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SUPPLIER-RETAILER PRODUCTION AND INVENTORY MODELS WITH DEFECTIVE ITEMS AND INSPECTION ERRORS IN NON-COOPERATIVE AND COOPERATIVE ENVIRONMENTS

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Abstract. This paper proposes single-supplier single-retailer production and inventory models for maximizing the supplier and retailer's profits in non-cooperative and cooperative environments. The effect of defective items and inspection errors are considered in the proposed models. In addition, we consider that the supplier offers the retailer a quantity threshold to absorb transportation costs for promoting the economies of scale of transport. Mathematical analyses are conducted, and optimal equilibrium production and replenishment strategies for the supplier and retailer are derived under non-cooperative and cooperative situations. Subsequently, we establish two algorithms to explain the optimal equilibrium solutions for these cases. Finally, several numerical examples and a sensitivity analysis with respect to major parameters are presented to demonstrate the theoretical results, compare the distinct solutions, and derive managerial insights.

Mathematics Subject Classification. 90B05.

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1. Introduction

Product quality is an important and crucial issue for companies or supply chains in production and sales activities. On the production side, quality will affect cost and efficiency. In sales, customer satisfaction is primarily induced by quality. Traditional models of economic production quantity (EPQ) and economic order quantity (EOQ) are developed with a perfect production process and do not address product quality. However, such a situation is unattainable; in practice, defective items are inevitable in any production environment. Therefore, deriving the optimal production and order quantities from traditional EPQ and EOQ models is unrealistic because the influence of defective goods on production and order quantities is overlooked. Porteus [32] was the first to use the EPQ model to explore how an out-of-control production process can produce defective items. Considering that the purpose of this model is to determine the optimal production batch, Porteus hypothesized that manufacturers can invest in improving process quality to reduce the output of defective items. Lee and

Keywords. Inventory, Defective items, Inspection errors, Non-cooperative, Cooperative.

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Rosenblatt [21] developed an economic manufacturing quantity model that accounts for defective items, in which the production cycle length and equipment maintenance time serve as the decision variables. Lee and Rosenblatt [22] subsequently explored the EPQ model in an imperfect production process by considering the costs of machinery maintenance and repair. Zhang and Gerchak [57] developed an EOQ model with a random defective rate for identifying the optimal lot size and the fraction of goods to be inspected. Sana [37] proposed an economic production lot size model by assuming that the percentage of imperfect products varies with the production rate and production run time. Further, Sana [38] developed an EOQ model considering stochastic demand, where a certain percentage of nonconforming quality goods after the screening process are returned/bought back by the supplier at a reduced price. Other studies on production quality include [5, 33, 44, 53]. These studies have focussed on the effects of defective items on optimal production or order strategies without addressing how manufacturers should manage defective items.

To ensure product quality and maintain a favourable reputation, business owners perform quality inspections before selling their goods and implement procedures for managing defective items. Salameh and Jaber [35] conducted a full inspection on purchased goods and suggested selling defective items at a lower price. Chan *et al.* [4] developed an EPQ model that accounts for a full inspection of goods, selling defective items at lower prices, and reworking or rejecting products. Assuming that defective items occur randomly and can be reworked or rejected, Chiu [7] developed an EPQ model that considers out-of-stock situations. Papachristos and Konstantaras [30] extended Salameh and Jaber's [35] model to the case in which withdrawing occurs at the end of the planning horizon. Jaber *et al.* [17] extends the work of Salameh and Jaber [35] by assuming the percentage defective per lot reduces according to a learning curve. Further, Konstantaras *et al.* [20] extended the model of Jaber *et al.* [17] by assuming shortages and unequal lot sizes over a finite planning horizon. Other relevant studies include [8,9,16,25,27,34,36,40,42,43,58] and their references. These studies determined optimal production or order strategies by adopting various methods to manage defective items based on the assumption that there is no human error in the screening process.

However, inspection results may be erroneous because of staff negligence, old equipment, or ineffective inspection technology. In other words, a proportion of non-defective items might be misclassified as defective ones (Type I inspection error) and a proportion of defective items might be misclassified as non-defective ones (Type II inspection error) during the inspection process. Yoo *et al.* [54] developed an EPQ model that accounts for defective items and inspection errors, in which items deemed defective or defective items returned by consumers are sold at a lower price or at the original price after modification. Wang *et al.* [49] constructed an EPQ model that reports Type I and II errors during raw material inspection and confirmed that partial inspection is more effective than full or no inspection. Khan *et al.* [19] expanded the EOQ model developed by Salameh and Jaber [35], which accounts for defective items, to include instances that may involve erroneous inspections. Yoo *et al.* [55] further proposed an EPQ model in which manufacturers invest money to improve imperfect production and enhance inspection quality. The purpose of their paper is to determine optimal production and inspection strategies for minimizing total quality cost and maximizing total profits. Hsu and Hsu [14] developed an EOQ model in which defective items and inspection errors are accounted for, out-of-stock and complete backorders are tolerated, and defective items may be returned. Mohammadi *et al.* [24] studied an inventory-production system with process deterioration and imperfect inspection where elapsed time of the system in the in-control state is a random variable. Recently, Zhou et al. [59] developed a synergic EOQ model involving trade credit, shortages, imperfect quality and inspection errors.

As mentioned, studies on defective items and inspection errors have involved exploring optimal production or order strategies only from a supplier or retailer perspective. Because of economic liberalization, market internationalization, and product diversification, business owners must integrate supply chain systems, improve operational efficiency, quickly respond to customer demands, and reduce inventory costs to survive in the fiercely competitive and rapidly changing global market. Recently, the concept of supply chain management has received considerable attention. Goyal [10] first proposed a single-seller single-buyer integrated inventory model. Banerjee [1] proposed an integrated inventory model in which suppliers manufacture and ship products according to the order quantity requested by retailers. Goyal [11] reported that when the setup cost borne by suppliers

TABLE 1. The major issues compared the above-mentioned studies with present paper.

References	Individual/supply chain perspective	Defective item	Inspection error	Quantity-dependent freight
[20, 25]	EOQ	Constant	No	No
[16, 30, 33, 57]	EOQ	Random	No	No
[14, 19, 59]	EOQ	Random	Yes	No
[29]	EPQ	No	No	No
[4, 5, 8, 17, 27, 40, 42, 43, 58]	EPQ	Constant	No	No
[7, 21, 22, 32, 35–38, 44, 53]	EPQ	Random	No	No
[34]	EPQ	Fuzzy	No	No
[19, 24, 49, 54, 55]	EPQ	Random	Yes	No
[1, 2, 6, 10–13, 23, 28, 39, 41, 45, 47, 51, 52, 60]	Integration supply chain	No	No	No
[9]	Integration supply chain	Constant	No	No
[18]	Integration supply chain	Random	Yes	No
[15, 26, 31, 50, 56]	Competitive supply chain	No	No	No
[3]	Competitive/cooperative supply chain	No	No	No
Present paper	Competitive/cooperative supply chain	Random	Yes	Yes

is considerably higher than the ordering cost borne by retailers, the suppliers should manufacture products in integral multiples of the quantity ordered by the retailers and ship them in equal quantities. Compared with the total cost per unit time yielded by the method recommended by Banerjee [1], in which suppliers manufacture the exact quantity of products ordered by retailers, the return by the method proposed by Goyal [11] is lower. Subsequently, Lu [23] applied the model implemented by Goyal [11] to an integration problem involving a single supplier and multiple retailers by using a relaxed assumption that products could be shipped before all production processes were completed. Assuming that supplier productivity exceeds the market demand rate, suppliers can begin shipping when the manufactured products reach the quantity ordered by retailers; afterward, shipments can be made in equal quantities. Other relevant studies of supply chain production and inventory models include [2, 6, 12, 13, 39, 41, 45, 47, 51, 52, 60]. Recently, Khan *et al.* [18] reported a production-inventory model that integrates suppliers and retailers and accounts for inspection errors on the sales side and learning effects on the production side. The authors designed the model to identify production and order strategies that minimize the total supply chain cost. These studies have been conducted with the assumption that suppliers and retailers fully cooperate to form a single system in which both parties jointly determine the optimal production and order quantities that yield the maximal total profit or minimal total cost in the supply chain system.

