined by the experts. Parameters defining a category can be: (1) high season-low season indicator, (2) weekday-weekend indicator, (3) indicator of the length of stay (up to 7 nights and more than 8 nights), (4) room type (1, 2, 3), and (5) number of nights between reservation and arrival (1–7 nights, 8–30 nights and more than 30 nights). The set of the specific values of these parameters defines the corresponding demand category. For the suggested parameter set, there are 72 demand categories. The demand value in each category for each night in the future within the planning horizon will be further represented as a function of the corresponding price. These functions will be the input for the optimization stage.

The other input parameters are the planning horizon, the room operation cost for each room type, the lower and upper bounds on the price values in the demand categories, the reference price for each demand category, percent of the rooms of each type, which can be transformed to rooms of the adjacent types, and the corresponding transformation costs. Usually, reference price for a demand category is the price of the last sale in this category. However, it can also account for earlier prices of the same hotel and hotels-competitors, see Viglia *et al.* [115] for a discussion of the reference price concept. We assume that the pairs of adjacent room types are (1,2) and (2,3), that is, rooms of types 1 and 3 are adjacent to a room of type 2. With respect to the price ranges and the reference prices, we suggest two input options: giving lower and upper bounds for each demand category, or giving a reference price for each demand category. If the decision maker gives lower and upper bounds, then the reference price is calculated as the average of these bounds. If the decision maker gives the reference price, which can be the last actual price in this demand category adjusted for the prices of competitors, then the lower and upper bounds on the price values are the result of 50% deviation from the reference price. The decision variables are prices for rooms in each demand category and each night of the planning horizon.

4.2. Data input

The hotel can store the reservation history in different formats. We extract from the historical data the number of realized arrivals for each demand category and each night in the historical period, as well as the length of stay associated with each arrival. Input parameters, time series of the numbers of realized arrivals for each category and time series of lengths of stay for each category are the input data for our revenue management method. Note that the season-offseason and the weekday-weekend indicators uniquely determine the sequence of dates in the time series of each category. For example, the sequence of dates in the category characterized by high season and weekends will consist of Fridays, Saturdays and Sundays of high season months (June, July, August and September). Due to the presence of these indicators, there is no need to take into account the seasonality factor of the time series.

4.3. Forecasting

We forecast the number of arrivals and the length of stay associated with each arrival for each demand category and each night in the planning horizon based on the category dependent time series. Note that disaggregation of the demand into categories can make data in historical periods too sparse and lead to big forecasting errors. Therefore, for sparse data, we suggest to use simple forecasting methods such as moving average with a modification for handling non-integer number of forecasted arrivals. If the historical data are saturated, we suggest to use Holt's forecasting method of double exponential smoothing. We consider data as saturated if at least one arrival is present in the arrival time series for each category and each day of the historical period. For the long term planning horizons such as more than 3 months ahead, we apply a modified "the same day last year" forecasting method. The method takes the value of the same day in the last year and adds to it the trend value calculated by Holt's double exponential smoothing method on the data of the latest month.

Determination of the demand-price relations requires the forecasted number of occupied rooms for each demand category and each night in the planning horizon. This information is obtained by means of arithmetic manipulations with the forecasted arrivals and lengths of stay.

4.4. Determination of demand-price relations

Consider an arbitrary demand category c. We restrictively assume that the demand for this category in any night τ is a linear function $f_{\tau,c}$ of the price $p_{\tau,c}$ for this night and category, and that the slope of this function is the same for all nights in the planning horizon: $f_{\tau,c}(p_{\tau,c}) = a_{\tau,c} - b_c p_{\tau,c}$, where $b_c > 0$ is the category dependent slope of the demand function, which is also called *elasticity coefficient* in the demand-price studies, and $a_{\tau,c} > 0$ is a constant. The elasticity coefficient b_c can be determined by the regression analysis, which operates with the numbers of rooms occupied in the historical period. The elasticity coefficient does not depend on τ because it tends to change only in a long term under the influence of external political and economic factors. The constant $a_{\tau,c}$ is calculated as $a_{\tau,c} = \hat{d}_{\tau,c} + bp_c^0$, where p_c^0 is the reference price for category c and $\hat{d}_{\tau,c}$ is the forecasted number of occupied rooms for this category in day τ.

