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A FUZZY IMPERFECT PRODUCTION AND REPAIR INVENTORY MODEL WITH TIME DEPENDENT DEMAND, PRODUCTION AND REPAIR RATES UNDER INFLATIONARY CONDITIONS

Shalini Jain[1], Sunil Tiwari[2],
Leopoldo Eduardo Cárdenas-Barrón[3,*], Ali Akbar Shaikh[3]
and Shiv Raj Singh[4]

Abstract. This research work derives an integrated inventory model for imperfect production/remanufacturing process with time varying demand, production and repair rates under inflationary environment. This inventory model deals with the joint manufacturing and remanufacturing options. There is a collection process devoted to collect used items with the aim to remanufacture them. Both production and repair runs generate imperfect items. The repair process remanufactures used and imperfect items. Further, it is also considered that the remanufactured item that is classified as good has exactly same quality as that of new one. Demand rate is supposed as time dependent. The production rate is assumed to be demand dependent and therefore it is also time dependent. The repair rate is supposed to be a function of time. All system costs are contemplated in uncertain environment. Therefore, the costs are considered as fuzzy nature. Theoretical results are illustrated thru a numerical example. Finally, a sensitivity analysis is performed in order to know the impact of different parameters on the optimal policy.

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1. Introduction and literature review

The concept of reverse logistics is utilized and defined in many distinct forms depending on the viewpoint of researchers and academicians. For example, Kokkinaki *et al.* [19] discussed a closed-loop supply chain model considering reverse logistics. They discussed operations related to the reuse of products and materials.

Recently, the significance of reverse logistics has augmented so much. Now, the organizations are becoming aware of the benefits that these can acquire from reverse logistics operations. Sometimes reverse logistic

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[1] Department of Mathematics, Sh. K.K.J. (PG) College, Khatauli 251 201, India.
[2] The Logistics Institute – Asia Pacific, National University of Singapore, 21 Heng Mui Keng Terrace, Singapore 119613, Singapore.
[3] School of Engineering and Sciences, Tecnológico de Monterrey, E. Garza Sada 2501 Sur, C.P. 64849 Monterrey, Nuevo León, Mexico.
[4] Department of Mathematics, C.C.S. University, Meerut 250 002, India.

[*] Corresponding author: lecarden@itesm.mx

operations can be highly hazardous for some organizations; nonetheless staking a worthy management of reverse logistic operations can lead to a noteworthy increment in profits. It is important to remark that reverse logistics operations are eco-friendly due to the fact that the reuse, refurbish and recycle are good options; putting the landfill as the last choice. According to Rogers and Tibben-Lembke [32] reverse logistics is also defined "as the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods, and related information from consumption point to the starting point for the purpose to capture value or to have a good disposal of the products". In other words, Wikipedia [12] says, "reverse logistics is the logistics process of removing new or used products from any point in a supply chain, for example, returns from consumers, overstocked inventory, or out-of-date goods, and redistributing these utilizing disposition management procedures that will give a maximized value at the expiration of the product's useful life". It is important to mention that reverse logistics process includes the returning and recycling of items.

Schrady [37] was the first to consider a repair–inventory system. Basically, he derived an EOQ inventory model for repairable items by considering no direct disposal cost for the manufacturing and recovery (repair) rates. Later, Nahmias and Rivera [26] revisited and extended Schrady's [37] inventory model to permit a finite repair rate. Richter [29, 30] discussed the production systems having two different shops. The first shop is dedicated to production and recovery, whereas the second shop is devoted to collect used/returned products. In a subsequent paper, Richter [31] extended the cost analysis of his former two papers [29, 30]. The papers [29, 30] analyzed EOQ inventory models for repair and waste disposal by considering variable setups for production and repair. Afterwards, Dobos and Richter [7] developed a production/recycling model for known demand assuming a single repair and a single production lot per interval. In a next paper, Dobos and Richter [8] generalized their previous work [7] by considering multiple repair and production batches in a time interval. Further, Dobos and Richter [9] extended their previous research work [8] by considering the quality of returned products. Later, El-Saadany and Jaber [10] extended Dobos and Richter [7, 8] works by considering that the return rate of used items is according to a demand function dependent on both the acceptance quality degree and purchasing price of returned items where acceptance quality degree and the price are taken as decision variables. Some other interesting research papers in this line are those of Inderfurth *et al.* [13], Jaber and Saadany [14, 15], Jaber and Rosen [16], Konstantaras and Papachristos [20, 21, 22, 23], Konstantaras and Skouri [24], Konstantaras *et al.* [25], just to name few relevant works.

Chung and Wee [5] derived an integrated production inventory model for deteriorating items considering green-component life-cycle and remanufacturing. In their model, they showed that the flow of returned product depends on both the price and quality of items. After that, Chung and Wee [6] extended their work [5] for finite planning horizon. In the same year, Alamri [1] proposed an integrated production model for new products and remanufacturing of returned products. Hsueh [11] examined inventory control policies in a manufacturing/remanufacturing system by considering that both demand rate and return rate of products follow a normal distribution. Singh and Saxena [38] recommended a closed loop structure with remanufacturing for decaying products. Their inventory model is considered for a single product with two different quality standards. Shortages are permitted and completely backlogged. Sarkar and Moon [34] addressed an imperfect production process in order to study the improved quality, setup cost reduction and variable backordering costs. Sarkar *et al.* [36] analyzed the economic production quantity (EPQ) inventory model in a single stage manufacturing with rework process including planned backorders of finished products. In many practical production systems, imperfect items can be reworked which significantly reduces the overall production-inventory costs (Cárdenas-Barrón *et al.* [3], Kim and Sarkar [18], Sarkar and Saren [35], Sarkar *et al.* [36], Taleizadeh *et al.* [40, 42, 43], Tayyab and Sarkar [44]). Sarkar and Mahapatra [33], Taleizadeh *et al.* [41], and Taleizadeh *et al.* [39] discussed fuzzy inventory models by considering different scenarios.

Benkherouf and Omar [2] proposed a finite-horizon and time changing demand rate to determine the optimal manufacturing lot size with rework. They study two policies for addressing with defective items. The first one consists in that defective items fabricated in a given production run are remanufactured within the same period. The second policy considers to accumulate defective items generated during several production runs and then these defective items are remanufactured in a dedicated repair run.

Omar and Yeo [27] formulated a production and repair inventory model considering time-varying demand over a finite planning horizon. They consider that there are multiples production runs, multiples ordering raw material lots, and multiples repair runs. They assume that there is no collection of used items during the repair runs. In a subsequent article, Omar and Yeo [28] presented an inventory model for a manufacturing system which covers a continuous time changing demand within a finite planning horizon. They suppose that there is no collection of used items in the repair cycle. In both Omar and Yeo [27, 28] the production and repair rates are constant and known. It is worth mentioning that production and repair rates are not always constant and in several real life manufacturing systems are time changing. Moreover, the product quality is not always perfect meaning that a proportion of the manufactured products can be found to be defective. Therefore, this paper proposes a reverse logistics inventory model for imperfect production/remanufacturing process with time varying demand, production and repair rates. Demand rate and repair rate are exponentially increasing function of time. Production rate depends on the demand rate and consequently it also depends on time. The collection of used items occurs always in both production run and repair run. Used and imperfect products are accumulated together and remanufactured in the repair run. Only one type of raw material (here after referred to as raw material 1) is needed to produce the finished product. After an order is placed, raw material 1 is immediately replenished. There is a single production run, a single repair run, and a single replenishment of raw material 1 per cycle. Inflation is also contemplated in this inventory model.

The rest of this article is structured as follows. Section 2 provides the problem definition of inventory model, assumptions and notation. Section 3 presents the mathematical formulation of the imperfect production and repair inventory model. Section 4 formulates the fuzzy imperfect production and repair inventory model. Section 5 gives a numerical example. Section 6 makes a sensitivity analysis. Finally, Section 7 presents some conclusions.

2. Problem definition, assumptions and notation

The objective of proposed inventory model is to determine jointly the duration of production run, repair run, and cycle time such that the total cost of the inventory system is minimized. The inventory model assumes a continuous time-varying demand over the cycle time. For simplicity, it is assumed that only one type of raw material (raw material 1) is necessary to manufacture the finished product. A single production run, a single repair run and a single replenishment of raw material 1 per cycle are considered. The general material flow of the inventory model is given in Figure 1.

The following assumptions are used in the development of the inventory model.

2.1. Assumptions

(i) A single type of products inventory system is considered over the planning horizon.
(ii) The demand rate $D(t)$ is an exponentially increasing function of time; it is given by $D(t) = ae^{bt}$ where a, b are constants and $a > b$.
(iii) The production rate $P(t) = kD(t) = kae^{bt}$ is demand dependent and $P(t) > D(t)$ for all t.
(iv) The repair rate $R(t)$ is an exponentially increasing function of time t; it is expressed as $R(t) = ce^{dt}$ and $R(t) > D(t)$ for all t; where c, d are constants.
(v) The collection rate of the used items from customers, $C(t)$, is proportional to the demand rate; it is determined with the following expression: $C(t) = \varphi D(t)$, $0 \le \varphi \le 1$. The collection rate occurs during entire planning horizon.
(vi) Both used and imperfect items are accumulated together and these are remanufactured in the repair run.
(vii) Both production and repair processes generate imperfect items at rate δ.
(viii) All fabricated and remanufactured items are inspected and after the screening process are classify as good or imperfect items.
(ix) Used and imperfect items are remanufactured during repair run and after the reparation process if these are catalogued as good then they are considered as new.

(x) Only one type of raw material (named as raw material 1) is needed to manufacture the finished product. After an order is placed, raw material 1 is immediately replenished.
(xi) There is a single production run, a single repair run, and a single replenishment of raw material 1 per cycle. See Omar and Yeo [28].
(xii) New produced or repaired items are immediately ready to satisfy the demand.
(xiii) Inflation is also considered in the inventory model.
(xiv) Shortages are not allowed during the planning horizon.

2.2. Notation

The following notation is used throughout this paper.