In practice, not all suppliers and retailers are willing to cooperate to form a single supply chain system. Under such circumstances, game theory can be employed to understand the interaction between the strategies adopted by both parties, thereby yielding an equilibrium solution. Parlar [31], Wang and Parlar [50], Cachon and Zipkin [3], Netessine *et al.* [26], Yu and Huang [56] and Huang and Huang [15] have used the Nash equilibrium to obtain the optimal solution of various production and inventory problems when suppliers and retailers are unwilling to cooperate. However, these studies have overlooked the possibility of defective items existing among the goods manufactured by upstream enterprises (suppliers or manufacturers).

The major issues considered in the above-mentioned studies and present paper are summarized in Table 1. Based on Table 1, it is obvious that he superior differences of this paper against previous studies are: (1) Goods delivered by the supplier include defective items (random defective rate) and the retailer inspects each

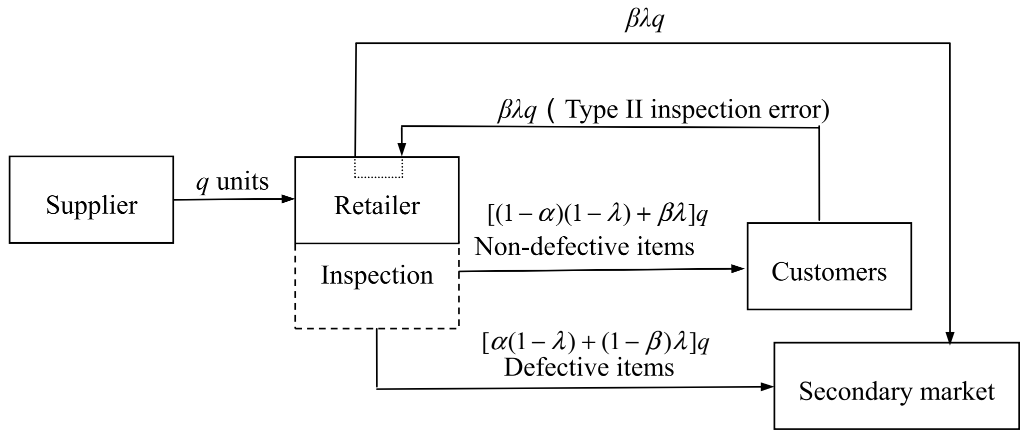

FIGURE 1. Supplier's production and the retailer's inventory systems.

lot of the purchased goods, although the inspection results may be erroneous; (2) we assume that the supplier encourages the retailer to purchase products in bulk quantities by offering a choice of unit shipping prices relative to the order quantity (*i.e.*, the unit shipping price decreases as the order quantity increases); and (3) the optimal production and order strategies are provided for the retailer and the supplier under non-cooperative and cooperative cases. In a non-cooperative supply chain system, because the supplier (retailer) is difficult to know its retailer' (supplier's) complete information but only know their decision results, then adopting Nash game is more convincing in this case. On the contrary, the two parties are willing to cooperate and negotiate with each other in a cooperative supply chain system and we use the cooperative Pareto equilibrium to identify the optimal equilibrium production and order strategies for the supply chain. First, we developed two mathematical models and conducted a mathematical analysis to obtain equilibrium solutions. Subsequently, on the basis of the theoretical results, two algorithms are designed to facilitate problem solving. We described the problem solving process by using numerical examples and compared the advantages and disadvantages of the non-cooperative and cooperative decisions. Finally, we proposed managerial implications according to the results of this study.

2. Problem description

In this model, one supplier and one retailer are considered as the members of the supply chain. At the outset, the retailer orders Q units with ordering cost A and unit wholesale price v. In turn, the supplier produces Q units and delivers $q = Q/n$ units in n shipments, where $n$is a positive integer. Furthermore, each shipment contains certain defective items at a defect rate of λ, and 100% inspection is conducted. Because of erroneous inspections, a portion of nondefective items may be misclassified as defective because of Type I inspection error and a portion of defective items may be misclassified as nondefective because of Type II inspection error. Type I and Type II inspection error occurs at proportions of α and β, respectively. All non-defective items (including misclassified items) are sold at the unit selling price, p, and the classified defective items are sold to a secondary market with a lower unit price, k, in a single batch after inspection. Over time, customers sequentially return defective items misclassified as non-defective because of Type II inspection error. For these returned ones, the retailer not only must provide refunds to each customer according to the unit selling price but also generate a treatment cost (including warranty cost) per unit. To reduce losses, these returned defective items are also sold to a secondary market at the end of each cycle. The entire process is repeated. Figure 1 shows the supplier and retailer's production and inventory systems.

Notation.

D	Retailer's demand rate
P	Supplier's production rate, where $P > D$
A	Retailer's ordering cost per order
K	Supplier's setup cost per setup
F	Fixed transportation cost per shipment
r_ℓ	The corresponding all-unit freight charged per unit, where ℓ =1, 2,. . . , ε
c	Supplier's unit production cost
v	Retailer's unit wholesale price, where $v > c$
p	Retailer's unit selling price of non-defective items, where $p > v$
k	Retailer's unit selling price of defective items in the secondary market, where $k < v$
w	Retailer's unit treatment cost of defective items (including warranty cost)
h_v	Supplier's holding cost per unit per unit time
h_1	Retailer's holding cost per non-defective item per unit time
h_2	Retailer's holding cost per defective item per unit time, where $h_2 \leqslant h_1$
s	Retailer's unit inspection cost
x	Retailer's inspection rate
λ	Defective rate per shipment, a random variable with $p.d.f.$ $f_1(\lambda)$, where $0 < \lambda < 1$
α	Proportion of Type I inspection error, a random variable with $p.d.f.$ $f_2(\alpha)$, where $0 < \alpha < 1$
β	Proportion of the Type II inspection error, a random variable with $p.d.f.$ $f_3(\beta)$, where $0 < \beta < 1$
Q	Retailer's order quantity per order, a decision variable
q	Supplier's shipment quantity per delivery, a decision variable
t	Retailer's inspection time during a replenishment cycle, that is$t = q/x$
T	Retailer's replenishment cycle length, a decision variable
n	Number of shipment from the supplier to the retailer per production run, an integer decision variable
$TPB(\cdot)$	Retailer's total profit per replenishment cycle
$TPV(\cdot)$	Supplier's total profit per production cycle
$E(\cdot)$	Expected value

TABLE 2. All-unit freight charged schedule.

ℓ	q	r_ℓ
1	$q_1 < q < q_2$	r_1
2	$q_2 \leqslant q < q_3$	r_2
...	...	...
ε	$q_\varepsilon \leqslant q$	r_ℓ

Assumption

(1) The production-inventory models consider a single supplier, single retailer, and single commodity. Further, no dominating firm exists in this supply chain system.
(2) The retailer orders Q units per order and allows the supplier to deliver $q$units in n shipments. That is, $Q = nq$
(3) The freight charged per unit r_ℓ is dependent on the shipment quantity q and has the following all-unit freight charged schedule (Tab. 2):

To develop the inventory system mathematical model, the notation is used throughout this paper as follows: where, $0 = q_1 < q_2 < \ldots < q_\ell$ and $r_1 > r_2 > \ldots > r_\ell > 0$

(4) Each shipment contains certain defective items with a defective rate of λ, and 100% inspection is conducted. During the inspection process, Type I and Type II inspection errors may occur at proportions of α and β, respectively. Table 3 shows the results of the retailer's product inspection process.