4.5. Optimization

We will maximize the total profit of the hotel *via* solving the following constrained mathematical programming problem.

$$\begin{aligned}\max \sum_{c=1}^{k} \sum_{\tau=t+1}^{t+T} (a_{\tau,c} - b_c p_{\tau,c})(p_{\tau,c} - h_c) - o_{1,2} \sum_{\tau=t+1}^{t+T} x_{\tau,1,2} \\ - o_{2,1} \sum_{\tau=t+1}^{t+T} x_{\tau,2,1} - o_{2,3} \sum_{\tau=t+1}^{t+T} x_{\tau,2,3} - o_{3,2} \sum_{\tau=t+1}^{t+T} x_{\tau,3,2} - W \sum_{c=1}^{k} \sum_{\tau=t+1}^{t+T} y_{\tau,c},\end{aligned} \tag{4.1}$$

subject to

$$L_{\tau,c} \le p_{\tau,c},\ \tau = t+1, \ldots, t+T,\ c = 1, \ldots, k, \tag{4.2}$$

$$p_{\tau,c} \le U_{\tau,c} + y_{\tau,c},\ \tau = t+1, \ldots, t+T,\ c = 1, \ldots, k, \tag{4.3}$$

$$a_{\tau,c} \ge b_c p_{\tau,c},\ \tau = t+1, \ldots, t+T,\ c = 1, \ldots, k, \tag{4.4}$$

$$p_{\tau,c} \ge h_c,\ \tau = t+1, \ldots, t+T,\ c = 1, \ldots, k, \tag{4.5}$$

$$\sum_{j=1}^{3} \sum_{c \in M_j} (a_{\tau,c} - b_c p_{\tau,c}) \le \sum_{j=1}^{3} R_{\tau,j},\ \tau = t+1, \ldots, t+T, \tag{4.6}$$

$$\sum_{c \in M_1} (a_{\tau,c} - b_c p_{\tau,c}) = x_{\tau,1} + x_{\tau,2,1},\ \tau = t+1, \ldots, t+T, \tag{4.7}$$

$$\sum_{c \in M_2} (a_{\tau,c} - b_c p_{\tau,c}) = x_{\tau,2} + x_{\tau,1,2} + x_{\tau,3,2},\ \tau = t+1, \ldots, t+T, \tag{4.8}$$

$$\sum_{c \in M_3} (a_{\tau,c} - b_c p_{\tau,c}) = x_{\tau,3} + x_{\tau,2,3},\ \tau = t+1, \ldots, t+T, \tag{4.9}$$

$$x_{\tau,1,2} \le \lfloor \frac{r_1 R_{\tau,1}}{100} \rfloor,\ \tau = t+1, \ldots, t+T, \tag{4.10}$$

$$x_{\tau,2,1} + x_{\tau,2,3} \le \lfloor \frac{r_2 R_{\tau,2}}{100} \rfloor,\ \tau = t+1, \ldots, t+T, \tag{4.11}$$

$$x_{\tau,3,2} \leq \lfloor \frac{r_3 R_{\tau,3}}{100} \rfloor, \ \tau = t+1, \ldots, t+T, \tag{4.12}$$

$$x_{\tau,1} + x_{\tau,1,2} \leq R_{\tau,1}, \ \tau = t+1, \ldots, t+T, \tag{4.13}$$

$$x_{\tau,2} + x_{\tau,2,1} + x_{\tau,2,3} \leq R_{\tau,2}, \ \tau = t+1, \ldots, t+T, \tag{4.14}$$

$$x_{\tau,3} + x_{\tau,3,2} \leq R_{\tau,3}, \ \tau = t+1, \ldots, t+T, \tag{4.15}$$

$$p_{\tau,c_1} \leq p_{\tau,c_2}, \ c_1 \in M_1, \ c_2 \in M_2, \ \tau = t+1, \ldots, t+T, \tag{4.16}$$

$$p_{\tau,c_2} \leq p_{\tau,c_3}, \ c_2 \in M_2, \ c_3 \in M_3, \ \tau = t+1, \ldots, t+T, \tag{4.17}$$

$$p_{\tau,c} \geq 0, \ x_{\tau,j} \geq 0, \ x_{\tau,j,i} \geq 0, \ y_{\tau,c} \geq 0, \ \forall \ \tau, \ c, \ i, \ j. \tag{4.18}$$

where the variables are $p_{\tau,c}$, $y_{\tau,c}$, $x_{\tau,j,i}$ and

- $[t+1, t+T]$ is the planning horizon,
- $L_{\tau,c}$ is the lower bound on the price $p_{\tau,c}$,
- $U_{\tau,c}$ is the upper bound on the price $p_{\tau,c}$,
- h_c is the operational cost of a room in category c,
- $R_{\tau,j}$ is the number of rooms of type j available in day τ,
- $x_{\tau,j}$ is the number of rooms of type j with no transformation,
- $x_{\tau,j,i}$ is the number of rooms of type j to be transformed to the adjacent type i,
- r_j is the percent of rooms of type j that can be transformed into rooms of the adjacent types,
- $o_{j,i}$ is the cost of transforming a room of type j into a room of type i,
- $y_{\tau,c}$ is the auxiliary variable which allows price upper bounds to be violated,
- W is a sufficiently large number that is greater than the optimal value of the problem in which all variables y are equal to zero (price upper bounds are not violated). For example, W can be set as the sum of maximal values of the functions $(a_{\tau,c} - b_c p_{\tau,c})(p_{\tau,c} - h_c)$ with respect to the variables $p_{\tau,c}$ for all τ and c,
- M_j is the set of all categories that include room type j. We assume that the sets M_j are numbered in the non-decreasing order of the room prices.