Parameter	**Description**	**Units**
C_P	Setup cost per setup of the production run	(\$/setup)
C_R	Setup cost per setup of the repair run	(\$/setup)
C_1	Ordering cost per order of raw material 1	(\$/order)
h_P	Inventory holding cost of finished items	(\$/unit/time unit)
h_R	Inventory holding cost of used and imperfect items	(\$/unit/time unit)
h_1	Inventory holding cost raw material 1	(\$/unit/time unit)
c_2	Production cost	(\$/unit)
u_R	Returned used item cost	(\$/unit)
l	Inspection cost	(\$/unit)
i	Remanufacturing cost	(\$/unit)
k	Production coefficient	$k > 1$
φ	Collection coefficient	$0 \leq \varphi \leq 1$
r	Inflation rate	(%)
δ	Rate of imperfect production and repair runs	(%)
q_1	Quantity of raw material 1 needed to manufacture one unit of the finished product	(units)
$I_{ui}(t)$	Inventory level of used and imperfect items at any time t	(units)
$I_f(t)$	Inventory level of finished items at any time t	(units)
$I_R(t)$	Inventory level of remanufactured items at any time t	(units)
$I_r(t)$	Inventory level of raw material 1 at any time t	(units)
$TC(t_1, T)$	Total cost per unit of time	(\$/time units)
Dependent variables		
t_2	Total elapsed time up to the start of repair run	(time units)
t_3	Total elapsed time up to the end of repair run	(time units)
Decision variables		
t_1	Total elapsed time up to the end of production run	(time units)
T	Length of cycle time	(time units)

3. Mathematical formulation of the imperfect production and repair inventory model

The imperfect production and repair inventory model is depicted in Figure 2. Production run is within of the interval $[0,\ t_1]$, during this time the inventory level of raw material 1 decreases. The demand $D(t)$ occurs during the period $[0,\ T]$. In the interval $[0,\ t_2]$ the inventory level of imperfect items and collected used items are being accumulated. As a result, the repair process of these items starts at t_2 and finishes at t_3. Notice that collection of used items occurs always from 0 to t_3. The imperfect items are generated during the intervals $[0,\ t_1]$ and

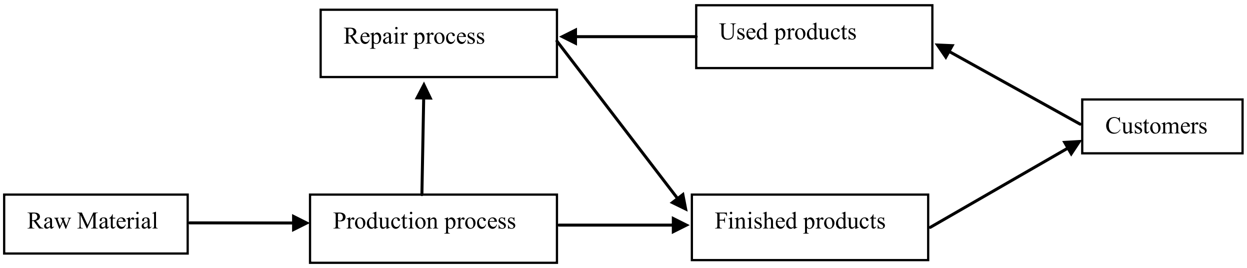

Figure 1. Material flow of the inventory model.

$[t_2,\ t_3]$. During the repair run there is not required raw material 1 because the used and imperfect items are remanufactured on themselves. It is easy to see in Figure 2 that the duration of production run is t_1, repair run is $t_3 - t_2$, production cycle t_2, repair cycle $T - t_2$, used and imperfect items cycle is t_3, raw material 1 cycle is t_1, and cycle time is T.

The differential equations that govern the inventory levels are as follows:

$$I'_{ui}(t) = \phi D(t) + \delta P\,(t) \quad 0 \le t \le t_1 \tag{3.1}$$

$$I'_{ui}(t) = \phi D(t) \quad t_1 \le t \le t_2 \tag{3.2}$$

$$I'_{ui}(t) = \phi D(t) - (1-\delta)\,R(t) \quad t_2 \le t \le t_3 \tag{3.3}$$

$$I'_{f}(t) = (1-\delta)P(t) - D(t) \quad 0 \le t \le t_1 \tag{3.4}$$

$$I'_{f}(t) = -D(t) \quad t_1 \le t \le t_2 \tag{3.5}$$

$$I'_{R}(t) = (1-\delta)R(t) - D(t) \quad t_2 \le t \le t_3 \tag{3.6}$$

$$I'_{R}(t) = -D(t) \quad t_3 \le t \le T \tag{3.7}$$

$$I'_{r}(t) = -q_1 P \quad 0 \le t \le t_1. \tag{3.8}$$

With boundary conditions

$$I_{ui}(0) = 0, I_{ui}(t_3) = 0, \quad I_f(0) = 0, \quad I_f(t_2) = 0, \quad I_R(T) = 0, \quad I_R(t_2) = 0, \quad I_r(t_1) = 0. \tag{3.9}$$

Solutions of these equations are:

$$I_{ui}\,(t) = \frac{(\phi + k\delta)\,a}{b}\left(e^{bt} - 1\right) \quad 0 \le t \le t_1 \tag{3.10}$$

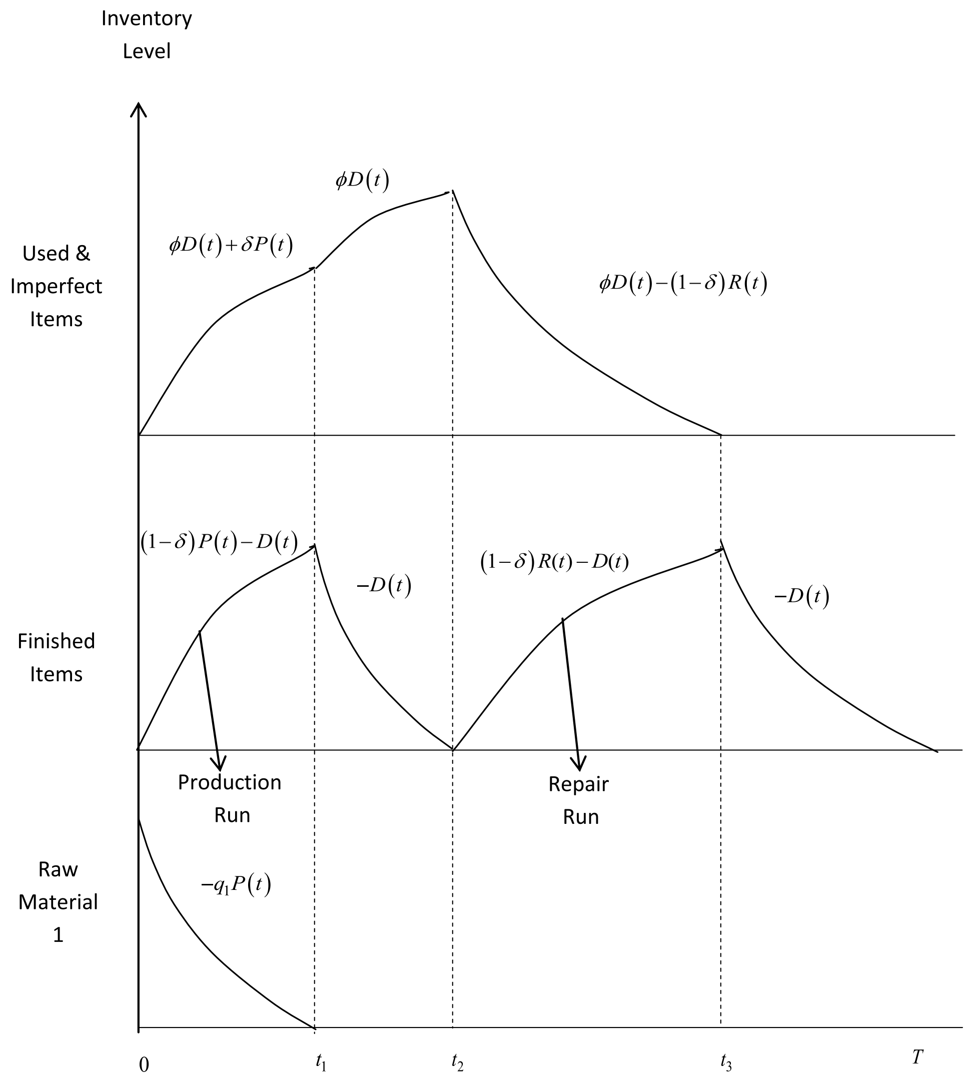

FIGURE 2. Inventory behavior thru time.

$$I_{ui}(t)=\frac{\phi a}{b}\left(e^{bt}-1\right)+\frac{k\delta a}{b}\left(e^{bt_1}-1\right)\quad t_1\le t\le t_2 \tag{3.11}$$

$$I_{ui}(t)=\frac{\phi a}{b}\left(e^{bt}-e^{bt_3}\right)-\frac{(1-\delta)c}{d}\left(e^{dt}-e^{dt_3}\right)\quad t_2\le t\le t_3 \tag{3.12}$$

$$I_f(t)=\{k(1-\delta)-1\}\frac{a}{b}\left(e^{bt}-1\right)\quad 0\le t\le t_1 \tag{3.13}$$

$$I_f(t)=\frac{a}{b}\left(e^{bt_2}-e^{bt}\right)\quad t_1\le t\le t_2 \tag{3.14}$$

$$I_R(t) = \frac{(1-\delta)c}{d}\left(e^{dt} - e^{dt_2}\right) - \frac{a}{b}\left(e^{bt} - e^{bt_2}\right) \quad t_2 \le t \le t_3 \tag{3.15}$$

$$I_R(t) = \frac{a}{b}\left(e^{bT} - e^{bt}\right) \quad t_3 \le t \le T \tag{3.16}$$

$$I_r(t) = \frac{q_1 k a}{b}\left(e^{bt_1} - e^{bt}\right) \quad 0 \le t \le t_1. \tag{3.17}$$

Considering the continuity of $I_f(t)$ at $t = t_1, I_{ui}(t)$ at $t = t_2$ and $I_R(t)$ at $t = t_3$ from equations (3.13)–(3.14), (3.11)–(3.12) and from equations (3.15) to (3.16), respectively, it follows that

$$\{k(1-\delta) - 1\}\left(e^{bt_1} - 1\right) = \left(e^{bt_2} - e^{bt_1}\right) \tag{3.18}$$

$$\frac{\varphi a}{b}\left(e^{bt_2} - 1\right) + \frac{k\delta a}{b}\left(e^{bt_1} - 1\right) = \frac{\varphi a}{b}\left(e^{bt_2} - e^{bt_3}\right) - \frac{(1-\delta)c}{d}\left(e^{dt_2} - e^{dt_3}\right) \tag{3.19}$$

$$\frac{(1-\delta)c}{d}\left(e^{dt_3} - e^{dt_2}\right) - \frac{a}{b}\left(e^{bt_3} - e^{bt_2}\right) = \frac{a}{b}\left(e^{bT} - e^{bt_3}\right). \tag{3.20}$$

Simplifying equations (3.18), (3.19) and (3.20),

$$\begin{aligned}
t_1 &= \frac{1}{b}\ln\left|\frac{1}{k(1-\delta)}\left\{e^{bt_2} + \{k(1-\delta) - 1\}\right\}\right|;\\
t_2 &= \frac{1}{b}\ln\left|\left\{e^{bt_1} + \{k(1-\delta) - 1\}\left(e^{bt_1} - 1\right)\right\}\right|; \quad \text{and}\\
t_3 &= \frac{1}{b}\ln\left|e^{bt_2} + \frac{ad}{(1-\varphi)bc}\left(e^{bT} - e^{bt_2}\right)\right|.
\end{aligned} \tag{3.21}$$

From equation (3.21) one can easily observed that $t_1, t_2 \& t_3$ are mutually dependent among them. So any one from $t_1, t_2 \& t_3$ can be act as decision variable. Here, t_1 is selected as decision variable.