TABLE 3. The results of stock inspection for the retailer

	Inspection results		Total
	Non-defective	Defective	
Real non-defective	$(1-\alpha)(1-\lambda)q$	$\alpha(1-\lambda)q$ (Type I error)	$(1-\lambda)q$
Real defective	$\beta\lambda q$ (Type II error)	$(1-\beta)\lambda q$	λq
Total	$[(1-\alpha)(1-\lambda)+\beta\lambda]q$	$[\alpha(1-\lambda)+(1-\beta)\lambda]q$	q

(5) After inspection, all classified defective items are immediately sold in a single batch in a secondary market at a lower price, k. In addition, as to the defective items successively returned by customers because of a Type II inspection error, they are stored and then sold to a secondary market in a single batch at the end of each cycle.
(6) The defective rate of item λ, the proportion of the Type I inspection error (α), and the proportion of the Type II inspection error (β) are independent, random variables with probability density functions $f_1(\lambda)$, $f_2(\alpha)$, and $f_3(\beta)$, respectively.
(7) Replenishments are instantaneous, and the lead time is zero.
(8) Shortages are not allowed to occur.

3. Model formulation

Now, we initially establish the retailer's total profit function per replenishment cycle and then establish the supplier's total profit function per production run base on above notation and assumptions.

3.1. Retailer's total profit function

The retailer's total profit per replenishment cycle includes the sales revenue, ordering cost, purchasing cost, freight cost, inspection cost, holding cost, and treatment cost for the customer-returned defective items. These components are calculated as follows:

(1) Sales revenue (SR): The sales revenue per replenishment cycle includes real non-defective items ($(1-\alpha)(1-\lambda)q$ units with unit price$p$), classified defective items ($[\alpha(1-\lambda)+(1-\beta)\lambda]q$ units with unit price k), and defective items returned by customers ($\beta\lambda q$ units with unit price k). Therefore, the sales revenue per replenishment cycle is expressed as $SR = p(1-\alpha)(1-\lambda)q + k[\alpha(1-\lambda)+\lambda]q$
(2) Ordering cost (OC): The retailer's ordering cost per replenishment cycle is $OC = A/n$
(3) Purchasing cost (PC): The retailer's purchasing cost per replenishment cycle is $PC = \quad vq$.
(4) Freight cost (FC_ℓ): The retailer's freight cost per replenishment cycle includes fixed cost F and various costs associated with the shipment quantity per delivery. Namely, when the shipment quantity is $q \in [q_\ell,\ q_{\ell+1})$, ℓ =1, 2, ..., ε, the retailer's freight cost per replenishment cycle is $FC_\ell = F + r_\ell q$, where r_ℓ is shown in Table 1.
(5) Inspection cost (IC): With a unit inspection cost, s, the inspection cost per replenishment cycle is IC= sq.
(6) Holding cost (HC): Figure 2 illustrates the retailer's inventory level in a replenishment cycle. Before defective items are inspected, they are stored in a warehouse until they are examined and classified as defective following which they are managed. Hence, the retailer's holding cost includes non-defective and defective items and is calculated as follows:

Figure 2 shows that because the quantity of classified non-defective items is $[(1-\alpha)(1-\lambda)+\beta\lambda]q$ and the quantity of defective items before identification is $[\alpha(1-\lambda)+(1-\beta)\lambda]q$, the retailer's cost of holding non-defective items per replenishment cycle can be calculated according to the holding cost per non-defective item per unit time h_1 as

$$h_1\left\{\frac{1}{2}\left[(1-\alpha)(1-\lambda)+\beta\lambda\right]qT+\frac{1}{2}\left[\alpha(1-\lambda)+(1-\beta)\lambda\right]q\left(\frac{q}{x}\right)\right\}$$

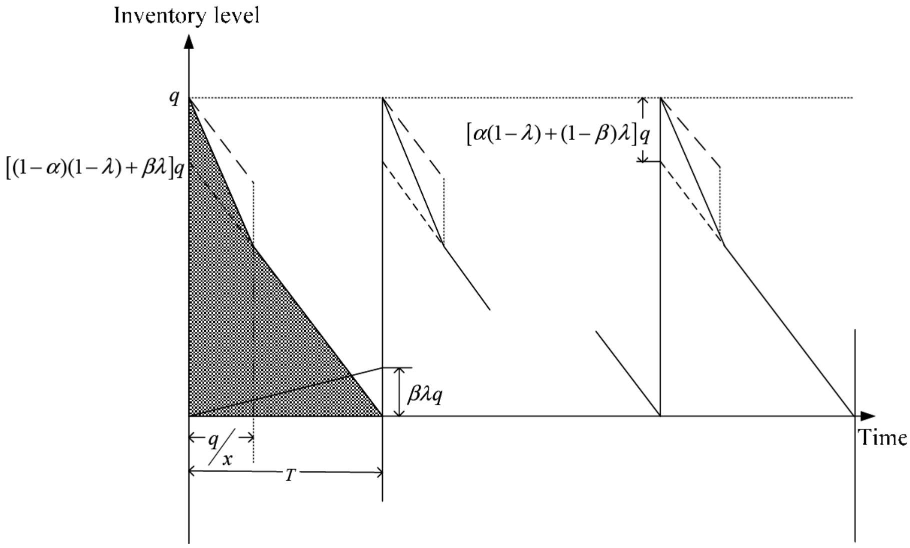

FIGURE 2. Retialer inventory level per replenishment cycle.

By contrast, the retailer's cost of holding defective items per replenishment cycle can be calculated according to the holding cost per defective item per unit time h_2 as

$$h_2\left\{\frac{1}{2}\left[\alpha\left(1-\lambda\right)+\left(1-\beta\right)\lambda\right]q\left(\frac{q}{x}\right)+\frac{1}{2}\beta\lambda qT\right\}$$

In summary, the retailer's total holding cost per replenishment cycle is

$$HC=\frac{1}{2}\left\{h_1\left[\left(1-\alpha\right)\left(1-\lambda\right)+\beta\lambda\right]+h_2\beta\lambda\right\}qT+\frac{q^2}{2x}\left(h_1+h_2\right)\left[\alpha\left(1-\lambda\right)+\left(1-\beta\right)\lambda\right]$$

(7) Treatment cost for the returned defective items (TC): Because of Type II inspection error, $\beta\lambda q$ units of defective products are overlooked during inspection and sold to customers incurring defective item treatment costs (including warranty cost, reverse logistics from customers to the retailer, and loss of goodwill) in addition to w per unit. Therefore, the treatment cost for the customer-returned defective items per replenishment cycle is $TC = w\beta\lambda q$.

Based on the above-mentioned components, the retailer's total profit function in a replenishment cycle for a given value of r_ℓ is expressed as

$$\begin{aligned}TPB_\ell(q) &= SR-OC-PC-FC_\ell-IC-HC-TC\\ &= p\left(1-\alpha\right)\left(1-\lambda\right)q+k\left[\alpha\left(1-\lambda\right)+\lambda\right]q\\ &\quad-\left\{\frac{A}{n}+F+\gamma_\ell q+sq+v\,q+w\beta\lambda q+\frac{1}{2}qT\right.\\ &\quad\times\left\{h_1\left[\left(1-\alpha\right)\left(1-\lambda\right)+\beta\lambda\right]+h_2\beta\lambda\right\}\\ &\quad\left.+\frac{q^2}{2x}\left(h_1+h_2\right)\left[\alpha\left(1-\lambda\right)+\left(1-\beta\right)\lambda\right]\right\}\end{aligned} \tag{3.1}$$

Because λ, α, and β are independent random variables with probability density functions $f_1(\lambda)$, $f_2(\alpha)$, and $f_3(\beta)$, respectively, for a given value of r_ℓ, the retailer's expected total profit in a replenishment cycle is

$$\begin{aligned} E[TPB_\ell(q)] =& p\ (1-\mu_\lambda)(1-\mu_\alpha)q + k\left[\mu_\alpha(1-\mu_\lambda)+\mu_\lambda\right]q \\ &- \frac{A}{n} - F - (r_\ell - +s - +v - +w\mu_\lambda\mu_\beta)q \\ &- \frac{qE(T)}{2}[h_1(1-\mu_\lambda)(1-\mu_\alpha)+(h_1+h_2)\mu_\lambda\mu_\beta] \\ &- \frac{q^2}{2x}(h_1+h_2)[\mu_\alpha(1-\mu_\lambda)+(1-\mu_\beta)\mu_\lambda]. \end{aligned} \tag{3.2}$$

where $\mu_\lambda = E(\lambda)$, $\mu_\alpha = E(\alpha)$, and $\mu_\beta = E(\beta)$.