The objective function (4.1) includes total profit with the positive sign and room transformation costs and price upper bounds extension costs with the negative sign. The inequalities (4.2) and (4.3) address price lower and upper bounds, respectively. Note that the positive values of the variables y provide an extra information to the decision maker. The inequalities (4.4) guarantee that the demand, that is, the number of rooms to be occupied, takes non-negative values only. The inequalities (4.5) require that the room price is not less than the room operational cost. The inequalities (4.6), the equalities (4.7)–(4.9) and the inequalities (4.10)–(4.15) address the room transformation constraints. The inequalities (4.6) ensure that the sum of the requested number of rooms of each type in different categories in day τ does not exceed the sum of numbers of available rooms of these types. The equalities (4.7)–(4.9) guarantee that the requested number of rooms of each type in different categories in day τ is equal to the number of rooms of this type that are transformed and not transformed. The inequalities (4.10)–(4.12) restrict the number of rooms to be transformed from each type to the number of rooms in accordance with the corresponding percent value. The inequalities (4.13)–(4.15) ensure that the number of rooms to be transformed from each type does not exceed the available number of rooms of this type. The inequalities (4.16)–(4.17) secure the price hierarchy of room types. The problem (4.1)–(4.18) is a mathematical programming problem with a concave quadratic objective function and linear constraints. The objective function is concave because it is the sum of concave quadratic functions of one variable. The problem (4.1)–(4.18) can be solved by a standard optimization software such as CPLEX or MATLAB.

Note that the problem (4.1)–(4.18) can be decomposed into T subproblems. Each such a subproblem considers one day τ and, hence, index τ of the variables and the input parameters of the subproblem is fixed, $\tau = t+1, \ldots, t+T$. An optimal solution of the original problem is determined by the optimal solutions of the subproblems.

4.6. Output

Solution of the problem (4.1)–(4.18) is the optimal price for each category c in each day τ of the planning horizon, the values of the room transformation variables x and the values of the price upper bound slack variables y. Given optimal prices $p^*_{\tau,c}$, we can compute corresponding demands $a_{\tau,c} - b_c p^*_{\tau,c}$. These values are estimates of the hotel occupancy for each day and room type and they can be used for planning service activities. Solution of the problem (4.1)–(4.18) can be analyzed and approved or modified by the decision makers. An approved solution can be made accessible by the potential clients in a friendly format. There can be two booking policies based on the solution of the problem. The first policy is to accept every incoming request and update solution after each booking. The second policy is to accept as many requests from each category as determined by the optimal demand values $a_{\tau,c} - b_c p^*_{\tau,c}$. The excessive requests will be rejected. The efficiency of the second policy strongly depends on the demand forecast quality.

4.7. Computer experiments

We are currently in the process of developing and implementing a computer system that realizes our revenue management approach. Initial computer experiments with real hotel data were performed. In the experiments, the number of demand categories was 72, with 3 room types. The actual revenue of the example hotel in the past period of 90 days, obtained by employing a static pricing strategy, was compared with the modeled potential revenue generated by our approach for the same past period, assuming that the demand behaves according to the suggested linear elastic model. A modified moving average forecasting method was used. The experiments were run on a conventional PC. Three types of scenarios were considered for the selection of the historical input data. In the *steady* scenario, daily number of check-ins in the historical period does not much differ from the daily number of check-ins in the planning horizon. *Low-grow* and *high-grow* scenarios are characterized by low and high growth, respectively, of daily check-ins in the planning horizon in comparison with the same values in the historical period. Experiments demonstrated that our dynamic pricing approach is efficient because each run required only few seconds. Regarding the solution quality, the average revenue of the hotel in comparison with the static pricing strategy was increased by about 3-5%, with distribution of 3%, 4% and 5% for the steady, low-grow and high-grow input data scenarios.

5. Conclusion

We gave several definitions of the Hotel Revenue Management (HRM) and pointed advantages and disadvantages of the HRM systems. A special attention has been paid to the dynamic pricing. Processes of a HRM system are described and a detailed overview of the research of the forecasting and optimization processes is provided. We discussed what has to be forecasted, described main forecasting methods and measures to asses accuracy of the forecast. For optimization, we reviewed seat inventory control models, gave equivalent notions of air transportation and hotel business and interpreted airlines seat inventory control models in terms of the hotel revenue management. We also introduced our own revenue management method for the hotel business. This method disaggregates the demand into several categories, forecasts the demand in each category, determines demand-price relations assuming their elasticity and finds optimal prices for categories by solving a mathematical programming problem with a concave quadratic objective function and linear constraints. The method allows room transformations.