Thus, the present worth of the total cost consists of the following elements:

(i) The present worth of holding cost for used and imperfect items is

$$\begin{aligned}
HC_R &= h_R\left[\int_0^{t_1} I_{ui}(t)e^{-rt}\mathrm{d}t + \int_{t_1}^{t_2} I_{ui}(t)e^{-rt}\mathrm{d}t + \int_{t_2}^{t_3} I_{ui}(t)e^{-rt}\mathrm{d}t\right]\\
HC_R &= h_R\left[\frac{(\varphi + k\delta)a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_1} - 1\right\} + \frac{1}{r}\left\{e^{-rt_1} - 1\right\}\right\}\right.\\
&\quad + \frac{\varphi a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_2} - e^{(b-r)t_1}\right\} + \frac{1}{r}\left\{e^{-rt_2} - e^{-rt_1}\right\}\right\} - \frac{ka\delta}{b}\left(e^{bt_1} - 1\right)\left\{e^{-rt_2} - e^{-rt_1}\right\}\\
&\quad + \frac{\varphi a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_3} - \frac{1}{(b-r)}e^{(b-r)t_2} + \frac{1}{r}e^{(b-r)t_3} - \frac{1}{r}e^{bt_3}e^{-rt_2}\right\}\\
&\quad \left. - \frac{(1-\delta)c}{d}\left\{\frac{1}{(d-r)}e^{(d-r)t_3} - \frac{1}{(d-r)}e^{(d-r)t_2} + \frac{1}{r}e^{(d-r)t_3} - \frac{1}{r}e^{dt_3}e^{-rt_2}\right\}\right].
\end{aligned} \tag{3.22}$$

(ii) The present worth of holding cost for finished goods is

$$HC_P = h_P\left[\int_0^{t_1} I_f(t)e^{-rt}\mathrm{d}t + \int_{t_1}^{t_2} I_f(t)e^{-rt}\mathrm{d}t + \int_{t_2}^{t_3} I_R(t)e^{-rt}\mathrm{d}t + \int_{t_3}^{T} I_R(t)e^{-rt}\mathrm{d}t\right]$$

$$HC_P = h_P\begin{bmatrix} \left[\{k(1-\delta)-1\}\frac{a}{b}\left[\frac{1}{(b-r)}\left\{e^{(b-r)t_1}-1\right\}+\frac{1}{r}\left\{e^{-rt_1}-1\right\}\right]\right. \\ +\frac{a}{b}\left[\frac{1}{-r}e^{(b-r)t_2}+\frac{e^{bt_2}e^{-rt_1}}{r}-\frac{1}{(b-r)}e^{(b-r)t_2}+\frac{e^{(b-r)t_1}}{(b-r)}\right] \\ +\frac{c}{d}\left[\frac{1}{(d-r)}e^{(d-r)t_3}-\frac{1}{(d-r)}e^{(d-r)t_2}+\frac{e^{dt_2}e^{-rt_3}}{r}-\frac{1}{r}e^{(d-r)t_2}\right] \\ +\frac{a}{b}\left[\frac{1}{(b-r)}e^{(b-r)t_3}-\frac{1}{(b-r)}e^{(b-r)t_2}+\frac{e^{bt_2}e^{-rt_3}}{r}-\frac{1}{r}e^{(b-r)t_2}\right] \end{bmatrix}. \quad (3.23)$$

(iii) The present worth of holding cost for raw material 1 is

$$HC_1 = h_1\left[\int_0^{t_1} I_r(t)e^{-rt}\mathrm{d}t\right]$$

$$HC_1 = h_1 q_1\frac{ka}{b}\left[\frac{e^{bt_1}}{r}\left(1-e^{-rt_1}\right)+\frac{\left(1-e^{(b-r)t_1}\right)}{(b-r)}\right]. \quad (3.24)$$

Here HC_R, HC_P and HC_1 are the total inventory holding cost over the cycle time for the used and imperfect items, the finished items and the raw material 1, correspondingly.

(iv) The present worth of production cost is

$$PC = c_2\left[\int_0^{t_1} kae^{bt}e^{-rt}\mathrm{d}t\right]$$

$$PC = c_2 k\frac{a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_1}-\frac{1}{(b-r)}\right\}. \quad (3.25)$$

(v) The present worth of returned used items cost is

$$AC = u_R\left[\int_0^{t_3}\varphi D(t)e^{-rt}\mathrm{d}t\right]$$

$$AC = u_R\varphi a\left\{\frac{1}{(b-r)}e^{(b-r)t_3}-\frac{1}{(b-r)}\right\}. \quad (3.26)$$

(vi) The present worth of remanufacturing cost is

$$RC = i\int_{t_2}^{t_3} ce^{dt}e^{-rt}\mathrm{d}t$$

$$RC = \frac{ic}{(d-r)}\left\{e^{(d-r)t_3}-e^{(d-r)t_2}\right\}. \quad (3.27)$$

(vii) The present worth of inspection cost is

$$IC = l\left[\int_0^{t_1} P(t)\,e^{-rt}\mathrm{d}t + \int_{t_2}^{t_3} R(t)\,e^{-rt}\mathrm{d}t\right]$$

$$IC = l\left[\frac{ka}{(b-r)}\left\{e^{(b-r)t_1}-1\right\}+\frac{c}{(d-r)}\left\{e^{(d-r)t_3}-e^{(d-r)t_2}\right\}\right]. \quad (3.28)$$

Thus, the present worth of the total cost is equal to the sum of all costs $C_R + C_P + C_1 + HC_R + HC_P + HC_1 + PC + AC + RC + IC$.

Therefore, the total cost per unit of time for the cycle is given by

$$TC\left(t_1, T\right) = \frac{X}{T}, \tag{3.29}$$

where

$$\begin{aligned}
X = {} & C_R + C_P + C_1 + h_R\left[\frac{(\varphi + k\delta)\, a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_1} - 1\right\} + \frac{1}{r}\left\{e^{-rt_1} - 1\right\}\right\}\right. \\
& + \frac{\varphi a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_2} - e^{(b-r)t_1}\right\} + \frac{1}{r}\left\{e^{-rt_2} - e^{-rt_1}\right\}\right\} - \frac{ka\delta}{b}\left(e^{bt_1} - 1\right)\left\{e^{-rt_2} - e^{-rt_1}\right\} \\
& + \frac{\varphi a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_3} - \frac{1}{(b-r)}e^{(b-r)t_2} + \frac{1}{r}e^{(b-r)t_3} - \frac{1}{r}e^{bt_3}e^{-rt_2}\right\} \\
& \left. - \frac{(1-\delta)\, c}{d}\left\{\frac{1}{(d-r)}e^{(d-r)t_3} - \frac{1}{(d-r)}e^{(d-r)t_2} + \frac{1}{r}e^{(d-r)t_3} - \frac{1}{r}e^{dt_3}e^{-rt_2}\right\}\right] \\
& + h_P\left[\begin{array}{l}
\left[\{k(1-\delta) - 1\}\, \frac{a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_1} - 1\right\} + \frac{1}{r}\left\{e^{-rt_1} - 1\right\}\right\}\right. \\
+ \frac{a}{b}\left\{\frac{1}{-r}e^{(b-r)t_2} + \frac{e^{bt_2}e^{-rt_1}}{r} - \frac{1}{(b-r)}e^{(b-r)t_2} + \frac{e^{(b-r)t_1}}{(b-r)}\right\} \\
+ \frac{c}{d}\left\{\frac{1}{(d-r)}e^{(d-r)t_3} - \frac{1}{(d-r)}e^{(d-r)t_2} + \frac{e^{dt_2}e^{-rt_3}}{r} - \frac{1}{r}e^{(d-r)t_2}\right\} \\
\left. + \frac{a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_3} - \frac{1}{(b-r)}e^{(b-r)t_2} + \frac{e^{bt_2}e^{-rt_3}}{r} - \frac{1}{r}e^{(b-r)t_2}\right\}\right.
\end{array}\right] \\
& + h_1 q_1 \frac{ka}{b}\left\{\frac{e^{bt_1}}{r}\left(1 - e^{-rt_1}\right) + \frac{\left(1 - e^{(b-r)t_1}\right)}{(b-r)}\right\} + c_2 k\frac{a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_1} - \frac{1}{(b-r)}\right\} \\
& + u_R\varphi a\left\{\frac{1}{(b-r)}e^{(b-r)t_3} - \frac{1}{(b-r)}\right\} + \frac{ic}{(d-r)}\left\{e^{(d-r)t_3} - e^{(d-r)t_2}\right\} \\
& + l\left\{\frac{ka}{(b-r)}\left\{e^{(b-r)t_1} - 1\right\} + \frac{c}{(d-r)}\left\{e^{(d-r)t_3} - e^{(d-r)t_2}\right\}\right\}.
\end{aligned} \tag{3.30}$$

Note that total cost per unit of time is highly non-linear.

4. Mathematical formulation of the fuzzy imperfect production and repair inventory model

Some basics from fuzzy set theory are required to be defined in order to make the development of inventory model self-contained. In this sense, the concepts related to fuzzy set theory applied in this paper are given in Appendix A. In order to show the fuzzy performance rates and the fuzzy availabilities of the components, triangular fuzzy numbers are used.

Fuzziness and randomness appear in setup cost for repair and production, ordering cost of raw material 1, holding cost for used and imperfect items, finished items and raw material 1, production cost, returned used item cost, remanufacturing cost, and inspection cost. Consequently, the expected fuzzy total cost of the integrated system is given by

$$T\tilde{C} = \frac{\left[\tilde{C}_R + \tilde{C}_P + \tilde{C}_1 + \tilde{h}_R + \tilde{h}_p + \tilde{h}_1 + \tilde{c}_2 + \tilde{u}_R + \tilde{i} + \tilde{l}\right]}{T} \tag{4.1}$$

All the costs are considered as a triangular fuzzy numbers such as $\tilde{C}_R = [C_R - \Delta_1, C_R, C_R + \Delta_2]$, where $0 < \Delta_1 < C_R$ and $\Delta_1, \Delta_2 > 0$, $\tilde{C}_p = [C_p - \Delta_3, C_p, C_p + \Delta_4]$, where $0 < \Delta_3 < C_p$ and $\Delta_3, \Delta_4 > 0$, $\tilde{C}_1 = [C_1 - \Delta_5, C_1, C_1 + \Delta_6]$, where $0 < \Delta_5 < C_1$ and $\Delta_5, \Delta_6 > 0$, $\tilde{h}_R = [h_R - \Delta_7, h_R, h_R + \Delta_8]$ where $0 < \Delta_7 < h_R$ and $\Delta_7, \Delta_8 > 0$, $\tilde{h}_p = [h_p - \Delta_9, h_p, h_p + \Delta_{10}]$ where $0 < \Delta_9 < h_p$ and $\Delta_7, \Delta_8 > 0$, $\tilde{h}_1 = [h_1 - \Delta_{11}, h_1, h_1 + \Delta_{12}]$ where $0 < \Delta_{11} < h_1$ and $\Delta_{11}, \Delta_{12} > 0$, $\tilde{c}_2 = [c_2 - \Delta_{13}, c_2, c_2 + \Delta_{14}]$ where $0 < \Delta_{13} < c_2$ and $\Delta_{13}, \Delta_{14} > 0$, $\tilde{u}_R = [u_R - \Delta_{15}, u_R, u_R + \Delta_{16}]$ where $0 < \Delta_{15} < u_R$ and $\Delta_{15}, \Delta_{16} > 0$, $\tilde{i} = [i - \Delta_{17}, i, i + \Delta_{18}]$ where $0 < \Delta_{17} < i$ and $\Delta_{17}, \Delta_{18} > 0$, $\tilde{l} = [l - \Delta_{19}, l, l + \Delta_{20}]$ where $0 < \Delta_{19} < l$ and $\Delta_{19}, \Delta_{20} > 0$.