3.2. Supplier's total profit function

The supplier's total profit per production run includes the sales revenue, setup cost, production cost, and holding cost. These components are calculated as follows:

(1) Sales revenue (SR): The supplier's sales revenue per production run is expressed as $SR = vnq$.
(2) Setup cost (SC): The supplier's setup cost per production run is $SC = K$
(3) Production cost (PC): The supplier's production cost per production run is $PC =cnq$.
(4) Holding cost (HC): Figure 3 shows the supplier's inventory level in a production run. From Figure 3, the supplier's cumulative inventory quantity in the production run is

$$\begin{aligned} &nq\left\{\frac{q}{P}+\frac{(n-1)\left[(1-\alpha)(1-\lambda)+\beta\lambda\right]q}{D}\right\}-\frac{n^2q^2}{2P}-\left[1+2+\ldots+(n-1)\right]q\left[(1-\alpha)(1-\lambda)+\beta\lambda\right]\frac{q}{D} \\ &\qquad = \frac{nq^2}{2}\left\{\frac{2-n}{P}+\frac{(n-1)[(1-\alpha)(1-\lambda)+\beta\lambda]}{D}\right\} \end{aligned}$$

Hence, the supplier's holding cost per production run is

$$HC = \frac{h_v nq^2}{2}\left\{\frac{2-n}{P}+\frac{(n-1)[(1-\alpha)(1-\lambda)+\beta\lambda]}{D}\right\}$$

$$q/P(n-1)[(1-\alpha)(1-\lambda)+\beta\lambda]q/Dnqnq/Pq$$

Therefore, the supplier's total profit function in a production run can be derived as follows:

$$\begin{aligned} TPV(n) &= SR - SC - PC - HC \\ &= vnq - K - cnq - \frac{h_v nq^2}{2}\left\{\frac{2-n}{P}+\frac{(n-1)[(1-\alpha)(1-\lambda)+\beta\lambda]}{D}\right\} \end{aligned} \tag{3.3}$$

Similarly, the supplier's expected total profit in a production run is

$$E[TPV(n)] = vnq - K - cnq - \frac{h_v nq^2}{2}\left\{\frac{2-n}{P}+\frac{(n-1)[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]}{D}\right\}. \tag{3.4}$$

4. Theoretical results

The following subsections explain the retailer's and the supplier's optimal strategies in non-cooperative and cooperative environments

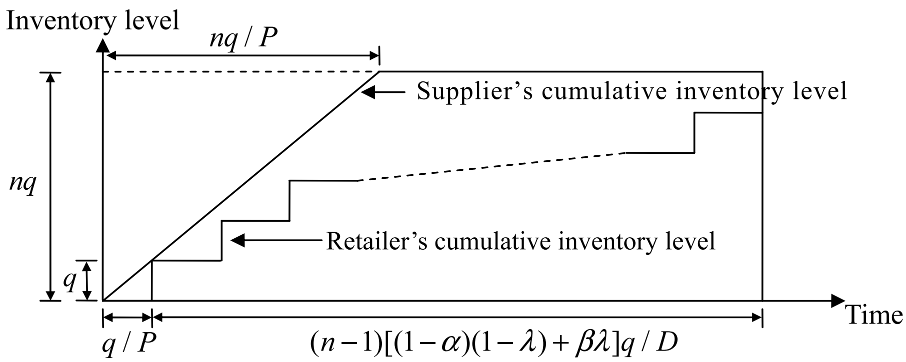

FIGURE 3. Supplier's inventory level in a production run.

4.1. Non-cooperative Nash equilibrium solution

Table 2 shows that $[(1-\alpha)(1-\lambda)+\beta\lambda]q$ units are classified as non-defective after inspection. These units meet the market demand in a replenishment cycle; namely, $[(1-\alpha)(1-\lambda)+\beta\lambda]q = DT$. Therefore, the length of the retailer's replenishment cycle can be expressed as $T = [(1-\alpha)(1-\lambda)+\beta\lambda]q/D$ and hence the expected length of the retailer's replenishment cycle time is

$$\begin{aligned} E(T) &= E\left\{\left[(1-\alpha)\left(1-\lambda\right)+\beta\lambda\right]q/D\right\} \\ &= \left[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta\right]q/D. \end{aligned}$$

For a given value of r_ℓ, the Renewal-Reward Theorem can be used to derive the retailer's expected total profit per unit time as

$$\begin{aligned} E\left[TPUB_\ell(q)\right] &= \frac{E\left[TPB_\ell(q)\right]}{E(T)} \\ &= \frac{D}{(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta}\left\{p(1-\mu_\lambda)(1-\mu_\alpha)+k\ \left[\mu_\alpha(1-\mu_\lambda)+\mu_\lambda\right]\right. \\ &\quad \left. -\left\{r_\ell+s+v+wm_1m_3+q(h_1+h_2)[\mu_\alpha(1-\mu_\lambda)+(1-\mu_\beta)\mu_\lambda]/(2x)\right\}\right\} \\ &\quad -\frac{D(A/n+F)}{\left[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta\right]q}-\frac{q}{2}\left[h_1(1-\mu_\lambda)(1-\mu_\alpha)+(h_1+h_2)\mu_\lambda\mu_\beta\right]. \end{aligned} \tag{4.1}$$

and the supplier's expected total profit per unit time as

$$\begin{aligned} E\left[TPUV(n)\right] &= \frac{E\left[TPV(n)\right]}{E(nT)} \\ &= \frac{D(v-c)}{(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta}-\frac{DK}{n\left[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta\right]q} \\ &\quad -\frac{Dh_v(2-n)q}{2P\left[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta\right]}-\frac{h_v(n-1)q}{2} \end{aligned} \tag{4.2}$$

The objective of this case is to determine the optimal batch quantity, q^*, and the number of shipments per production cycle, n^*, for maximizing the retailer's and the supplier's expected total profits per unit time in a competitive environment. For the retailer, for a given value of r_ℓ, $\ell = 1,\ 2, \ldots,\ \varepsilon$, the first-order necessary condition required for maximizing $E\,[TPUB_\ell(q)]$ is $dE\,\,[TPUB_\ell(q)]/dq = 0$, which leads to

$$\frac{(h_1+h_2)D[\mu_\alpha(1-\mu_\lambda)+(1-\mu_\beta)\mu_\lambda]}{2x[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]}+\frac{h_1(1-\mu_\lambda)(1-\mu_\alpha)+(h_1+h_2)\mu_\lambda\mu_\beta}{2}=\frac{D(A/n+F)}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]q^2}. \tag{4.3}$$

Furthermore, the second-order sufficient condition is

$$\frac{d^2E[TPUB_\ell(q)]}{dq^2}=-\frac{2D(A/n+F)}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]q^3}<0.$$

Therefore, for a given value of r_ℓ, $\ell = 1,\ 2, \ldots,\ \varepsilon$, $E[TPUB_\ell(q)]$ is a concave function of q; therefore, there is a unique value of q that maximizes $E[TPUB_\ell(q)]$. By solving equation (4.3), the following is derived:

$$q=\sqrt{\frac{2xD(A/n+F)}{H}}, \tag{4.4}$$

where

$$H=(h_1+h_2)D[\mu_\alpha(1-\mu_\lambda)+(1-\mu_\beta)\mu_\lambda]+x\,[h_1(1-\mu_\lambda)(1-\mu_\alpha)+(h_1+h_2)\mu_\lambda\mu_\beta][(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]>0.$$

Note that the value of q in equation (4.4) is independent of r_ℓ.

