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FUZZY GREEN VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS PICKUP – DELIVERY AND TIME WINDOWS

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Abstract. In this paper, we propose a fuzzy green vehicle routing problem with simultaneous pickup and delivery and time windows (F-GVRP-SPDTW), in which the amounts of fuel consumption and emission are estimated by a comprehensive modal emission model. A mixed integer nonlinear programming model is proposed to minimize the cost of fuel consumption and emissions of vehicles. Moreover, the fuzzy approach with credibility measure is applied under conditions in which both pickup and delivery demands are uncertain. To solve the problem, we have proposed an adaptive large neighborhood search heuristic by applying new removal and insertion operators. Finally, computational experiments are conducted on a set of benchmark instances from the literature to evaluate the efficiency of the proposed solution technique. The results indicate that the proposed solution method is capable of finding high quality solutions in most of the instances.

Mathematics Subject Classification. Fuzzy green vehicle routing problem with simultaneous pickup, delivery and time windows.

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1. Introduction

Considerable attention has been paid to global warming due to the harmful effects of the greenhouse gases. Organizations and businesses pay more attention to the impact of their activities on the environment while governments consider regulations for reducing fuel emissions and other environmental pollutions. One of the important sectors is the transportation sector that plays a major role in the emissions of greenhouse gases. It has been reported that the transport sector alone accounts for about 14% of the total emissions and 80 percent of the emissions within this sector is contributed by road transport. Given that these emissions are hazardous to the environment and human health, it is important to find ways to significantly reduce them.

In real business situations, most of the distribution companies are trying to find ways to minimize their operational costs. In addition to the economic considerations, it is of high importance to consider green objectives in distribution operations regarding environmental issues. Vehicle Routing Problem (VRP) is one of the substantial subjects in transportation issues. Recently, green objectives are incorporated in VRP models to investigate transportation problems from an environmental point of view. The classical models of routing problems generally focus on minimizing operational costs, distance, fleet size or travel time. Green Vehicle Routing Problem

Keywords. Fuzzy, Green vehicle routing problem, ALNS, simultaneous pickup and delivery.

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(GVRP) can be regareded an extension of vehicle routing problem which aims to minimize total emissions and fuel consumption in addition to traditional costs.

In real world situations, it is often hard to exactly measure the parameters of the VRP problems in a deterministic way. For instance, parameters such as customer demands, time windows and traveling times are uncertain. Among various uncertain parameters, the uncertainty of customer demand is very common in practice. It is observed that the demand at a customer node usually can be expressed as "around 400 units", "between 90 and 110 units", "approximately 200 units" and so on. In these situations, the fuzzy approach can be applied to model problems under uncertainty.

The main contributions of the current paper are (1) applying the fuzzy approach for Green Vehicle Routing Problem with Simultaneous Pickup and Delivery and Time Windows (GVRP-SPDTW), and (2) proposing a heuristic algorithm for the investigated problem. In the proposed model, it is assumed that travel times depend on the speed of vehicles due to the time windows. The proposed problem intends to minimize the fuel consumption and emissions. To this end, the approach of Comprehensive Modal Emission Model (CMEM), which is as an exact estimation model, is applied to estimate the fuel consumption. Finally, the Adaptive Large Neighborhood Search (ALNS) method by considering new removal and insertion heuristics is exerted to solve the problem.

The paper is organized as follows. Section 2 investigates the literature on GVRP. In Section 3, mathematical model for the problem is presented. Section 4 describes the proposed solution algorithm. Computational results are provided in Section 5 and finally, Section 6 describes conclusion along with directions for future research.

2. Literature review

The traditional objective of VRP is reducing the total distance that can result in the reduction of fuel consumption. However, to reduce the fuel consumption and emissions, accurate investigations should be conducted [23]. The amount of CO_2 emission from a vehicle depends on some factors such as the speed, acceleration, load, road slope. A broad range of models have been presented to estimate the amount of fuel consumption and CO_2 emission. In the case of transportation planning, minimizing pollution has been rarely investigated [2]. Palmer in 2007 [29] proposed an integrated model of routing and emissions for freight vehicles. They considered the influence of speed in reducing the emissions of CO_2 according to the various factors like congestion and time window constraints. Kara *et al.* in 2007 [14] introduced a new extension of the VRP, which is called Energy-Minimizing VRP that aims to minimize a weighted load function instead of distance. Maden *et al.* in 2010 [24] discussed a VRP with time windows constraints and also considered a nonlinear relationship between the travel time of the vehicle and speed.