The signed distance of $\tilde{C}_R$ to $\tilde{0}$ is given by the relation $d(\tilde{C}_R, \tilde{0}) = C_R + \frac{1}{3}(\Delta_2 - \Delta_1)$ where $d(\tilde{C}_R, \tilde{0}) > 0$ and $d(\tilde{C}_R, \tilde{0}) \in [C_R - \Delta_1, C_R, C_R + \Delta_2]$. Similarly, other parameters are defined as above.

Now, the triangular fuzzy rule is used for the fuzzification of total cost per unit of time. Thus the fuzzyfied total cost per unit time is given by

$$FTC(\tilde{C}_R + \tilde{C}_P + \tilde{C}_1 + \tilde{h}_R + \tilde{h}_p + \tilde{h}_1 + \tilde{c}_2 + \tilde{u}_R + \tilde{i} + \tilde{l}) = (F_1 + F_2 + F_3)/T \quad \text{[See Appendix B]}.$$

Now, defuzzified cost is given by $T\tilde{C}(t_1, T) = (F_1 + 2F_2 + F_3)/4T$.

4.1. Optimality condition

To minimize total cost per unit time (TC), the optimal values of t_1 and T are obtained by solving the following equations simultaneously

$$\frac{\partial TC}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC}{\partial T} = 0. \tag{4.2}$$

Also, the following conditions must be satisfied

$$\begin{gathered}\frac{\partial^2 TC}{\partial t_1^2} > 0, \quad \frac{\partial^2 TC}{\partial T^2} > 0 \\ \left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2 > 0\end{gathered} \tag{4.3}$$

As, it can be easily observed that the objective function is highly non-linear (see Appendix C), so it is proved the optimality graphically.

5. Numerical example

This section is devoted to solve a numerical example with the aim to illustrate and validate the proposed inventory model. The data for the input parameters are given in Table 1. It is easy to see that the total cost per unit of time is highly non-linear. So, the optimality of the objective function is proved graphically instead of analytically. For this, the mathematical software Lingo 10 is used to obtain the optimal solution of the objective solution in consideration.

The optimal solution to numerical example is given in Table 2 and the convexity of the total cost function is shown graphically in Figure 3.

6. Sensitivity analysis

Sensitivity analysis with respect to some input parameters of the inventory model is shown in Table 3. For the different parameters, different levels are chosen as follows: −10%, −5%, 5% and 10% respectively.

TABLE 1. Input values for the parameters.

Parameters	**Value**
Demand rate a, b (units/ years)	500, 0.4
Repair rate c, d (units/ years)	1000,0.1
Setup cost of production run, C_P (\$/setup)	150
Setup cost of repair run, C_R (\$/setup)	390
Ordering cost of raw material 1, C_1 (\$/order)	200
Holding cost for finished items, h_P (\$/unit/ years)	2.5
Holding cost for used and imperfect items, h_R (\$/unit/ years)	0.9
Holding cost for raw material 1, h_1 (\$/unit/ years)	1.1
Production cost, c_2 (\$/unit)	20
Returned used item cost u_R (\$/unit)	3
Inspection cost, l (\$/unit)	2
Remanufacturing cost, i (\$/unit)	12
Production coefficient, k	50
Collection coefficient, ϕ	0.7
Imperfect rate, δ	0.1
Inflation rate, r	0.04
Quantity of raw material 1, q_1 (units)	1

TABLE 2. Optimal solution.

t_1^*	t_2^*	t_3^*	T^*	TC^*
0.004638236 years	0.2006430 years	0.3593000 years	0.4579766 years	\$20796.11

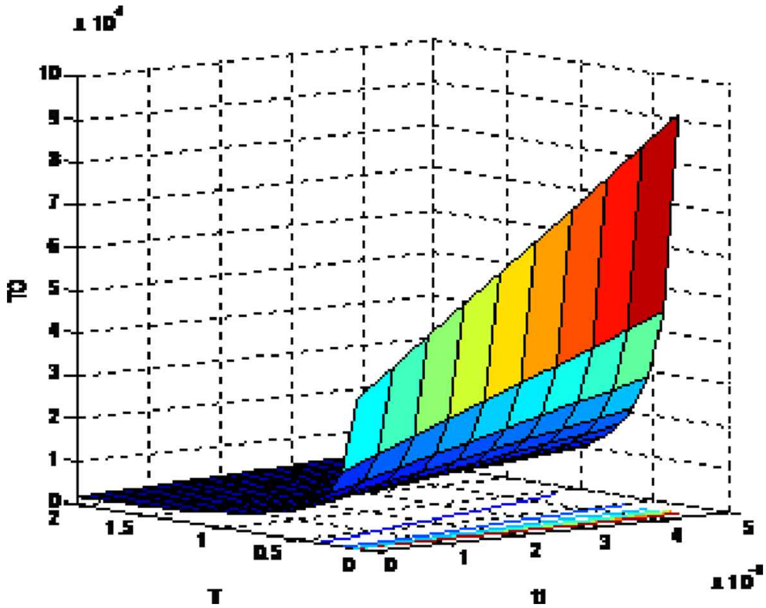

FIGURE 3. Convexity of total cost with respect to t_1 and T.

TABLE 3. Sensitivity analysis of different parameters.

Parameter	Change in parameter	t_1^*	t_2^*	t_3^*	T^*	TC^*
a	−50%	0.007160628	0.3034991	0.3930457	0.5829200	11933.34
	−20%	0.005384689	0.2315137	0.3601481	0.4894856	17511.02
	−10%	0.0049850	0.2150316	0.3581034	0.4715996	19199.37
	−5%	0.0048059	0.2076124	0.3583138	0.4643011	20009.53
	5%	0.0044803	0.1940650	0.3610479	0.4525620	21558.32
	10%	0.0043310	0.1878286	0.3635585	0.4480121	22295.32
	20%	0.004053920	0.1762155	0.3709454	0.4414087	23689.70
	50%	–	–	–	–	–
b	−50%	0.005006860	0.2204878	0.3849701	0.5042857	32465.39
	−20%	0.004813262	0.2095764	0.3716531	0.4785259	23655.11
	−10%	0.0047273	0.2051427	0.3655949	0.4682961	22058.32
	−5%	0.0046829	0.2028947	0.3624656	0.4631338	21392.01
	5%	0.0045932	0.1983972	0.3561152	0.4528448	20260.52
	10%	0.0045483	0.1961645	0.3529251	0.4477539	19776.99
	20%	0.004458847	0.1917605	0.3465705	0.4377406	18939.98
	50%	–	–	–	–	–
c	−50%	–	–	–	–	–
	−20%	0.004389028	0.1902533	0.4215002	0.4897604	19794.46
	−10%	0.0045336	0.1962888	0.3842105	0.4704308	20371.31
	−5%	0.0045898	0.1986278	0.3706401	0.4635992	20598.66
	5%	0.0046804	0.2023983	0.3496710	0.4532654	20969.23
	10%	0.0047175	0.2039414	0.3413863	0.4492588	21122.28
	20%	0.004779880	0.2065296	0.3278467	0.4428059	21380.72
	50%	0.004906355	0.2117744	0.3016514	0.4306629	21910.89
d	−50%	0.004707560	0.2035257	0.3687531	0.4672240	20743.23
	−20%	0.004664929	0.2017534	0.3629455	0.4615423	20775.27
	−10%	0.0046514	0.2011915	0.3611014	0.4597385	20785.74
	−5%	0.0046447	0.2009156	0.3601955	0.4588524	20790.93
	5%	0.004632	0.2003737	0.3584147	0.4571109	20801.25
	10%	0.0046253	0.2001075	0.3575395	0.4562550	20806.38
	20%	0.004612795	0.1995843	0.3558183	0.4545720	20816.56
	50%	0.004576779	0.1980847	0.3508753	0.4497404	20846.54
C_P	−50%	–	–	–	–	–
	−20%	0.004542007	0.1966361	0.3518917	0.4489693	20729.95
	−10%	0.0045903	0.1986505	0.3556147	0.4534980	20763.19
	−5%	0.0046143	0.1996494	0.3574620	0.4557435	20779.69
	5%	0.0046619	0.2016313	0.3611288	0.4601976	20812.44
	10%	0.0046856	0.2026144	0.3629487	0.4624067	20828.70
	20%	0.004732577	0.2045653	0.3665620	0.4667897	20860.99
	50%	0.004870787	0.2103005	0.3771993	0.4796697	20956.08

(continued)

Table 3. continued.