Regarding the supplier, the first-order necessary condition required for maximizing the supplier's expected total profit per unit time $E[TPUV(n)]$ is $dE[TPUV(n)]/dn = 0$. This implies that

$$\frac{DK}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]n^2q}-\frac{\{P[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]-D\}h_vq}{2P[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]}=0. \tag{4.5}$$

Because the second-order derivative of $E[TPUV(n)]$, with respect to n, is

$$\frac{d^2E[TPUV(n)]}{dn^2}=\frac{-2DK}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]n^3q}<0,$$

$E[TPUV(n)]$ is a concave function of n; therefore, there is a unique value of n that maximizes $E[TPUV(n)]$. By solving equation (4.5), the following expression is derived:

$$n=\sqrt{\frac{2PDK}{\{P[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]-D\}h_vq^2}}. \tag{4.6}$$

Notice that the optimal solution for n must be a positive integer because this variable represents the number of shipments per production run from the supplier to the retailer. We can summarize the preceding arguments and develop an algorithm to obtain the Nash equilibrium solution for the supplier and retailer in a non-cooperative environment. The flowchart is also shown as in Figure 4.

Algorithm 1.

Step 1. Start with $j = 1$ and the initial value of $n^*_{(j)} = 1$
Step 2. Find $q_{(j)}$ from equation (4.4).
Step 3.

(i) If $q_{(j)} \in [q_\ell,\ q_{\ell+1})$ for some $\ell = 1,\ 2,\ \ldots,\ \varepsilon-1$, then substitute $(q_{(j)},\ r_\ell)$, and $(q_{\ell+1},\ r_{\ell+1})$, $(q_{\ell+2},\ r_{\ell+2}),\ldots,$ $(q_\varepsilon,\ r_\varepsilon)$ into equation (4.1) to separately evaluate $E[TPUB_\ell(q_{(j)})]$ and $E[TPUB_i(q_i)]$, where $i = \ell+1,\ \ell+2,\ \ldots,\ \varepsilon$. Go to Step 4.

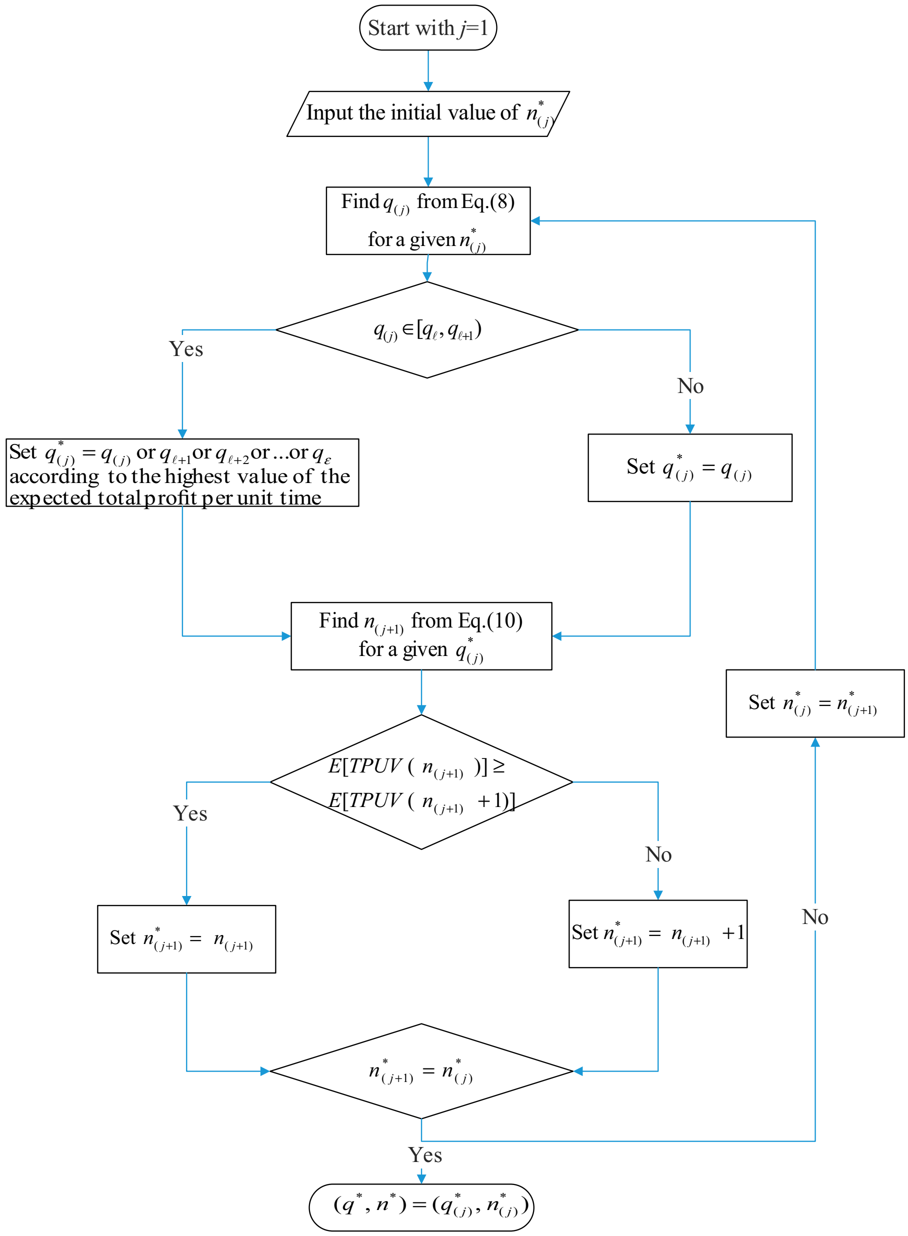

FIGURE 4. The flowchart of Algorithm 1.
Note: $\lfloor n_{(j+1)} \rfloor$ denote the great integer less than or equal to $n_{(j+1)}$

(ii) If $q_{(j)} \in [q_\varepsilon, \infty)$, then set $q^*_{(j)} = q_{(j)}$ and substitute $(q^*_{(j)}, \gamma_\varepsilon)$ into equation (4.1) to evaluate $E[TPUB_\varepsilon(q^*_{(j)})]$. Go to Step 5.

Step 4. Find Max$\{E[TPUB_\ell(q_{(j)})], E[TPUB_i(q_i)], i = \ell+1, \ell+2, \ldots, \varepsilon\}$ and set $q^*_{(j)} = q_{(j)}$ or q_i according to the highest value of the retailer's expected total profits per unit time

Step 5. For a given $q^*_{(j)}$ solve for the value of n(set $n_{(j+1)}$) from equation (4.6). Let $\lfloor n_{(j+1)} \rfloor$ denote the great integer less than or equal to $n_{(j+1)}$ and separately substitute $\lfloor n_{(j+1)} \rfloor$ and $\lfloor n_{(j+1)} \rfloor + 1$ in equation (4.2).

(i) If $E[TPUV(\lfloor n_{(j+1)} \rfloor)] \geqslant E[TPUV(\lfloor n_{(j+1)} \rfloor + 1)]$, then set $n^*_{(j+1)} = \lfloor n_{(j+1)} \rfloor$.
(ii) If $E[TPUV(\lfloor n_{(j+1)} \rfloor)] < E[TPUV(\lfloor n_{(j+1)} \rfloor + 1)]$, then set $n^*_{(j+1)} = \lfloor n_{(j+1)} \rfloor + 1$.

Step 6. If $n^*_{(j+1)} = n^*_{(j)}$, then set $n^* = n^*_{(j)}$ and $q^* = q^*_{(j)}$. Therefore, $(q^*,\ n^*)$ is the Nash equilibrium solution. Otherwise, set $j = j + 1$ and return to Step 2.

Once the optimal solution $(q^*,\ n^*)$ is obtained, the optimal retailer's order quantity $Q^* = n^* q^*$ follows.