This review revealed that most of the studies concentrate either on forecasting or on optimization and that a combined revenue management approach is rarely described. The existing practical systems are sophisticated and expensive software tools, which are employed by the major hotel chains. There is a need of complete solution approaches which are easy to implement, computationally efficient and can be employed by small and medium size hotels. We believe that our method described in this paper possesses these characteristics.

References

[1] G. Abrate and G. Viglia, Strategic and tactical price decisions in hotel revenue management. *Tour. Manag.* **55** (2016) 123–132.

[2] G. Abrate, G. Fraquelli and G. Viglia, Dynamic pricing strategies: evidence from European hotels. *Int. J. Hosp. Manag.* **31** (2012) 160–168.

[3] M. Anjos, R. Cheng and C. Currie, Optimal pricing policies for perishable products. *Eur. J. Oper. Res.* **166** (2005) 246–254.

[4] J. Armstrong and F. Collopy, Error measures for generalizing about forecasting methods: Empirical comparisons. *Int. J. Forecast.* **8** (1992) 69–80.

[5] H. Aziz, M. Saleh, M. Rasmy and H. ElShishiny, Dynamic room pricing model for hotel revenue management systems. *Egypt. Inf. J.* **12** (2011) 177–183.

[6] R. Badinelli, An optimal, dynamic policy for hotel yield management. *Eur. J. Oper. Res.* **121** (2000) 476–503.

[7] T. Baker and D. Collier, The benefits of optimizing prices to manage demand in hotel revenue management systems. *Prod. Oper. Manag.* **12** (2003) 502–518.

[8] P. Belobaba, *Air Travel Demand and Airline Seat Inventory Management*. Ph.D. thesis, MIT (1987).

[9] M. Ben-Akiva, Improving airline passenger forecasts using reservation data, in *Fall ORSA/TIMS Conference, St. Louis* (1987).

[10] D. Bertsimas and S. de Boer, Simulation-based booking limits for airline revenue management. *Oper. Res.* **53** (2005) 90–106.

[11] D. Bertsimas and R. Shioda, Restaurant revenue management. *Oper. Res.* **51** (2003) 472–486.

[12] D. Besanko and W. Winston, Optimal price skimming by a monopolist facing rational consumers. *Manag. Sci.* **36** (1990) 555–567.

[13] G. Bitran and R. Caldentey, An overview of pricing models for revenue management. *Manuf. Serv. Oper. Manag.* **5** (2003) 203–229.

[14] G. Bitran and S. Monschein, An application of yield management to the hotel industry considering multiple day stays. *Oper. Res.* **43** (1995) 427–443.

[15] G. Bitran and S. Mondschein, Periodic pricing of seasonal product in retailing. *Manag. Sci.* **43** (1997) 427–443.

[16] E. Boyd and I. Bilegan, Revenue management and e-commerce. *Manag. Sci.* **49** (2003) 1363–1386.

[17] S. Brumelle and J. McGill, Airline seat allocation with multiple nested fare classes. *Oper. Res.* **41** (1993) 127–137.

[18] E. Brynjolfsson and M. Smith, Frictionless commerce? A comparison of internet and conventional retailers. *Manag. Sci.* **46** (1999) 563–585.

[19] C. Burger, M. Dohnal, M. Kathrada and R. Law, A practitioners guide to time-series methods for tourism demand forecasting – a case study of Durban, South Africa. *Tour. Manag.* **22** (2001) 403–409.

[20] P. Cao, J. Li and H. Yan, Optimal dynamic pricing of inventories with stochastic demand and discounted criterion. *Eur. J. Oper. Res.* **217** (2012) 580–588.

[21] S. Carvell and D. Quan, Exotic reservations – low price guarantee. *Int. J. Hosp. Manag.* **27** (2008) 162–169.

[22] E. Casado and J.-C. Ferrer, Consumer price sensitivity in the retail industry: latitude of acceptance with heterogeneous demand. *Eur. J. Oper. Res.* **228** (2013) 418–426.

[23] R. Chatwin, Optimal dynamic pricing of perishable products with stochastic demand and a finite set of prices. *Eur. J. Oper. Res.* **125** (2000) 149–174.

[24] Y. Chen and V.F. Farias, Simple policies for dynamic pricing with imperfect forecasts. *Oper. Res.* **61** (2013) 612–624.

[25] C. Chen and S. Kachani, Forecasting and optimisation for hotel revenue management. *J. Revenue Pricing Manag.* **6** (2007) 163.

[26] V. Chen, D. Gunther and E. Johnson, *A Markov Decision Problem Based Approach to the Airline YM Problem*. Tech. rep., Georgia Institute of Technology, The Logistics Institute (1998).