Parameter	Change in parameter	t_1^*	t_2^*	t_3^*	T^*	TC^*
C_R	−50%	0.003972876	0.1728089	0.3080535	0.3953178	20339.15
	−20%	0.004383811	0.1900353	0.3397106	0.4341219	20621.24
	−10%	0.0045127	0.1954168	0.3496394	0.4462274	20709.84
	−5%	0.0045759	0.1980485	0.3545018	0.4521447	20753.25
	5%	0.0046997	0.2032018	0.3640364	0.4637265	20838.42
	10%	0.0047605	0.2057262	0.3687134	0.4693975	20880.21
	20%	0.004879867	0.2106768	0.3778980	0.4805146	20962.33
	50%	0.005222131	0.2248225	0.4042330	0.5122483	21198.02
C_1	−50%	0.004309434	0.1869260	0.3339825	0.4271240	20570.15
	−20%	0.004509491	0.1952808	0.3493883	0.4459216	20707.60
	−10%	0.0045743	0.1979815	0.3543780	0.4519941	20752.15
	−5%	0.0046063	0.1993171	0.3568473	0.4549964	20774.20
	5%	0.0046698	0.2019596	0.3617364	0.4609353	20817.87
	10%	0.0047013	0.2032669	0.3641570	0.4638729	20839.50
	20%	0.004763624	0.2058547	0.3689517	0.4696862	20882.34
	50%	0.004945963	0.2134147	0.3829844	0.4866601	21007.82
h_P	−50%	0.004785946	0.2067814	0.3706697	0.4717676	20712.85
	−20%	0.004695402	0.2030204	0.3637005	0.4633190	20763.15
	−10%	0.00466651	0.2018194	0.3614770	0.4606203	20779.68
	−5%	0.0046523	0.2012282	0.3603828	0.4592917	20787.91
	5%	0.0046243	0.2000638	0.3582284	0.4566748	20804.28
	10%	0.0046105	0.1994904	0.3571678	0.4553860	20812.42
	20%	0.004583404	0.1983607	0.3550788	0.4528464	20828.62
	50%	0.004505175	0.1951008	0.3490559	0.4455169	20876.61
h_R	−50%	0.004660162	0.2015551	0.3609878	0.4600264	20782.37
	−20%	0.004646967	0.2010063	0.3599721	0.4587930	20790.62
	−10%	0.0046425	0.2008244	0.3596356	0.4583842	20793.36
	−5%	0.0046404	0.2007336	0.3594676	0.4581803	20794.74
	5%	0.0046360	0.2005525	0.3591325	0.4577732	20797.48
	10%	0.0046338	0.2004622	0.3589654	0.4575702	20798.85
	20%	0.004629555	0.2002818	0.3586318	0.4571649	20801.58
	50%	0.004616630	0.1997439	0.3576367	0.4559559	20809.78
h_1	−50%	0.004638759	0.2006648	0.3593403	0.4580256	20795.78
	−20%	0.004638445	0.2006517	0.3593161	0.4579962	20795.98
	−10%	0.0046383	0.2006474	0.3593080	0.4579864	20796.04
	−5%	0.0046383	0.2006452	0.3593040	0.4579815	20796.07
	5%	0.0046381	0.2006408	0.3592959	0.4579717	20796.14
	10%	0.0046381	0.2006387	0.3592919	0.4579668	20796.17
	20%	0.004638026	0.2006343	0.3592839	0.4579570	20796.24
	50%	0.004637712	0.2006212	0.3592597	0.4579277	20796.43

(*continued*)

TABLE 3. continued.

Parameter	Change in parameter	t_1^*	t_2^*	t_3^*	T^*	TC^*
c_2	−50%	0.005256186	0.2262257	0.4068529	0.5153938	14439.71
	−20%	0.004861276	0.2099062	0.3764673	0.4787845	18258.87
	−10%	0.0047462	0.2051327	0.3676134	0.4680643	19528.31
	−5%	0.0046913	0.2028540	0.3633923	0.4629450	20162.40
	5%	0.0045866	0.1984968	0.3553306	0.4531526	21429.42
	10%	0.0045366	0.1964125	0.3514786	0.4484665	22062.37
	20%	0.004440891	0.1924190	0.3441061	0.4394849	23327.17
	50%	0.004183965	0.1816722	0.3243180	0.4152937	27113.62
u_R	−50%	0.004716148	0.2038827	0.3652974	0.4652562	20355.99
	−20%	0.004668908	0.2019189	0.3616611	0.4608438	20620.16
	−10%	0.0046534	0.2012777	0.3604744	0.4594029	20708.15
	−5%	0.0046458	0.2009596	0.3598856	0.4586880	20752.13
	5%	0.0046306	0.2003281	0.3587173	0.4572688	20840.07
	10%	0.0046231	0.2000147	0.3581377	0.4565645	20884.03
	20%	0.004608193	0.1993928	0.3569872	0.4551665	20971.92
	50%	0.004564263	0.1975634	0.3536052	0.4510541	21235.42
l	−50%	0.004717229	0.2039276	0.3653806	0.4653571	20189.98
	−20%	0.004669349	0.2019372	0.3616950	0.4608850	20553.76
	−10%	0.0046537	0.2012869	0.3604914	0.4594237	20674.95
	−5%	0.0046459	0.2009642	0.3598942	0.4586984	20735.53
	5%	0.0046305	0.2003234	0.3587087	0.4572583	20856.67
	10%	0.0046229	0.2000054	0.3581204	0.4565436	20917.23
	20%	0.004607742	0.1993740	0.3569525	0.4551243	21038.32
	50%	0.004563121	0.1975158	0.3535173	0.4509472	21401.40
i	−50%	0.005018954	0.2164347	0.3886009	0.4934369	18673.67
	−20%	0.004779274	0.2065045	0.3701562	0.4711456	19949.27
	−10%	0.0046537	0.2012869	0.3604914	0.4594237	20674.95
	−5%	0.0046459	0.2009642	0.3598942	0.4586984	20735.53
	5%	0.0046305	0.2003234	0.3587087	0.4572583	20856.67
	10%	0.0046229	0.2000054	0.3581204	0.4565436	20917.23
	20%	0.004509538	0.1952827	0.3493919	0.4459260	21640.39
	50%	0.004335893	0.1880325	0.3360203	0.4296147	22902.53
δ	−50%	0.004695981	0.2138554	0.3574498	0.4591402	20426.09
	−20%	0.004663906	0.2060135	0.3584071	0.4584465	20646.64
	−10%	0.0046515	0.2033429	0.3588265	0.4582123	20721.13
	−5%	0.0046449	0.2019966	0.3590563	0.4580946	20758.56
	5%	0.0046312	0.1992820	0.3595577	0.4578582	20833.77
	10%	0.0046239	0.1979134	0.3598300	0.4577394	20871.56
	20%	0.004608763	0.1951535	0.3604192	0.4575006	20947.49
	50%	0.004556632	0.1866863	0.3625704	0.4567750	21178.08

(*continued*)

Table 3. continued.

Parameter	Change in parameter	t_1^*	t_2^*	t_3^*	T^*	TC^*
q_1	−50%	0.004638759	0.2006648	0.3593403	0.4580256	20795.78
	−20%	0.004638445	0.2006517	0.3593161	0.4579962	20795.98
	−10%	0.0046383	0.2006474	0.3593080	0.4579864	20796.04
	−5%	0.0046382	0.2006452	0.3593040	0.4579815	20796.07
	5%	0.0046381	0.2006408	0.3592959	0.4579717	20796.14
	10%	0.0046381	0.2006387	0.3592919	0.4579668	20796.17
	20%	0.004638026	0.2006343	0.3592839	0.4579570	20796.24
	50%	0.004637712	0.2006212	0.3592597	0.4579277	20796.43

6.1. Observations

(i) As the demand parameters a and b increase that means the total demand increases, then the cycle time (T) decreases and total cost (TC) increases.

(ii) With the increment in repair parameters c and d, the cycle time (T) decreases and total cost (TC) of the inventory model increases. This is because the total repair cost increases.

(iii) With the increment in setup cost of production run (C_P), setup cost of repair run (C_R), and ordering cost of raw material $1(C_1)$, the total cost (TC) increases, which is obvious as the costs associated with each of these factors also increase, and the cycle time (T) also increases.

(iv) With the increase of holding cost of finished items (h_P), holding cost of raw material $1(h_1)$, and holding cost of used items (h_R), the cycle time (T) decreases and the total cost (TC) increases. This is for an obvious reason, which consist in that the total holding cost of the inventory model increases, which eventually increases the total cost.

(v) With the increase of production cost (c_2), returned used item cost (u_R), the inspection cost (l) and remanufacturing cost (i), the cycle time (T) decreases and the total cost (TC) increases, because the costs associated with each factors increase which results in an increment in the total cost (TC).

(vi) As the rate of imperfect production (δ) increases, this means that the total number of imperfect items also increases therefore the cost associated to remanufacturing increments. Consequently, the total cost (TC) also increases, and the cycle time (T) decreases.

(vii) As the quantity of raw material, 1 (q_1) increases the cycle time (T) is slightly decreased but the total cost (TC) is slightly increased.

7. Conclusion

This paper proposes an integrated inventory model for imperfect production/remanufacturing process with time varying demand, production and repair rates under inflationary environment. In the integrated inventory model, the manufacturing and remanufacturing rates are directly related to the time dependent demand rates. The used items are returned back thru a collection process and then these enter to the remanufacturing process. Various costs of the inventory system considered here include ordering, setup, production, remanufacturing, inspection and holding costs. The main objective of this integrated inventory model is to determine the optimal value of total cost. The integrated inventory model is numerically illustrated and a sensitivity analysis is performed.

For future research, it would be interesting to extend the proposed inventory model under one or two level trade credit policy. The inventory model may also be explored for a two warehousing inventory system. Finally, it would be also interesting to consider learning effect, multi-product and the multi-stage supply chain. Also, it can be considered price discount policy.

Appendix A

The preliminaries required are similar to those utilized by Chang [4].

Definition A.1. Consider the fuzzy set $\tilde{A} = (a, b, c)$ where $a < b < c$ and defined on R, which is named as triangular fuzzy number. Thus, the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a) & a \leq x \leq b \\ (c-x)/(c-b) & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases} \tag{A.1}$$

Definition A.2. Let B be a fuzzy set on R and $0 \leq \alpha \leq 1$. The α-cut of $\tilde{B}$ is all the points x such that $\mu_{\tilde{B}}(x) \geq \alpha$,

$$B(\alpha) = \{x | \mu_{\tilde{B}}(x) \geq \alpha\} \tag{A.2}$$

According to Kaufmann and Gupta [17], the interval operations are as follows. For any $a, b, c, d, k \in R$ and $a, c > 0$.

$$\begin{aligned} &\text{(i)}\ \ [a,b]\,(+)[c,d] = [a+c, b+d] \\ &\text{(ii)}\ [a,b](-)[c,d] = [a-d, b-c] \\ &\text{(iii)}\ k(.)[c,d] = \begin{cases} [kc, kd], & k > 0 \\ [kd, kc], & k < 0 \end{cases} \\ &\text{(iv)}\ [a,b](.)[c,d] = [a.c, b.d] \\ &\text{(v)}\ [a,b](\div)[c,d] = \left[\frac{a}{d}, \frac{b}{c}\right]. \end{aligned} \tag{A.3}$$

In order to find non-fuzzy values for the inventory model in the next section, it is required to use some distance measures as in Chang [4], also the signed distance given in Kauffman and Gupta [17] is utilized.

Definition A.3. For any α and $0 \in R$, the signed distance from a to 0 is $d_0(a, 0) = a$. And if $\alpha < 0$, the distance from a to 0 is $-a = -d_0(a, 0)$.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R for which the α-cut $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$ exists for every $\alpha \in [0, 1]$, and both $B_L(\alpha)$ and $B_U(\alpha)$ are continuous functions on $\alpha \in [0, 1]$. Then, for any $\tilde{B} \in \Omega$, the following is obtained from Yao and Wu [45]

$$\tilde{B} = \bigcup_{0 \leq \alpha \leq 1} [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \tag{A.4}$$

From Chang [4] it can be established (by results from Kauffman and Gupta [17]) how to compute the signed distances.