4.2. Cooperative Pareto equilibrium solution

If the supplier and retailer cooperate through consultations and adopt a Pareto equilibrium model, then they can jointly determine the optimal strategies to maximize the weighted expected total profit per unit time. The weight of the retailer's expected total profit per unit time is δ, whereas that of the supplier's expected total profit per unit time is $1-\delta$, where $0<\delta<1$. Hence, for a given value of r_ℓ, the weighted expected total profit per unit time is

$$\begin{aligned}E[JTPU_\ell(q,\ n)] &= \delta \cdot E\left[TPUB_\ell(q)\right] + (1-\delta)\cdot E\left[TPUV(n)\right]\\ &= \frac{D}{(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta}\ \{\delta\,\{\,p(1-\mu_\lambda)(1-\mu_\alpha) + k\ [\mu_\alpha(1-\mu_\lambda)+\mu_\lambda]\ -\{r_\ell\ + s\\ &\quad + v + w\,\mu_\lambda\mu_\beta + q(h_1+h_2)[\mu_\alpha(1-\mu_\lambda) + (1-\mu_\beta)\mu_\lambda]/(2x)\} + (1-\delta)(v-c)\}\\ &\quad - \frac{D\{[\delta A + (1-\delta)K]/n + \delta\, F\}}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]\, q} - \frac{q}{2}\{\delta\,[h_1(1-\mu_\lambda)(1-\mu_\alpha) + (h_1+h_2)\mu_\lambda\mu_\beta]\\ &\quad + (1-\delta)h_v(n-1)\} - \frac{(1-\delta)Dh_v(2-n)q}{2P[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]}. \end{aligned} \tag{4.7}$$

The objective of this case is to determine the optimal batch quantity, q^*, and the number of shipments per production run, n^*, to maximize the weighted expected total profit per unit time in a cooperative environment. The first-order necessary conditions required for maximizing $E[JTPU_\ell(q,\ n)]$ are $\partial E[JTPU_\ell(q,\ n)]/\partial q = 0$ and $\partial E[JTPU_\ell(q,\ n)]/\partial n = 0$, where $\ell = 1,\ 2,\ \ldots,\ \varepsilon$ which lead to

$$\begin{aligned}\frac{\partial E[JTPU_\ell(q,\ n)]}{\partial q} &= \frac{-\delta\, D(h_1+h_2)[\mu_\alpha(1-\mu_\lambda)+(1-\mu_\beta)\mu_\lambda]}{2x\,[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]} + \frac{D\{[\delta A + (1-\delta)K]/n + \delta\, F\}}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]\, q^2}\\ &\quad - \frac{\delta\,[h_1(1-\mu_\lambda)(1-\mu_\alpha)+(h_1+h_2)\mu_\lambda\mu_\beta] + (1-\delta)h_v(n-1)}{2}\\ &\quad - \frac{(1-\delta)Dh_v(2-n)}{2P\,[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]} = 0,\end{aligned} \tag{4.8}$$

and

$$\begin{aligned}\frac{\partial\, E[JTPU_\ell(q,\ n)]}{\partial\, n} &= \frac{D\,[\delta\, A + (1-\delta)K]}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]\, n^2 q} - \frac{(1-\delta)h_v q}{2}\\ &\quad + \frac{(1-\delta)Dh_v q}{2P\,[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]} = 0.\end{aligned} \tag{4.9}$$

Deriving the closed-form solution of $(q,\ n)$ from equations (4.8) and (4.9) is difficult. In addition, because of the high-power expression of the decision variables, the concavity property of the weighted expected total profit per unit time in equation (4.7) cannot be proved using the Hessian Matrix. Instead, we solve the problem by using the following search procedure: First, for fixed q and r_ℓ ($\ell = 1,\ 2,\ \ldots,\ \varepsilon$), examining the effect of n on the weighted expected total profit per unit time $E[JTPU_\ell(q,\ n)]$ and obtaining the second-order derivative of $E[JTPU_\ell(q,\ n)]$ with respect to n yields

$$\frac{d^2E[JTPU_\ell(q,\ n)]}{dn^2} = \frac{-2D[\delta A + (1-\delta)K]}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]\ n^3 q} < 0.$$

Hence, $E[JTPU_\ell(q,\ n)]$ is a concave function of n for given q and r_ℓ ($\ell = 1, 2, \ldots, \varepsilon$). Consequently, the search for the optimal number of shipments n is narrowed to derive a local maximum.

For any given integer n, the condition required for maximizing the weighted expected total profit per unit time $E[JTPU_\ell(q,\ n)]$ is $d\,E[JTPU_\ell(q,\ n)]/dq = 0$, which implies that

$$\begin{aligned}\frac{\delta\, D(A+nF)+(1-\delta)DK}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]nq^2} &= \frac{(1-\delta)h_v\{D(2-n)+P[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta](n-1)\}}{2P\,[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]}\\ &\quad + \frac{\delta\, D(h_1+h_2)[\mu_\alpha(1-\mu_\lambda)+(1-\mu_\beta)\mu_\lambda]}{2x\,[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]}\\ &\quad + \frac{\delta\, h_1(1-\mu_\lambda)(1-\mu_\alpha)+(h_1+h_2)\mu_\lambda\mu_\beta}{2}.\end{aligned} \tag{4.10}$$

Next, for any given n, obtaining the second-order derivative of $E[JTPU_\ell(q,\ n)]$ with respect to q yields

$$\frac{d^2E[JTPU_\ell(q,\ n)]}{dq^2} = -\frac{2D[\delta\,(A+nF)+(1-\delta)\,K]}{[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta]\,n\,q^3} < 0.$$

Consequently, for any given n, $E[JTPU_\ell(q,\ n)]$ is a concave function of q and hence there is a unique value of q (denoted by $q_{(n)}$) that maximizes $E[JTPU_\ell(q,\ n)]$ as follows:

$$q_{(n)} = \sqrt{\frac{2[\delta\, D(A/n+F)+(1-\delta)D(K/n)]}{H'}}, \tag{4.11}$$

where

$$\begin{aligned}H' = &\frac{(1-\delta)h_v\{D(2-n)+[(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta](n-1)P\}}{P}\\ &+ \frac{\delta\, D(h_1+h_2)[\mu_\alpha(1-\mu_\lambda)+(1-\mu_\beta)\mu_\lambda]}{x}\\ &+ [(1-\mu_\lambda)(1-\mu_\alpha)+\mu_\lambda\mu_\beta[\delta\, h_1(1-\mu_\lambda)(1-\mu_\alpha)+(h_1+h_2)\mu_\lambda\mu_\beta] > 0.\end{aligned}$$

Similarly, notice that the value $q_{(n)}$ in equation (4.11) is independent of r_ℓ.

An algorithm can be developed to obtain the optimal solution for the supplier and retailer in a cooperative environment. The flowchart is similar to Algorithm 1, and hence we omit it here.

Algorithm 2.

Step 1. Start with the initial value of $n = 1$.
Step 2. Find $q_{(n)}$ from equation (4.11).
Step 3.

(1) If $q_{(n)} \in [q_\ell,\ q_{\ell+1})$ for some $\ell = 1, 2, \ldots, \varepsilon - 1$, then substitute $(q_{(n)}, r_\ell)$ and $(q_{\ell+1},\ r_{\ell+1})$, $(q_{\ell+2},\ r_{\ell+2}), \ldots$, $(q_\varepsilon,\ r_\varepsilon)$ in equation (4.7) to separately evaluate $E[JTPU_\ell(q_{(n)},\ n)]$ and $E[JTPU_i(q_i, n)]$, where $i = \ell + 1,\ \ell+2, \ldots, \varepsilon$. Go to Step 4.
(2) If $q_{(n)} \in [q_\varepsilon,\ \infty)$, then substitute $(q_{(n)}, r_\varepsilon)$ in equation (4.7) to evaluate $E[JTPU_\varepsilon(q_{(n)},\ n)]$, and set $E[JTPU_\varepsilon(q^*_{(n)},\ n)] = E[JTPU_\varepsilon(q_{(n)},\ n)]$. Go to Step 5.

TABLE 4. Freight terms for Example 5.1.