[27] J. Chen, J. Wang and P. Bell, Lease expiration management for a single lease term in the apartment industry. *Eur. J. Oper. Res.* **238** (2014) 233–244.

[28] Y. Chen, R. Levi and C. Shi, Revenue management of reusable resources with advanced reservations. *Prod. Oper. Manag.* **26** (2017) 836–859.

[29] W. Chiang, J. Chen and X. Xu, Overview of research on revenue management: current issues and future research. *Int. J. Revenue Manag.* **1** (2007) 97–128.

[30] W. Cooper and T.H. de Mello, *A Class of Hybrid Methods for Revenue Management*. Tech. rep., Northwestern University, Department of Industrial Engineering and Manag. Sci. (2003).

[31] R. Cross, Revenue Management. Broadway Books (1997).

[32] R. Curry, Optimal airline seat allocation with fare classes nested by origin and destinations. *Transp. Sci*. **24** (1990) 193–204.

[33] S. Dasu and C. Tong, Dynamic pricing when consumers are strategic: analysis of posted and contingent pricing schemes. *Eur. J. Oper. Res.* **204** (2010) 662–671.

[34] S. de Boer, R. Freling and N. Piersma, Mathematical programming for network revenue management revisited. *Eur. J. Oper. Res.* **137** (2002) 72–92.

[35] T. Demirciftci, C. Cobanoglu, S. Beldona and P. Cummings, Room rate parity analysis across different hotel distribution channels in the U.S. *J. Hosp. Mark. Manag.* **19** (2010) 295–308.

[36] K. Donaghy, U. McMahon and D. McDowell, Yield management: an overview. *Int. J. Hosp. Manag.* **44** (1995) 139–150.

[37] W. Elmaghraby and P. Keskinocak, Dynamic pricing in the presence of inventory considerations: research overview, current practices, and future directions. *Manag. Sci.* **49** (2003) 1287–1305.