Definition A.4. For $\tilde{B} \in \Omega$ define the signed distance of $\tilde{B}$ to $\tilde{0}_1$, as

$$d(\tilde{B}, \tilde{0}_1) = \frac{1}{2} \int_0^1 [B_L(\alpha) + B_U(\alpha)] \mathrm{d}\alpha. \tag{A.5}$$

The definition in equation (A.5) yields several properties of which the most significant is

Property A.5. *Consider the triangular fuzzy number* $\tilde{A} = (a, b, c)$*; the* α*-cut of* $A(\alpha) = [A_L(\alpha), A_U(\alpha)]$*; for* $\alpha \in [0, 1]$ *where* $A_L(\alpha) = a + (b - a)\alpha$ *and* $A_U(\alpha) = c - (c - b)\alpha$*, the signed distance of* $\tilde{A}$ *to* $\tilde{0}_1$*, is given by*

$$d(\tilde{A}, \tilde{0}_1) = \frac{1}{4}(a + 2b + c) \tag{A.6}$$

Appendix B

$$F_1 = \left[\begin{array}{l}
(C_R - \Delta_1) + (C_P - \Delta_3) + (C_1 - \Delta_5) + (h_R - \Delta_7) \left[\frac{(\varphi + k\delta)a}{b} \left\{ \frac{1}{(b-r)} \left\{ e^{(b-r)t_1} - 1 \right\} + \frac{1}{r} \left\{ e^{-rt_1} - 1 \right\} \right\} \right. \\
\quad + \frac{\varphi a}{b} \left\{ \frac{1}{(b-r)} \left\{ e^{(b-r)t_2} - e^{(b-r)t_1} \right\} + \frac{1}{r} \left\{ e^{-rt_2} - e^{-rt_1} \right\} \right\} - \frac{ka\delta}{b} \left(e^{bt_1} - 1 \right) \left\{ e^{-rt_2} - e^{-rt_1} \right\} \\
\quad + \frac{\varphi a}{b} \left\{ \frac{1}{(b-r)} e^{(b-r)t_3} - \frac{1}{(b-r)} e^{(b-r)t_2} + \frac{1}{r} e^{(b-r)t_3} - \frac{1}{r} e^{bt_3} e^{-rt_2} \right\} \\
\quad \left. - \frac{(1-\delta)c}{d} \left\{ \frac{1}{(d-r)} e^{(d-r)t_3} - \frac{1}{(d-r)} e^{(d-r)t_2} + \frac{1}{r} e^{(d-r)t_3} - \frac{1}{r} e^{dt_3} e^{-rt_2} \right\} \right] \\
\quad + (h_P - \Delta_9) \left[\begin{array}{l}
\left[\{ k(1-\delta) - 1 \} \frac{a}{b} \left\{ \frac{1}{(b-r)} \left\{ e^{(b-r)t_1} - 1 \right\} + \frac{1}{r} \left\{ e^{-rt_1} - 1 \right\} \right\} \right. \\
+ \frac{a}{b} \left\{ \frac{1}{-r} e^{(b-r)t_2} + \frac{e^{bt_2} e^{-rt_1}}{r} - \frac{1}{(b-r)} e^{(b-r)t_2} + \frac{e^{(b-r)t_1}}{(b-r)} \right\} \\
+ \frac{c}{d} \left\{ \frac{1}{(d-r)} e^{(d-r)t_3} - \frac{1}{(d-r)} e^{(d-r)t_2} + \frac{e^{dt_2} e^{-rt_3}}{r} - \frac{1}{r} e^{(d-r)t_2} \right\} \\
+ \frac{a}{b} \left\{ \frac{1}{(b-r)} e^{(b-r)t_3} - \frac{1}{(b-r)} e^{(b-r)t_2} + \frac{e^{bt_2} e^{-rt_3}}{r} - \frac{1}{r} e^{(b-r)t_2} \right\}
\end{array} \right] \\
\quad + (h_1 - \Delta_{11}) \, q_1 \frac{ka}{b} \left\{ \frac{e^{bt_1}}{r} \left(1 - e^{-rt_1} \right) + \frac{\left(1 - e^{(b-r)t_1} \right)}{(b-r)} \right\} + (c_2 - \Delta_{13}) \, k \frac{a}{b} \left\{ \frac{1}{(b-r)} e^{(b-r)t_1} - \frac{1}{(b-r)} \right\} \\
\quad + (u_R - \Delta_{15}) \, \varphi a \left\{ \frac{1}{(b-r)} e^{(b-r)t_3} - \frac{1}{(b-r)} \right\} + \frac{(i - \Delta_{17})c}{(d-r)} \left\{ e^{(d-r)t_3} - e^{(d-r)t_2} \right\} \\
\quad + (l - \Delta_{19}) \left\{ \frac{ka}{(b-r)} \left\{ e^{(b-r)t_1} - 1 \right\} + \frac{c}{(d-r)} \left\{ e^{(d-r)t_3} - e^{(d-r)t_2} \right\} \right\}
\end{array} \right] \tag{B.1}$$

$$F_2 = \left[\begin{array}{l}
C_R + C_P + C_1 + h_R \left[\frac{(\varphi + k\delta)a}{b} \left\{ \frac{1}{(b-r)} \left\{ e^{(b-r)t_1} - 1 \right\} + \frac{1}{r} \left\{ e^{-rt_1} - 1 \right\} \right\} \right. \\
+ \frac{\varphi a}{b} \left\{ \frac{1}{(b-r)} \left\{ e^{(b-r)t_2} - e^{(b-r)t_1} \right\} + \frac{1}{r} \left\{ e^{-rt_2} - e^{-rt_1} \right\} \right\} - \frac{ka\delta}{b} \left(e^{bt_1} - 1 \right) \left\{ e^{-rt_2} - e^{-rt_1} \right\} \\
+ \frac{\varphi a}{b} \left\{ \frac{1}{(b-r)} e^{(b-r)t_3} - \frac{1}{(b-r)} e^{(b-r)t_2} + \frac{1}{r} e^{(b-r)t_3} - \frac{1}{r} e^{bt_3} e^{-rt_2} \right\} \\
\left. - \frac{(1-\delta)c}{d} \left\{ \frac{1}{(d-r)} e^{(d-r)t_3} - \frac{1}{(d-r)} e^{(d-r)t_2} + \frac{1}{r} e^{(d-r)t_3} - \frac{1}{r} e^{dt_3} e^{-rt_2} \right\} \right] \\
+ h_P \left[\begin{array}{l}
\left[\{ k(1-\delta) - 1 \} \frac{a}{b} \left\{ \frac{1}{(b-r)} \left\{ e^{(b-r)t_1} - 1 \right\} + \frac{1}{r} \left\{ e^{-rt_1} - 1 \right\} \right\} \right. \\
+ \frac{a}{b} \left\{ \frac{1}{-r} e^{(b-r)t_2} + \frac{e^{bt_2} e^{-rt_1}}{r} - \frac{1}{(b-r)} e^{(b-r)t_2} + \frac{e^{(b-r)t_1}}{(b-r)} \right\} \\
+ \frac{c}{d} \left\{ \frac{1}{(d-r)} e^{(d-r)t_3} - \frac{1}{(d-r)} e^{(d-r)t_2} + \frac{e^{dt_2} e^{-rt_3}}{r} - \frac{1}{r} e^{(d-r)t_2} \right\} \\
+ \frac{a}{b} \left\{ \frac{1}{(b-r)} e^{(b-r)t_3} - \frac{1}{(b-r)} e^{(b-r)t_2} + \frac{e^{bt_2} e^{-rt_3}}{r} - \frac{1}{r} e^{(b-r)t_2} \right\}
\end{array} \right] \\
+ h_1 q_1 \frac{ka}{b} \left\{ \frac{e^{bt_1}}{r} \left(1 - e^{-rt_1} \right) + \frac{\left(1 - e^{(b-r)t_1} \right)}{(b-r)} \right\} + c_2 k \frac{a}{b} \left\{ \frac{1}{(b-r)} e^{(b-r)t_1} - \frac{1}{(b-r)} \right\} \\
+ u_R \varphi a \left\{ \frac{1}{(b-r)} e^{(b-r)t_3} - \frac{1}{(b-r)} \right\} + \frac{ic}{(d-r)} \left\{ e^{(d-r)t_3} - e^{(d-r)t_2} \right\} \\
+ l \left\{ \frac{ka}{(b-r)} \left\{ e^{(b-r)t_1} - 1 \right\} + \frac{c}{(d-r)} \left\{ e^{(d-r)t_3} - e^{(d-r)t_2} \right\} \right\}
\end{array} \right] \tag{B.2}$$

$$F_3 = \left[\begin{array}{l} (C_R+\Delta_2)+(C_P+\Delta_4)+(C_1+\Delta_6)+(h_R+\Delta_8)\left[\frac{(\varphi+k\delta)a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_1}-1\right\}+\frac{1}{r}\left\{e^{-rt_1}-1\right\}\right\}\right. \\ +\frac{\varphi a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_2}-e^{(b-r)t_1}\right\}+\frac{1}{r}\left\{e^{-rt_2}-e^{-rt_1}\right\}\right\}-\frac{ka\delta}{b}\left(e^{bt_1}-1\right)\left\{e^{-rt_2}-e^{-rt_1}\right\} \\ +\frac{\varphi a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_3}-\frac{1}{(b-r)}e^{(b-r)t_2}+\frac{1}{r}e^{(b-r)t_3}-\frac{1}{r}e^{bt_3}e^{-rt_2}\right\} \\ \left.-\frac{(1-\delta)c}{d}\left\{\frac{1}{(d-r)}e^{(d-r)t_3}-\frac{1}{(d-r)}e^{(d-r)t_2}+\frac{1}{r}e^{(d-r)t_3}-\frac{1}{r}e^{dt_3}e^{-rt_2}\right\}\right] \\ +(h_P+\Delta_{10})\left[\begin{array}{l}\left[\{k(1-\delta)-1\}\frac{a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_1}-1\right\}+\frac{1}{r}\left\{e^{-rt_1}-1\right\}\right\}\right. \\ +\frac{a}{b}\left\{\frac{1}{-r}e^{(b-r)t_2}+\frac{e^{bt_2}e^{-rt_1}}{r}-\frac{1}{(b-r)}e^{(b-r)t_2}+\frac{e^{(b-r)t_1}}{(b-r)}\right\} \\ +\frac{c}{d}\left\{\frac{1}{(d-r)}e^{(d-r)t_3}-\frac{1}{(d-r)}e^{(d-r)t_2}+\frac{e^{dt_2}e^{-rt_3}}{r}-\frac{1}{r}e^{(d-r)t_2}\right\} \\ +\frac{a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_3}-\frac{1}{(b-r)}e^{(b-r)t_2}+\frac{e^{bt_2}e^{-rt_3}}{r}-\frac{1}{r}e^{(b-r)t_2}\right\}\end{array}\right] \\ +(h_1+\Delta_{12})\,q_1\frac{ka}{b}\left\{\frac{e^{bt_1}}{r}\left(1-e^{-rt_1}\right)+\frac{\left(1-e^{(b-r)t_1}\right)}{(b-r)}\right\}+(c_2+\Delta_{14})\,k\frac{a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_1}-\frac{1}{(b-r)}\right\} \\ +(u_R+\Delta_{16})\,\varphi a\left\{\frac{1}{(b-r)}e^{(b-r)t_3}-\frac{1}{(b-r)}\right\}+\frac{(i+\Delta_{18})c}{(d-r)}\left\{e^{(d-r)t_3}-e^{(d-r)t_2}\right\} \\ +(l+\Delta_{20})\left\{\frac{ka}{(b-r)}\left\{e^{(b-r)t_1}-1\right\}+\frac{c}{(d-r)}\left\{e^{(d-r)t_3}-e^{(d-r)t_2}\right\}\right\} \end{array}\right] \tag{B.3}$$