Ubeda *et al.* in 2011 [36] conducted a case study to minimize both the distances and pollutant emissions as the objective functions. The results showed that backhauling is very effective in controlling the emissions. Therefore, companies can reduce their negative environmental impact by initiating backhauling. The authors considered minimizing GHG emissions for VRP with backhauls. Suzuki in 2011 [30] proposed a model and showed that more saving can be achieved by delivering heavy items at the beginning of the route and delivering the light items at the end. Faulin *et al.* in 2011 [10] presented a Capacitated Vehicle Routing Problem (CVRP) by considering environmental criteria. Three types of costs are considered including (1) the traditional economic costs, (2) the environmental costs which are caused by pollution emissions, and (3) other environmental costs derived from the noise, congestion and depreciation of infrastructure. Bektas and Laporte in 2011 [2], introduced an extension of Vehicle Routing Problem with Time Windows (VRPTW), which is called Pollution Routing Problem (PRP). In this problem, the objective function is to determine the speed variable on each arc of the route to minimize fuel consumption, emissions and driver costs. They used the model of CMEM for the fuel consumption estimation which is suitable for heavy vehicles and considered factors like the speed, load and road angle. The authors reported a 10% reduction in emissions for a model of PRP. In 2011, the authors presented the review on the green road freight transportation and compared the different models of fuel consumption [8]. Demir *et al.* in 2012 [9] proposed a heuristic solution method for the medium and large scales of PRP. Xio *et al.* in

2012 [37] presented a routing model by the assumption of fuel consumption rate in CVRP and called it FCVRP. In this paper, the regression model based on the real data of transportation in Japan was used for measuring emissions. Kopfer *et al.* [18] in 2014 considered VRP with different types of vehicles and also assumed a linear relation between the fuel consumption and speed. Franceschetti *et al.* [11], in 2013 developed a time-dependent PRP including the cost functions of fuel, emission and driver costs by considering the traffic congestion at the peak periods and vehicle speeds. Demir *et al.*, in 2014 presented a bi-objective PRP to minimize the consuming fuel and driving time. Molina *et al.* 2014 proposed a multi-objective vehicle routing problem by considering the cost and emission functions [26].

Koç *et al.* [15], in 2014 proposed a fleet size and mix pollution-routing problem. They applied a meta-heuristic on some realistic benchmark instances. Kramer *et al.*, 2014 [16] proposed a matheuristic approach for the Pollution-Routing Problem. In the mentioned study, the results were compared with previous algorithms in the literature. Koç *et al.*, in 2016 [17] investigated the impact of the depot location, fleet Composition and routing decisions on the vehicle emissions in the city logistics.

In GVRP similar to other variants of VRP, there may be not enough information about parameters. Therefore, in many cases, the parameters would be uncertain. One of the important approaches for dealing with the uncertain environment is fuzzy modelling. However, there are few studies on the fuzzy approach in the GVRP literature. Tillman in 1969 proposed an algorithm based on Clarke and Wright for the Stochastic Vehicle Routing Problem (SVRP) by considering multi depots. Stewart and Golden in 1983 presented a chance constrained programming model and also two expected value models. Gendreau *et al.*, 1996 [12] provided a comprehensive review of the scientific literature on stochastic vehicle routing problems. Yang *et al.*, 2000 [38], considered a SVRP in which customers' demands are assumed to be uncertain. Laporte *et al.* [20], 2002 considered the stochastic demands for CVRP. Hvattum *et al.* 2006 [13] presented a Dynamic and Stochastic VRP with a Sample Scenario Hedging heuristic as a solution method. Taş *et al.*, 2013 [35] proposed a vehicle routing problem with stochastic travel times including soft time windows and service costs. They used Tabu Search method to solve the model. Sarasola *et al.*, 2016 [35] considered the vehicle routing problem (VRP) with stochastic demand and dynamic requests. They proposed variable neighborhood search algorithm (VNS) to solve the problem.

There are a lot of studies on various types of SVRP such as Dror and Trudeau [6], Bodin *et al.* [1], Liu *et al.* [22]. Dror *et al.* [5], described a variety of SVRP models by considering different operating and service policies.

The current study extends the model of Green VRP by considering simultaneous pickup and delivery and time windows. A mathematical model is proposed under uncertain demand. To this end, fuzzy numbers for customers' demands are assumed and the credibility measure theory is applied to deal with the uncertainty. For solving the problem, a heuristic solution method is suggested.