[38] M. Emeksiz, D. Gursoy and O. Icoz, A yield management model for five-star hotels: computerized and non-computerized implementation. *Int. J. Hosp. Manag.* **25** (2006) 536–551.
[39] Y. Feng and G. Gallego, Optimal starting times for end-of-season sales and optimal stopping times for promotional fares. *Manag. Sci.* **41** (1995) 1371–1391.
[40] Y. Feng and G. Gallego, Perishable asset revenue management with Markovian time dependent demand intensities. *Manag. Sci.* **46** (2000) 941–956.
[41] Y. Feng and B. Xiao, A continuous-time yield management model with multiple prices and reversible price changes. *Manag. Sci.* **48** (2000) 644–657.
[42] Y. Feng and B. Xiao, Optimal policies of yield management with multiple predetermined prices. *Manag. Sci.* **48** (2000) 332–343.
[43] Y. Feng and B. Xiao, A continuous-time seat control model for single-leg flights with no-shows and optimal overbooking upper bound. *Eur. J. Oper. Res*. **174** (2006) 1298–1316.
[44] Y. Feng and B. Xiao, Integration of pricing and capacity allocation for perishable products. *Eur. J. Oper. Res.* **168** (2006) 17–34.
[45] R. Fildes and K. Ord, Forecasting competitions – their role in improving forecasting practice and research, in A Companion to Economic Forecasting. Blackwell Publishing (2002) 322–353.
[46] C. Gaimon, Simultaneous and dynamic price, production, inventory and capacity decisions. *Eur. J. Oper. Res.* **35** (1988) 426–441.
[47] G. Gallego and G. van Ryzin, Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Manag. Sci.* **40** (1994) 999–1020.
[48] F. Glover, R. Glover, J. Lorenzo and C. McMillan, The passenger mix problem in the scheduled airlines. *Interfaces* **12** (1982) 73–79.
[49] P. Goldman, R. Freling, K. Pak and N. Piersma, Models and techniques for hotel revenue management using a rolling horizon. *J. Revenue Pricing Manag.* **1** (2002) 207–219.
[50] X. Guo, L. Ling, C. Yang, Z. Li and L. Liang, Optimal pricing strategy based on market segmentation for service products using online reservation systems: an application to hotel rooms. *Int. J. Hosp. Manag.* **35** (2013) 274–281.
[51] R.E. Haddad, A. Roper and P. Jones, The impact of revenue management decisions on customers attitudes and behaviours: a case study of a leading UK budget hotel chain, in *EuroCHRIE 2008 Congress, Dubai, 11–14 October 2008, Emirates Hotel School* (2008).
[52] G. Hadjinicola and C. Panayi, The overbooking problem in hotels with multiple tour-operators. *Int. J. Oper. Prod. Manag.* **17** (1997) 874–885.
[53] R. Hanks, R. Cross and R. Noland, Discounting in the hotel industry, A new approach. *Cornell Hotel Restaur. Adm. Q.* **43** (2002) 94–103.
[54] S. Ivanov, Management of overbookings in the hotel industry – basic concepts and practical challenges. *Tour. Today* **6** (2006) 19–32.
[55] S. Ivanov, Dynamic overbooking limits for guaranteed and nonguaranteed hotel reservations. *Tour. Today* **7** (2007) 100–108.
[56] S. Ivanov, Hotel Revenue Management – From Theory to Practice. Zangador, Varna (2014).
[57] S. Ivanov and V. Zhechev, Hotel revenue management – a critical literature review. *Tourism* **60** (2012) 175–197.
[58] S. Jauncey, I. Mitchell and P. Slamet, The meaning and management of yield in hotels. *Int. J. Contemp. Hosp. Manag.* **4** (1995) 23–26.
[59] P. Jones and D. Hamilton, Yield management: putting people in the big picture. *Cornell Hotel Restaur. Adm. Q.* **33** (1992) 89–96.
[60] D. Kahneman, J. Knetsch and R. Thaler, Fairness and the assumptions of economics. *J. Bus.* **76** (1986) 728–741.
[61] D. Kahneman, J. Knetsch and R. Thaler, Fairness as a constraint on profit seeking: entitlements in the market. *Am. Econo. Rev.* **76** (1986) 728–741.
[62] S. Kimes, Perceived fairness of yield management. *Cornell Hotel Restaur. Adm.* **35** (1994) 22–24.
[63] S. Kimes, Revenue management: a retrospective. *Cornell Hotel Restaur. Adm. Q.* **44** (2004) 131–138.
[64] S. Kimes and R. Chase, The strategic levers of yield management. *J. Serv. Res.* **1** (1998) 156–166.
[65] S. Kimes and J. Wirtz, Has revenue management become acceptable? Findings from an international study on the perceived fairness of rate fences. *J. Serv. Res.* **6** (2003) 125–135.
[66] A. Kleywegt and J. Papastavrou, The dynamic and stochastic knapsack problem. *Oper. Res.* **46** (1998) 17–35.
[67] M. Koenig and J. Meissner, List pricing *versus* dynamic pricing: impact on the revenue risk. *Eur. J. Oper. Res.* **204** (2010) 505–512.
[68] T. Koide and H. Ishii, The hotel yield management with two types of room prices, overbooking and cancellations. *Int. J. Prod. Econ.* **93** (2005) 417–428.
[69] D. Koushik, J. Higbie and C. Eister, Retail price optimization at intercontinental hotels group. *Interfaces* **42** (2012) 45–47.
[70] C. Kwon, T. Freisz, R. Mookherjee and B.F.T. Yao, Non-cooperative competition among revenue maximizing service providers with demand learning. *Eur. J. Oper. Res.* **197** (2009) 981–996.
[71] S. Ladany, Optimal market segmentation of hotel rooms – the non-linear case. *Omega* **24** (1996) 29–36
[72] S. Ladany and A. Arbel, Optimal cruise-liner passenger cabin pricing policy. *Eur. J. Oper. Res.* **55** (1991) 136–147.
[73] K.-K. Lai and W.-L. Ng, A stochastic approach to hotel revenue optimization. *Comput. Oper. Res.* **32** (2005) 1059–1072.
[74] A. Lau and H. Lau, The newsboy problem with price dependent demand distribution. *IIE Trans.* **20** (1988) 168–175.