Appendix C

From equation (3.29),

$$TC\,(t_1,T)=\frac{X}{T}, \tag{C.1}$$

where

$$\begin{aligned} X = {} & C_R+C_P+C_1+h_R\left[\frac{(\varphi+k\delta)\,a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_1}-1\right\}+\frac{1}{r}\left\{e^{-rt_1}-1\right\}\right\}\right. \\ & +\frac{\varphi a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_2}-e^{(b-r)t_1}\right\}+\frac{1}{r}\left\{e^{-rt_2}-e^{-rt_1}\right\}\right\}-\frac{ka\delta}{b}\left(e^{bt_1}-1\right)\left\{e^{-rt_2}-e^{-rt_1}\right\} \\ & +\frac{\varphi a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_3}-\frac{1}{(b-r)}e^{(b-r)t_2}+\frac{1}{r}e^{(b-r)t_3}-\frac{1}{r}e^{bt_3}e^{-rt_2}\right\} \\ & \left.-\frac{(1-\delta)\,c}{d}\left\{\frac{1}{(d-r)}e^{(d-r)t_3}-\frac{1}{(d-r)}e^{(d-r)t_2}+\frac{1}{r}e^{(d-r)t_3}-\frac{1}{r}e^{dt_3}e^{-rt_2}\right\}\right] \\ & +h_P\left[\begin{array}{l}\left[\{k(1-\delta)-1\}\frac{a}{b}\left\{\frac{1}{(b-r)}\left\{e^{(b-r)t_1}-1\right\}+\frac{1}{r}\left\{e^{-rt_1}-1\right\}\right\}\right. \\ +\frac{a}{b}\left\{\frac{1}{-r}e^{(b-r)t_2}+\frac{e^{bt_2}e^{-rt_1}}{r}-\frac{1}{(b-r)}e^{(b-r)t_2}+\frac{e^{(b-r)t_1}}{(b-r)}\right\} \\ +\frac{c}{d}\left\{\frac{1}{(d-r)}e^{(d-r)t_3}-\frac{1}{(d-r)}e^{(d-r)t_2}+\frac{e^{dt_2}e^{-rt_3}}{r}-\frac{1}{r}e^{(d-r)t_2}\right\} \\ +\frac{a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_3}-\frac{1}{(b-r)}e^{(b-r)t_2}+\frac{e^{bt_2}e^{-rt_3}}{r}-\frac{1}{r}e^{(b-r)t_2}\right\}\end{array}\right] \\ & +h_1q_1\frac{ka}{b}\left\{\frac{e^{bt_1}}{r}\left(1-e^{-rt_1}\right)+\frac{\left(1-e^{(b-r)t_1}\right)}{(b-r)}\right\}+c_2k\frac{a}{b}\left\{\frac{1}{(b-r)}e^{(b-r)t_1}-\frac{1}{(b-r)}\right\} \\ & +u_R\varphi a\left\{\frac{1}{(b-r)}e^{(b-r)t_3}-\frac{1}{(b-r)}\right\}+\frac{ic}{(d-r)}\left\{e^{(d-r)t_3}-e^{(d-r)t_2}\right\} \\ & +l\left\{\frac{ka}{(b-r)}\left\{e^{(b-r)t_1}-1\right\}+\frac{c}{(d-r)}\left\{e^{(d-r)t_3}-e^{(d-r)t_2}\right\}\right\}. \end{aligned} \tag{C.2}$$

Differentiating the objective function with respect to t_1

$$\frac{\partial TC\left(t_1,T\right)}{\partial t_1}=\frac{1}{T}\frac{\partial X}{\partial t_1} \tag{C.3}$$

Again, differentiating with respect to t_1, thus

$$\frac{\partial^2 TC\left(t_1,T\right)}{\partial t_1^2}=\frac{1}{T}\frac{\partial^2 X}{\partial t_1^2} \tag{C.4}$$

Now, differentiating the equation (C.3) with respect to T, hence

$$\frac{\partial^2 TC\left(t_1,T\right)}{\partial T\partial t_1}=\frac{1}{T}\frac{\partial^2 X}{\partial T\partial t_1}-\frac{1}{T^2}\frac{\partial X}{\partial t_1} \tag{C.5}$$

Differentiating the objective function of the equation (C.1) with respect to T, then

$$\frac{\partial TC\left(t_1,T\right)}{\partial T}=\frac{T\frac{\partial X}{\partial T}-X}{T^2} \tag{C.6}$$

Now, differentiating the objective function of the equation (C.6) with respect to T, thus

$$\begin{aligned}\frac{\partial^2 TC\left(t_1,T\right)}{\partial T^2}&=\frac{T^2\left(\frac{\partial X}{\partial T}+T\frac{\partial^2 X}{\partial T^2}-\frac{\partial X}{\partial T}\right)-2T\left(T\frac{\partial X}{\partial T}-X\right)}{T^4}\\&=\frac{T^2\frac{\partial^2 X}{\partial T^2}-2T\frac{\partial X}{\partial T}+2X}{T^3}\end{aligned} \tag{C.7}$$