Shipment quantity (units)	Freight (\$/unit)
$q_1 = 0 \leqslant q < 5000 = q_2$	$r_1 = 0.5$
$q_2 = 5000 \leqslant q < 10\,000 = q_3$	$r_2 = 0.45$
$q_3 = 10\,000 \leqslant q$	$r_3 = 0.4$

TABLE 5. Results of using Algorithm 1 for Example 5.1.

j	n_j^*	$q_{(j)}^*$	$E[TPUB_j(q)]$	$E[TPUB_j(q)]$
1	1	10 000	158, 197	154, 446
2	2	10 000	158, 676	155, 311
3	2	10 000	158, 676	155, 311

Note: Boldface type expresses the optimal solution of in Example 5.1.

Step 4. Find $Mxa\{E[JTPU_\ell(q_{(n)},\ n)],\ E[JTPU_i(q_i,\ n)],\ i = \ell+1,\ \ell+2,\ldots,\varepsilon\}$, and set $E[JTPU(q_{(n)}^*,\ n)] = Mxa\{E[JTPU_\ell(q_{(n)},\ n)],\ E[JTPU_i(q_i,\ n)],\ i = \ell+1,\ \ell+2,\ldots,\varepsilon\}$.
Step 5. Set $n = n + 1$ and repeat Steps 2−4.
Step 6. If $E[JTPU(q_{(n)}^*,\ n)] \geqslant E[JTPU(q_{(n-1)}^*,\ n-1)]$, then return to Step 5; otherwise, execute Step 7.
Step 7. Let $(q^*, n^*) = (q_{(n-1)}^*,\ n-1)$; therefore, (q^*, n^*) is the optimal solution and the corresponding maximum weighted expected total profit per unit time is $E[JTPU(q^*,\ n^*)]$.

Once the optimal solution (q^*, n^*) is obtained, the retailer's optimal order quantity $Q^* = n^*\, q^*$ follows.

5. NUMERICAL EXAMPLES

Example 5.1. To illustrate the problem solving procedure, we consider an inventory system involving the following data: $D = 30000$ units/year, $P = 45000$ units/year, $A = \$300$/order, $K = \$1000$/setup, $F = \$100$/shipment, $c = \$3$/unit, $v = \$8$/unit, $p = \$15$/unit, $k = \$3$/unit, $w = \$2$/unit, $h_v = \$0.5$/unit/year, $h_1 = \$0.75$/unit/year, $h_2 = \$0.35$/unit/year, $s = \$0.75$/unit, $x = 150\,000$ units/year, $\lambda \sim U(0,\ 0.02)$, $\alpha \sim U(0,\ 0.1)$, $\beta \sim U(0,\ 0.1)$, implying that $E[\lambda] = \mu_\lambda = 0.01$, $E[\alpha] = \mu_\alpha = 0.05$, and $E[\beta] = \mu_\beta = 0.05$. Table 4 lists the freight terms offered by the supplier.

The optimal Nash equilibrium solution derived using Algorithm 1 is $(q^*, n^*) = (10\,000,\ 2)$ Hence, the retailer's optimal order quantity is $Q^* = n^* q^* = 20\,000$ units, the retailer's expected annual total profit is $E[TPUB(q^*)] = \$158{,}676$, the supplier's expected annual total profit is $E[TPUV(n^*)] = \$155{,}311$, and the sum of the retailer's and supplier's expected annual total profit is $\$313,987$ (*i.e.*, \$158,676 + \$155,311). The solution procedure is shown in Table 5. The results show that supplier can provide freight discounts to encourage the retailer to deliver more quantities each time such that the delivery times can be reduced effectively.

Example 5.2. When considering a cooperative environment, to understand the effect of δ on the retailer's expected annual total profit and the supplier's expected annual total profit, we compared the optimal expected annual total profit of the retailer with that of the supplier. The data are identical to those in Example 1, except that$\delta \in \{0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9\}$. Algorithm 2 is applied to obtain an optimal value of (q^*, n^*). Following, the retailer's expected annual total profit $E[TPUB(q^*)]$ and the supplier's expected annual total profit $E[TPUV(n^*)]$ are subsequently determined from equations (4.1) and (4.2), respectively, such that the weighted expected total profit per unit time (denoted by $JTPU^*$) has a maximum value. Table 6 shows the computation results for different values of δ.

TABLE 6. The retailer's and supplier's optimal solutions with various values of δ.

δ	(q^*, n^*)	$E[TPUB(q^*)]$	$E[TPUV(n^*)]$	$JTPU^*$
0.1	(1525, 14)	\$157, 433	\$156, 197	\$156, 320
0.2	(5000, 4)	\$159, 200	\$155, 832	\$156, 506
0.3	(5000, 4)	\$159, 200	\$155, 832	\$156, 842
0.4	(5000, 5)	\$159, 295	\$155, 786	\$157, 190
0.5	(5000, 5)	\$159, 295	\$155, 786	\$157, 541
0.6	(5000, 5)	\$159, 295	\$155, 786	\$157, 892
0.7	(5000, 5)	\$159, 295	\$155, 786	\$158, 243
0.8	(5000, 6)	\$159, 359	\$155, 635	\$158, 614
0.9	(10 000, 4)	\$159, 552	\$155, 113	\$159, 062

Note. $JTPU^* = \delta\, E[TPUB(q^*)] + (1-\delta)E[TPUV(n^*)]$.

TABLE 7. Optimal solutions of non-cooperative and cooperative policies.

	Non-cooperative	Cooperative	Allocated
Optimal number of shipments per production run	2	4	
Optimal batch quantity	10 000	10 000	
Optimal order quantity	20 000	40 000	
Retailer's expected annual total profit	\$158, 970	\$159, 552	\$159, 164
Supplier's expected annual total profit	\$155, 311	\$155, 113	\$155, 501
Joint expected annual total profit	\$314, 281	\$314, 665	\$314, 665

Table 5 shows that a higher weight δ results in a higher weighted expected annual total profit. Furthermore, from an economical perspective, a higher weight δ (which implies the retailer has a greater power or in a comparative position of the supply chain system) results in a larger order quantity and a higher retailer's expected annual total profit. However, when the weight δ increases, the supplier's expected annual total profit decreases.

Example 5.3. The data in this example are identical to those in Example 5.1. In addition, we set $\delta = 0.9$. The optimal Pareto equilibrium solution derived using Algorithm 2 is $(q^*, n^*) = (10\,000,\ 4)$. Hence, the retailer's optimal order quantity is $n^*q^* = 40,000$ units, and the maximum weighted expected annual total profit is \$159, 062. For a given value of weight $\delta(e.g.,\ \delta = 0.9)$, from an individual perspective, the retailer's maximum expected annual total profit is $E[TPUB(q^*)] =$ \$159,552, the supplier's maximum expected annual total profit is $E[TPUV(n^*)] =$ \$155,113, and the sum of the retailer's and supplier's maximum expected annual total profits is \$314, 665(*i.e.*, \$159,552 + \$155,113). Table 7 lists the optimal solutions of the decision variables in addition to the retailer and supplier's expected annual total profit in non-cooperative and cooperative environments, indicating that the cooperative policy jointly determined by the retailer and supplier improved supply chain performance. However, from each party's perspective, the retailer could gain an improved expected annual total profit from cooperation, but the cooperation is unfavourable for the supplier because of increased production quantity, which leads to an increase in the supplier's holding cost. Therefore, cooperation is never achieved. To encourage the supplier to cooperate with the retailer, the method of benefit-sharing suggested by various researchers, including Goyal [10] and Ouyang *et al.* [30], can be adopted. The expected total profit of \$ 314,665 can be allocated between the supplier and retailer according to the following evaluations:

The retailer: \$314,665×\$158, 970/\$314, 281 = \$159, 164, and
The supplier: \$314,665×\$155, 311/\$314, 281 = \$155, 501
The results of the allocated expected annual total profit are listed in the right-hand column of Table 7.

TABLE 8. Comparison of the optimal solutions for non-cooperative and cooperative situations under different parametric values.