[75] O. Lee, *Airline Reservations Forecasting: Probabilistic and Statistical Models of the Booking Process*. Ph.D. thesis. Massachusetts Institute of Technology (1990).
[76] T. Lee and M. Hersh, A model for dynamic airline seat inventory control with multiple seat bookings. *Transp. Sci.* **27** (1993) 252–265.
[77] R. Levi and C. Shi, Dynamic allocation problems in loss network systems with advanced reservation. Preprint arXiv:1505.03774v1[math.PR] (2015).
[78] C. Lim and F. Chan, An econometric analysis of hotel-motel room nights in New Zealand with stochastic seasonality. *Int. J. Revenue Manag.* **5** (2011) 63–83.
[79] C. Lim, C. Chang and M. McAleer, Forecasting hotel guest nights in New Zealand. *Int. J. Hosp. Manag.* **28** (2009) 228–235.
[80] K. Lin, Dynamic pricing with real-time demand learning. *Eur. J. Oper. Res.* **174** (2006) 522–538
[81] K. Lin and S. Sidbari, Dynamic price competition with discrete customer choices. *Eur. J. Oper. Res.* **197** (2009) 969–980.
[82] K. Littlewood, Forecasting and control of passenger bookings, in *AGIFORS Symposium Proc. 12, Nathanya* (1972).
[83] S. Liu, K. Lai, J. Dong and S. Wang, A stochastic approach to hotel revenue management considering multiple-day stays. *Int. J. Inf. Technol. Decision Mak.* **5** (2006) 545–556.
[84] S. Liu, K. Lai and S. Wang, Booking models for hotel revenue management considering multiple-day stays. *Int. J. Revenue Manag.* **2** (2008) 78–91.
[85] B. Maddah, L. Moussawi-Haidar, M. El-Taha and H. Rida, Dynamic cruise ship revenue management. *Eur. J. Oper. Res.* **207** (2010) 445–455.
[86] S. Makridakis, A. Andersen, R. Carbone, R. Fildes, M. Hibon, R. Lewandowski, et al. The accuracy of extrapolation (time series) methods: results of a forecasting competition. *J. Forecast.* **1** (1982) 111–153.
[87] J. McGill and G. van Ryzin, Revenue management: research overview and prospects. *Transp. Sci.* **33** (1999) 233–256.
[88] J. Meissner and A. Strauss, Improved bid prices for choice-based network revenue management. *Eur. J. Oper. Res.* **217** (2012) 417–427.
[89] J. Meissner and A. Strauss, Network revenue management with inventory-sensitive bid prices and customer choice. *Eur. J. Oper. Res.* **216** (2012) 459–468.
[90] S. Netessine and R. Shumsky, Introduction to the theory and practice of yield management. *INFORMS Trans. Edu.* **3** (2002) 34–44.
[91] I. Ng, A demand-based model for the advance and spot pricing of services. *J. Prod. Brand Manag.* **18** (2009b) 517–528.
[92] Y. Nguyen, Hotel revenue management: a necessary evil, but not sufficient for delivering profitably. Available at http://hotelmarketing.com/index.php/content/article/hotel_revenue_management_a_necessary_evil_but_not_sufficient_for_delivering (2013).
[93] C. Özkan, F. Karaesmen and S. Özekici, Structural properties of Markov modulated revenue management problems. *Eur. J. Oper. Res.* **225** (2013) 324–331.
[94] S. Padhi and V. Aggarwal, Competitive revenue management for fixing quota and price of hotel commodities under uncertainty. *Int. J. Hosp. Manag.* **30** (2011) 725–734.
[95] K. Pak and N. Piersma, *Airline Revenue Management: an Overview of OR Techniques 1982–2001*. Tech. rep., Erasmus University Rotterdam (2002).
[96] A. Palmer, U. McMahon-Beattie, Variable pricing through revenue management: a critical evaluation of affective outcomes. *Manag. Res. News* **31** (2008) 189–199.
[97] P. Pekgun, R.P. Menich, S. Acharya, P.G. Finch, F. Deschamps, K. Mallery, et al. Carlson Rezidor hotel group maximizes revenue through improved demand management and price optimization. *Interfaces* **43** (2013) 21–36.
[98] N. Phumchusri and P. Mongkolkul, Demand forecasting via observed reservation information, in *Proceedings of the Asia Pacific Industrial Engineering and Management Systems Conference, Asia Pacific Industrial Engineering and Management Society, Phuket, Thailand* (2012) 1978–1985.
[99] S. Pölt, Forecasting is difficult – especially if it refers to the future, in *Reservations and Yield Management Study Group Annual Meeting Proceedings, Melbourne, AGIFORS* (1998).
[100] M. Pullman and S. Rogers, Capacity management for hospitality and tourism: a review of current approaches. *Int. J. Hosp. Manag.* **29** (2010) 177–187.
[101] M. Rajopadhye, M. Ghalia, P. Wang, T. Baker and C. Eister, Forecasting uncertain hotel room demand. *Inf. Sci.* **132** (2001) 1–11.
[102] O. Rubel, Stochastic competitive entries and dynamic pricing. *Eur. J. Oper. Res.* **231** (2013) 381–392.
[103] K. Sato and K. Sawaki, A continuous-time dynamic pricing model knowing the competitor's pricing strategy. *Eur. J. Oper. Res.* **229** (2013) 223–229.
[104] S. Schnaars, Situational factors affecting forecast accuracy. *J. Mark. Res.* **21** (1984) 290–297.
[105] X. Shi, H. Shen, T. Wu and T. Cheng, Production planning and pricing policy in a make-to-stock system with uncertain demand subject to machine breakdowns. *Eur. J. Oper. Res.* **238** (2014) 122–129.
[106] S. Sibdari and D. Pyke, Dynamic pricing with uncertain production cost: an alternating-move approach. *Eur. J. Oper. Res.* **236** (2014) 218–228.
[107] D.D. Sierag, G.M. Koole, R. van der Mei, J.I. van der Rest and B. Zwart, Revenue management under customer choice behaviour with cancellations and overbooking. *Eur. J. Oper. Res.* **246** (2015) 170–185.
[108] C. Steinhardt and J. Gönsch, Integrated revenue management approaches for capacity control with planned upgrades. *Eur. J. Oper. Res.* **223** (2012) 380–391.