$$\begin{aligned}\frac{\partial X}{\partial t_1}=h_R&\left\{\begin{array}{l}\frac{(\varphi+k\delta)a}{b}\left(e^{(b-r)t_1}-e^{-rt_1}\right)+\frac{\varphi a}{b}\left(e^{(b-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}-e^{(b-r)t_1}-e^{-rt_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+e^{-rt_1}\right)\\-\frac{ka\delta}{b}\left\{-r\left(e^{bt_1}-1\right)\left(e^{-rt_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}-e^{-rt_1}\right)+be^{bt_1}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\\+\frac{\varphi a}{b}\left\{e^{(b-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-e^{(b-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+\frac{(b-r)}{r}e^{(b-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-\frac{1}{r}\left(be^{bt_3}e^{-rt_2}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-re^{bt_3}e^{-rt_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)\right\}\\-\frac{(1-\delta)c}{\mathrm{d}}\left\{e^{(d-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-e^{(d-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+\frac{(d-r)}{r}e^{(d-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-\frac{1}{r}\left(de^{dt_3}e^{-rt_2}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-re^{dt_3}e^{-rt_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)\right\}\end{array}\right\}\\&+h_P\left[\begin{array}{l}\left\{k(1-\delta)-1\right\}\frac{a}{b}\left(e^{(b-r)t_1}-e^{-rt_1}\right)\\+\frac{a}{b}\left\{-\frac{(b-r)}{r}e^{(b-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+\frac{1}{r}\left(be^{bt_2}e^{-rt_1}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}-re^{bt_2}e^{-rt_1}\right)-e^{(b-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+e^{(b-r)t_1}\right\}\\+\frac{c}{d}\left\{e^{(d-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-e^{(d-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+\frac{1}{r}\left(de^{dt_2}e^{-rt_3}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}-re^{dt_2}e^{-rt_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)-\frac{(d-r)}{r}e^{(d-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right\}\\+\frac{a}{b}\left\{e^{(b-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-e^{(b-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+\frac{1}{r}\left(be^{bt_2}e^{-rt_3}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}-re^{bt_2}e^{-rt_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)-\frac{(b-r)}{r}e^{(b-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right\}\end{array}\right]\\&+h_1q_1\frac{ka}{b}\left\{\frac{be^{bt_1}}{r}\left(1-e^{-rt_1}\right)+e^{bt_1}e^{-rt_1}-e^{(b-r)t_1}\right\}+c_2k\frac{a}{b}e^{(b-r)t_1}+u_R\varphi ae^{(b-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\\&+ic\left\{e^{(d-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-e^{(d-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right\}+l\left\{kae^{(b-r)t_1}+c\left(e^{(d-r)t_3}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}-e^{(d-r)t_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)\right\}\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 X}{\partial t_1^2} = h_R & \left[\begin{array}{l}
\frac{(\varphi+k\delta)a}{b}\left\{(b-r)\,e^{(b-r)t_1}+re^{-rt_1}\right\}+\frac{\varphi a}{b}\left\{\begin{array}{l}(b-r)\,e^{(b-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2+e^{(b-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}-(b-r)e^{(b-r)t_1}\\ -\left(-re^{-rt_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2+e^{-rt_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\right)-re^{-rt_1}\end{array}\right\}\\
-\frac{ka\delta}{b}\left\{\begin{array}{l}-rbe^{bt_1}\left(e^{-rt_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}-e^{-rt_1}\right)-r\left(e^{bt_1}-1\right)\left(-re^{-rt_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2+e^{-rt_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}+re^{-rt_1}\right)\\ +b^2e^{bt_1}\left(e^{-rt_2}-e^{-rt_1}\right)+be^{bt_1}\left(-re^{-rt_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+re^{-rt_1}\right)\end{array}\right\}\\
+\frac{\varphi a}{b}\left\{\begin{array}{l}(b-r)e^{(b-r)t_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(b-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}-(b-r)e^{(b-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-e^{(b-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\\ +\frac{(b-r)}{r}\left((b-r)e^{(b-r)t_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(b-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}\right)\\ -\frac{1}{r}\left(\begin{array}{l}b^2e^{bt_3}e^{-rt_2}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2-bre^{bt_3}e^{-rt_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}+be^{bt_3}e^{-rt_2}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}\\ -bre^{bt_3}e^{-rt_2}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+r^2e^{bt_3}e^{-rt_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-re^{bt_3}e^{-rt_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\end{array}\right)\end{array}\right\}\\
-\frac{(1-\delta)c}{\mathrm{d}}\left\{\begin{array}{l}(d-r)e^{(d-r)t_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(d-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}-(d-r)e^{(d-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-e^{(d-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\\ +\frac{(d-r)}{r}\left((d-r)e^{(d-r)t_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(d-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}\right)\\ -\frac{1}{r}\left(\begin{array}{l}\mathrm{d}^2e^{dt_3}e^{-rt_2}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2-rde^{dt_3}e^{-rt_2}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}+de^{dt_3}e^{-rt_2}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}\\ -rde^{dt_3}e^{-rt_2}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+r^2e^{dt_3}e^{-rt_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-re^{dt_3}e^{-rt_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\end{array}\right)\end{array}\right\}
\end{array}\right]\\
+h_P & \left[\begin{array}{l}
\{k(1-\delta)-1\}\,\frac{a}{b}\left\{(b-r)e^{(b-r)t_1}+re^{-rt_1}\right\}\\
+\frac{a}{b}\left\{\begin{array}{l}-\frac{(b-r)}{r}\left((b-r)e^{(b-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2+e^{(b-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\right)\\ +\frac{1}{r}\left(b^2e^{bt_2}e^{-rt_1}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-bre^{bt_2}e^{-rt_1}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+be^{bt_2}e^{-rt_1}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}-bre^{bt_2}e^{-rt_1}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}+r^2e^{bt_2}e^{-rt_1}\right)\\ -(b-r)e^{(b-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-e^{(b-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}+(b-r)e^{(b-r)t_1}\end{array}\right\}\\
+\frac{c}{d}\left\{\begin{array}{l}(d-r)e^{(d-r)t_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(d-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}-(d-r)e^{(d-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-e^{(d-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\\ +\frac{1}{r}\left(\begin{array}{l}\mathrm{d}^2e^{dt_2}e^{-rt_3}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-rde^{dt_2}e^{-rt_3}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}+de^{dt_2}e^{-rt_3}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\\ -dre^{dt_2}e^{-rt_3}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}+r^2e^{dt_2}e^{-rt_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2-re^{dt_2}e^{-rt_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}\end{array}\right)\\ -\frac{(d-r)}{r}\left\{(d-r)e^{(d-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2+e^{(d-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\right\}\end{array}\right\}\\
+\frac{a}{b}\left\{\begin{array}{l}(b-r)e^{(b-r)t_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(b-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t\mathrm{d}t_1^2}-(b-r)e^{(b-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-e^{(b-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\\ +\frac{1}{r}\left(\begin{array}{l}b^2e^{bt_2}e^{-rt_3}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-bre^{bt_2}e^{-rt_3}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}+be^{bt_2}e^{-rt_3}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}-bre^{bt_2}e^{-rt_3}\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\\ +r^2e^{bt_2}e^{-rt_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2-re^{bt_2}e^{-rt_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}\end{array}\right)\\ -\frac{(b-r)}{r}\left((b-r)e^{(b-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2+e^{(b-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\right)\end{array}\right\}
\end{array}\right]\\
& +h_1q_1\frac{ka}{b}\left\{\frac{b^2e^{bt_1}}{r}\left(1-e^{-rt_1}\right)-\frac{be^{bt_1}e^{-rt_1}}{r}+be^{bt_1}e^{-rt_1}-re^{bt_1}e^{-rt_1}-(b-r)e^{(b-r)t_1}\right\}+c_2k\frac{a(b-r)}{b}e^{(b-r)t_1}\\
& +u_R\varphi a\left\{(b-r)e^{(b-r)t_3}\left(\frac{\mathrm{d}t\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(b-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}\right\}+ic\left\{\begin{array}{l}(d-r)e^{(d-r)t_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(d-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}\\ -(d-r)e^{(d-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-e^{(d-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\end{array}\right\}\\
& +l\left\{ka(b-r)e^{(b-r)t_1}+c\left((d-r)e^{(d-r)t_3}\left(\frac{\mathrm{d}t_3}{\mathrm{d}t_1}\right)^2+e^{(d-r)t_3}\frac{\mathrm{d}^2t_3}{\mathrm{d}t_1^2}-(d-r)e^{(d-r)t_2}\left(\frac{\mathrm{d}t_2}{\mathrm{d}t_1}\right)^2-e^{(d-r)t_2}\frac{\mathrm{d}^2t_2}{\mathrm{d}t_1^2}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 X}{\partial T \partial t_1} = h_R & \left\{ \begin{array}{l} \frac{\phi a}{b} \left\{ \begin{array}{l} (b-r)e^{(b-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} \frac{\mathrm{d}t_3}{dt_1} + e^{(b-r)t_3} \frac{\partial}{\partial T} \left(\frac{\mathrm{d}t_3}{dt_1} \right) + \frac{(b-r)}{r} \left((b-r)e^{(b-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} \frac{\mathrm{d}t_3}{dt_1} + e^{(b-r)t_3} \frac{\partial}{\partial T} \left(\frac{\mathrm{d}t_3}{dt_1} \right) \right) \\ -\frac{1}{r} \left(b^2 e^{bt_3} e^{-rt_2} \frac{\mathrm{d}t_3}{dt_1} \frac{\mathrm{d}t_3}{\mathrm{d}T} + b e^{bt_3} e^{-rt_2} \frac{\mathrm{d}t_3}{dt_1} \frac{\partial}{\partial T} \left(\frac{\mathrm{d}t_3}{dt_1} \right) \right) \end{array} \right\} \\ -\frac{(1-\delta)c}{\mathrm{d}} \left\{ e^{(d-r)t_3} \frac{\mathrm{d}t_3}{dt_1} - e^{(d-r)t_2} \frac{\mathrm{d}t_2}{dt_1} + \frac{(d-r)}{r} e^{(d-r)t_3} \frac{\mathrm{d}t_3}{dt_1} - \frac{1}{r} \left(d e^{dt_3} e^{-rt_2} \frac{\mathrm{d}t_3}{dt_1} - r e^{dt_3} e^{-rt_2} \frac{\mathrm{d}t_2}{dt_1} \right) \right\} \end{array} \right\} \\
& + h_P \left[\begin{array}{l} \{k(1-\delta)-1\} \frac{a}{b} \left(e^{(b-r)t_1} - e^{-rt_1} \right) \\ + \frac{a}{b} \left\{ -\frac{(b-r)}{r} e^{(b-r)t_2} \frac{\mathrm{d}t_2}{dt_1} + \frac{1}{r} \left(b e^{bt_2} e^{-rt_1} \frac{\mathrm{d}t_2}{dt_1} - r e^{bt_2} e^{-rt_1} \right) - e^{(b-r)t_2} \frac{\mathrm{d}t_2}{dt_1} + e^{(b-r)t_1} \right\} \\ + \frac{c}{d} \left\{ e^{(d-r)t_3} \frac{\mathrm{d}t_3}{dt_1} + e^{(d-r)t_2} \frac{\mathrm{d}t_2}{dt_1} + \frac{1}{r} \left(d e^{dt_2} e^{-rt_3} \frac{\mathrm{d}t_2}{dt_1} - r e^{dt_2} e^{-rt_3} \frac{\mathrm{d}t_3}{dt_1} \right) - \frac{(d-r)}{r} e^{(d-r)t_2} \frac{\mathrm{d}t_2}{dt_1} \right\} \\ + \frac{a}{b} \left\{ e^{(b-r)t_3} \frac{\mathrm{d}t_3}{dt_1} - e^{(b-r)t_2} \frac{\mathrm{d}t_2}{dt_1} + \frac{1}{r} \left(b e^{bt_2} e^{-rt_3} \frac{\mathrm{d}t_2}{dt_1} - r e^{bt_2} e^{-rt_3} \frac{\mathrm{d}t_3}{dt_1} \right) - \frac{(b-r)}{r} e^{(b-r)t_2} \frac{\mathrm{d}t_2}{dt_1} \right\} \end{array} \right] \\
& + h_1 q_1 \frac{ka}{b} \left\{ \frac{b e^{bt_1}}{r} \left(1 - e^{-rt_1} \right) + e^{bt_1} e^{-rt_1} - e^{(b-r)t_1} \right\} + c_2 k \frac{a}{b} e^{(b-r)t_1} + u_R \phi a e^{(b-r)t_3} \frac{\mathrm{d}t_3}{dt_1} \\
& + ic \left\{ e^{(d-r)t_3} \frac{\mathrm{d}t_3}{dt_1} - e^{(d-r)t_2} \frac{\mathrm{d}t_2}{dt_1} \right\} + l \left\{ ka e^{(b-r)t_1} + c \left(e^{(d-r)t_3} \frac{\mathrm{d}t_3}{dt_1} - e^{(d-r)t_2} \frac{\mathrm{d}t_2}{dt_1} \right) \right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial X}{\partial T} = h_R & \left[\frac{\varphi a}{b} \left\{ e^{(b-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}t} + \frac{(b-r)}{r} e^{(b-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}t} - \frac{b e^{bt_3} e^{-rt_2}}{r} \frac{\mathrm{d}t_3}{\mathrm{d}T} \right\} - \frac{(1-\delta)\, c}{\mathrm{d}} \left\{ \begin{array}{l} e^{(d-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} + \frac{(d-r)}{r} e^{(d-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} \\ -\frac{d e^{dt_3} e^{-rt_2}}{r} \frac{\mathrm{d}t_3}{\mathrm{d}T} \end{array} \right\} \right] \\
& + h_P \left[\frac{c}{d} \left\{ e^{(d-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} - e^{dt_2} e^{-rt_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} \right\} + \frac{a}{b} \left\{ e^{(b-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} - e^{bt_2} e^{-rt_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} \right\} \right] + u_R \varphi a e^{(b-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} \\
& + ic e^{(d-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T} + lc e^{(d-r)t_3} \frac{\mathrm{d}t_3}{\mathrm{d}T}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 X}{\partial T^2} = h_R & \left[\begin{array}{l} \frac{\varphi a}{b} \left\{ \begin{array}{l} (b-r)e^{(b-r)t_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 + e^{(b-r)t_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} + \frac{(b-r)}{r} \left((b-r)e^{(b-r)t_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 + e^{(b-r)t_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} \right) \\ -\frac{b^2 e^{bt_3} e^{-rt_2}}{r} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 - \frac{b e^{bt_3} e^{-rt_2}}{r} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} \end{array} \right\} \\ -\frac{(1-\delta)c}{\mathrm{d}} \left\{ \begin{array}{l} (d-r)e^{(d-r)t_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 + e^{(d-r)t_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} + \frac{(d-r)}{r} \left((d-r)e^{(d-r)t_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 + e^{(d-r)t_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} \right) \\ -\frac{\mathrm{d}^2 e^{dt_3} e^{-rt_2}}{r} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 - \frac{d e^{dt_3} e^{-rt_2}}{r} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} \end{array} \right\} \end{array} \right] \\
& + h_P \left[\begin{array}{l} \frac{c}{d} \left\{ (d-r)e^{(d-r)t_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 + e^{(d-r)t_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} + r e^{dt_2} e^{-rt_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 - e^{dt_2} e^{-rt_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} \right\} \\ + \frac{a}{b} \left\{ (b-r)e^{(b-r)t_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 + e^{(b-r)t_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} + r e^{bt_2} e^{-rt_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 - e^{bt_2} e^{-rt_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} \right\} \end{array} \right] \\
& + u_R \varphi a \left\{ (b-r)e^{(b-r)t_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 + e^{(b-r)t_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} \right\} + (ic + lc) \left\{ (d-r)e^{(d-r)t_3} \left(\frac{\mathrm{d}t_3}{\mathrm{d}T} \right)^2 + e^{(d-r)t_3} \frac{\mathrm{d}^2 t_3}{\mathrm{d}T^2} \right\}
\end{aligned}
$$

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