Parameter	Value	Non-cooperative			Cooperative		
		(q^*, n^*)	$E[TPUB\,(q^*)]$	$E[TPUV(n^*)]$	(q^*, n^*)	$E[TPUB\,(q^*)]$	$E[TPUV(n^*)]$
μ_λ	0.005	(10 000, 2)	162, 700	152, 059	(5000, 5)	160, 348	155, 021
	0.0075	(10 000, 2)	160, 709	153, 668	(5000, 5)	159, 823	155, 403
	0.01	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	0.0125	(10 000, 2)	156, 598	156, 989	(5000, 5)	158, 765	156, 172
	0.015	(10 000, 2)	154,475	158, 703	(5000, 5)	157, 233	156, 559
μ_α	0.03	(10 000, 2)	162, 700	152, 059	(5000, 5)	163, 369	152, 473
	0.04	(10 000, 2)	160, 709	153, 668	(5000, 5)	161, 354	154, 112
	0.05	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	0.06	(10 000, 2)	156, 598	156, 989	(5000, 5)	157, 193	157, 496
	0.07	(10 000, 2)	154, 475	158, 703	(5000, 5)	155, 045	159, 242
μ_β	0.03	(10 000, 2)	158, 724	155, 344	(5000, 5)	159, 343	155, 821
	0.04	(10 000, 2)	158, 700	155, 328	(5000, 5)	159, 319	155, 803
	0.05	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	0.06	(10 000, 2)	158, 652	155, 294	(5000, 5)	159, 272	155, 796
	0.07	(10 000, 2)	158, 627	155, 277	(5000, 5)	159, 248	155, 752
q_2	3000	(10 000, 2)	158, 676	155, 311	(3000, 8)	159, 343	155, 821
	4000	(10 000, 2)	158, 676	155, 311	(4000, 6)	159, 574	156, 015
	5000	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 480	155, 910
	6000	(10 000, 2)	158, 676	155, 311	(6000, 4)	159, 295	155, 786
	7000	(10 000, 2)	158, 676	155, 311	(7000, 3)	159, 026	155, 702
q_3	6000	(6000, 4)	159, 557	155, 702	(6000, 4)	160, 620	155, 702
	8000	(8000, 2)	159, 236	155, 493	(8000, 3)	160, 033	155, 493
	10 000	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	12 000	(12 000, 2)	158, 195	155, 077	(5000, 5)	159, 295	155, 786
	14 000	(14 000, 2)	157, 646	154, 766	(5000, 5)	159, 295	155, 786
r_1	0.46	(10 000, 2)	158, 676	155, 311	(2638, 9)	159, 235	156, 057
	0.48	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	0.5	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	0.52	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	0.54	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
r_2	0.41	(10 000, 2)	158, 676	155, 311	(5000, 5)	160, 571	155, 786
	0.43	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 933	155, 786
	0.45	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	0.47	(10 000, 2)	158, 676	155, 311	(10 000, 2)	159, 313	155, 311
	0.49	(10 000, 2)	158, 676	155, 311	(10 000, 2)	159, 313	155, 311
r_3	0.36	(10 000, 2)	159, 951	155, 311	(10 000, 2)	160, 588	155, 311
	0.38	(10 000, 2)	159, 313	155, 311	(10 000, 2)	159, 951	155, 311
	0.4	(10 000, 2)	158, 676	155, 311	(5000, 5)	159, 295	155, 786
	0.42	(10 000, 2)	158, 038	155, 311	(5000, 5)	159, 295	155, 786
	0.44	(10 000, 2)	157, 400	155, 311	(5000, 5)	159, 295	155, 786

6. Sensitivity analysis

In this section, we study the effects of changes in the defective rate, inspection error and freight parameters μ_λ, μ_α, μ_β, q_2, q_3, r_1, r_2 and r_3 on the optimal solutions for non-cooperative and cooperative situations. The data in this example are identical to those in Example 5.1. In addition, we set $\delta = 0.5$ for cooperative environment. The comparison results are shown as in Table 8.

On the basis of the results in Table 8, the following observation can be made.

(a) When the value of μ_λ increases, the retailer's expected annual total profit decreases but the supplier's expected annual total profit increases whether in non-cooperative or cooperative environments. However, the optimal value of (q^*, n^*) remain constant because the quantities the retailer required per shipment always meet the supplier's freight discount threshold.

(b) Thought the optimal value of (q^*, n^*) still remain constant, the value of μ_α has a negative effect on the retailer's expected annual total profit but a positive effect on the supplier's expected annual total profit for the two situations. On the contrary, the value of μ_β has negative effects on the retailer's and the supplier's expected annual total profits.

(c) The optimal solutions are not affected by the supplier's freight discount threshold q_2 in non-cooperative situation due to the retailer's optimal shipping quantity is always equal to q_3 regardless of the value of q_2. However, when the value of q_3 increases, the retailer's optimal shipping quantity increases. Additionally, in cooperative case, the retailer's optimal shipping quantity increases but the number of shipment decreases as the value q_2 increases. Further, both the retailer's and the supplier's expected annual total profits increases first and then decreases with the increase on the value q_2.

(d) As to the impact of freight on the optimal solutions and expected annual total profits, because the retailer's optimal shipping quantity is always equal to q_3 in non-cooperative environment, only the value of r_3 has a negative effect on the retailer's expected annual total profit. In the case of cooperation, the retailer's optimal shipping quantity increases but the supplier's optimal number of shipment and expected annual total profit of both sides decrease as the value r_1 or r_2 increases. However, when the value r_3 increases, the retailer's optimal shipping quantity and expected annual total profit decrease but supplier's optimal number of shipment and expected annual total profits increase.

7. Conclusions

Different from the previous study, this paper proposed production and inventory models in non-cooperative and cooperative environments by considering the following situations simultaneously: (1) the defective items with random defective rate; (2) Type I and Type II inspection errors may occur randomly during the inspection process; and (3) the supplier offers freight discounts to encourage retailers to delivery large quantities per shipment The purpose of this study is to determine optimal equilibrium production and replenishment strategies for maximizing the retailer and supplier's expected total profits per unit time under non-cooperative and cooperative environments. Two algorithms are developed to determine the optimal solutions for various situations. Several management insights are obtained from the numerical results and a sensitivity analysis with respect to the defective rate, inspection error and freight parameters. First, the supplier can encourage the retailer to deliver more quantities each time such that the delivery times can be reduced effectively by providing freight discounts to. Second, when the retailer and the suppler are willing to cooperate with each other, the weight δ plays an important role in weighted expected total profit per unit time. From an individual perspective, a higher weight δ is favourable for the retailer but unfavourable for the supplier. Third, to ensure a cooperative solution could be achieved, we develop an allocation mechanism for profit sharing. Finally, from the results of sensitivity analysis, it is found that the changes of defective rate, inspection error and freight parameters (except the value of freight discount threshold q_3) are not sensitive to optimal solutions of decision variables in non-cooperative environment. In the case of cooperation, the design of the shipping discount does affect the optimal solutions for the both sides. It is our belief that our work will make some innovation and significant contributions for a supply chain to determine its optimal ordering and shipping polices form from the point of view of competition or cooperation.

The proposed model can be extended in several directions. For instance, it would be interesting to consider the supply chain system with multiple products (please see example [42,43,45,46]). Because product deterioration is a common phenomenon in real life such as vegetables, fruits, medicine, and gasoline *etc.* (for example Mohammadi *et al.* [24], Pal *et al.* [29]), we may generalize the proposed model to consider deteriorating items. Additionally, this model may also be extended by considering a variable demand rate (a function of price, time, or inventory level). Further, a constant unit wholesale price (purchase cost) is assumed in our study. In practice, suppliers sometimes face a known price increase (Taleizadeh and Pentico [48]). Hence, the effect of known price increase may be incorporated in the proposed model. Finally, this paper assumes no dominating firm exists in this supply chain system which implies Nash game is considered in non-cooperative environment, future work could model a Stackelberg game, with the retailer or the supplier as the leader.

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