[109] J. Subramanian, J. Stidham and C. Lautenbacher, Airline yield management with overbooking, cancellations and no-shows. *Transp. Sci.* **33** (1999) 147–167.

[110] K. Talluri and G. van Ryzin, An analysis of bid-price controls for network revenue management. *Manag. Sci.* **44** (1998) 1577–1593.

[111] K. Talluri and G. van Ryzin, A randomized linear programming method for computing network bid prices. *Transp. Sci*. **33** (1999) 207–216.

[112] K. Tranter, T. Stuart-Hill and J. Parker, Introduction to Revenue Management for the Hospitality Industry. Prentice Hall (2008).

[113] G. van Ryzin and G. Gallego, A multi-product dynamic pricing problem and its applications to network yield management. *Oper. Res.* **45** (1997) 24–41.

[114] G. van Ryzin and K.T. Talluri, Revenue management, in Handbook of Transportation Science. Kluwer Academic Publishers (2003) 599–659.

[115] G. Viglia, A. Mauri and M. Carricano, The exploration of hotel reference prices under dynamic pricing scenarios and different forms of competition. *Int. J. Hosp. Manag.* **52** (2016) 46–55.

[116] B. Vinod, Unlocking the value of revenue management in the hotel industry. *J. Revenue Pricing Manag.* **3** (2004) 178–190.

[117] L. Weatherford, A tutorial on optimization in the context of perishable-asset revenue management problems for the airline industry, in Operations Research in the Airline Industry. Kluwer Academic Publishers (1998) 68–100.

[118] L. Weatherford, Optimization of perishable-asset revenue management problems that allow prices as decision variables. *Int. J. Serv. Technol. Manag.* **2** (2001) 71–101.

[119] L. Weatherford and S. Bodily, A taxonomy and research overview of perishable-asset revenue management: yield management overbooking, and pricing. *Oper. Res.* **40** (1992) 831–843.

[120] L. Weatherford and S. Kimes, A comparison of forecasting methods for hotel revenue management. *Int. J. Forecast.* **19** (2003) 401–415.

[121] L. Weatherford, S. Kimes and D. Scott, Forecasting for hotel revenue management: testing aggregation against disaggregation. *Cornell Hotel Restaur. Adm. Q.* **42** (2001) 53–64.

[122] Y. Wei, *Airline O-D Control Using Network Displacement Concepts*. Master's thesis, MIT (1997).

[123] H. Williams, Model Building in Mathematical Programming. John Wiley and Sons (1999).

[124] E. Williamson, *Airline Network Seatinventory Control: Methodologies and Revenue Impacts*. Ph.D. thesis, MIT (1992).

[125] J. Wirtz, S. Kimes, J. Theng and P. Patterson, Revenue management: resolving potential customer conflicts. *J. Revenue Pricing* **3** (2003) 216–226.

[126] R. Wollmer, *Hub-spoke Seat Management Model*. Tech. rep., Douglas Aircraft Company, McDonnell Douglas Corporation, Long Beach, CA (1986).

[127] R. Wollmer, An airline seat management model for a single leg route when lower fare classes book first. *Oper. Res.* **40** (1992) 26–37.

[128] P. You, Dynamic pricing in airline seat management for flights with multiple flight legs. *Trans. Sci.* **33** (1999) 192–206.

[129] S. Yüksel, An integrated forecasting approach to hotel demand. *Math. Comput. Model.* **46** (2007) 1063–1070.

[130] A. Zakhary, N.E. Gayar and A. Atiya, A comparative study of the pickup method and its variations using a simulated hotel reservation data. *ICGST Int. J. Artif. Intel. Mach. Learn.* **8** (2008) 15–21.

[131] A. Zakhary, A. Atiya, H. El-Shishiny and N.E. Gayar, Forecasting hotel arrivals and occupancy using Monte Carlo simulation. *J. Revenue Pricing Manag.* 10 (2011) 344–366.

[132] M. Zhang and P. Bell, Fencing in the context of revenue management. *Int. J. Rev. Manag.* **4** (2010) 42–68.

[133] D. Zhang and Z. Lu, Assessing the value of dynamic pricing in network revenue management. *INFORMS J. Comput.* **25** (2013) 102–115.

[134] D. Zhang and L. Weatherford, Dynamic pricing for network revenue management: a new approach and application in the hotel industry. *INFORMS J. Comput.* **29** (2017) 18–35.

[135] W. Zhuang and M. Li, Dynamic pricing with two revenue streams. *Oper. Res. Lett.* **40** (2012) 46–51.