3. Problem definition

In this paper, the routing and scheduling problem for a homogenous fleet of vehicles is discussed. The vehicles with limited capacity deliver the deterministic amounts of goods from the central depot to customers and simultaneously pick up the goods that customers want to return to the depot. In the investigated problem, there is a constraint related to the time windows. It is related to the earliest and latest times of servicing the customers. It is assumed that late arrival is not allowed, and in the case of early arrival, the vehicle should wait until the service time starts. The main objective of the problem is to minimize the costs of fuel consumption and emissions. In the following, firstly we explain one of the most important fuel consumption models which is called CMEM, and afterward the mathematical model is presented.

3.1. Comprehensive modal emission model

In order to measure the fuel consumption and CO_2 emission, an accurate estimation method is applied. The CMEM is one of the estimation models of fuel consumption, which was developed by Barth *et al.* 2005 [3]. The mentioned model consists of three modules: engine power, engine speed and, fuel consumption rate. The Emission

TABLE 1. Literature review in scope of GVRP and Fuzzy VRP.

Article	Year	Problem	Solution method	Objective function	Distinctive Elements
Tillman	1969	Stochastic Vehicle Routing Problem (SVRP)	Clarke and Wright	Minimize cost	multi depots
Stewart and Golden	1983	Stochastic vehicle routing	chance constrained programming model	Minimize cost	two expected value models
Laporte *et al.*	2002	Capacitated Vehicle Routing Problem with Stochastic Demands	adaptive large neighborhood search heuristic	Minimize cost	Stochastic demand, capacitated vehicle
Palmer	2007	integrated model of routing and emissions	VRPTW Heuristics	Minimize cost	Speed, congestion and time window
Kara *et al.*	2007	Energy-Minimizing VRP	CPLEX	Minimize cost	Weighted load
Maden *et al.*	2010	Vehicle Routing Problem with time windows	C&W savings heuristic	Minimize distance	Travel time of vehicle and speed
Ubeda *et al.*	2011	Vehicle Routing Problem with Backhauls (VRPB)	delivery re scheduling, backhauling,	Minimize distance and pollutant emissions	Backhauling
Suzuki	2011	Time-dependent vehicle routing problem	Simulation	Minimize fuel consumption	Vehicle load, average speed,
Faulin *et al.*	2011	Capacitated Vehicle Routing Problem	Extended heuristic algorithms	Minimize traditional economic costs, and environmental costs	Noise, congestion and damage of infrastructure
Bektas and Laporte	2011	Pollution Routing Problem (PRP)	CPLEX	Minimize fuel consumption, emissions and driver costs	Speed, load and road angle
Demir *et al.*	2012	Pollution Routing Problem (PRP)	Adaptive Large Neighborhood Search Heuristic	Minimize fuel consumption, emissions and driver costs	Speed, load and road angle
Xio *et al.*	2012	Fuel consumption in CVRP (FCVRP)	CPLEX	Minimize fuel consumption	Regression model based on real data
Franceschetti *et al.*	2013	Time-dependent PRP	speed optimization algorithm	Minimize fuel consumption, emissions and driver costs	Traffic congestion at peak periods, speed
Ta^{O} *et al.*,	2013	Vehicle routing problem with stochastic travel times	Tabu Search method	Minimize distance	Time window
Kopfer *et al.*	2014	VRP with different types of vehicles	CPLEX	Minimize fuel consumption	Speed
Demir *et al.*,	2014	Bi-objective PRP	Pareto optimal solution	Minimize fuel consumption, emissions and driver costs	Speed, load and road angle
Molina *et al.*	2014	Multi-objective vehicle routing problem	C&W savings heuristic, Tchebycheff methods	Minimize traditional economic costs, and environmental costs	Cost and emissionfunctions
Koc *et al.*	2014	Fleet size and mix pollution-routing problem	evolutionary metaheuristic		Heterogeneous fleet
Kramer *et al.*,	2014	Pollution-Routing Problem	Matheuristic approach	Minimize fuel consumption, emissions and driver costs	Speed, load and road angle
Sarasola *et al.*,	2015	VRP with stochastic demand and/or dynamic requests	Variable Neighborhood Search Algorithm (VNS)	Minimize cost	Uncertain demand
Koç *et al.*,	2016	Location-routing Search Heuristic	Adaptive Large Neighborhood	Minimize fuel consumption, emissions, total depot, vehicle and routing cost	Speed, load and road angle
Current article	2016	Fuzzy Green VRP with simultaneously pickup and delivery and time windows	Adaptive Large Neighborhood Search Heuristic	Minimize fuel consumption	Speed, load and road angle

Rate (ER) (grams per second) for greenhouse gases (such as CO, HC or NOx) is related to the fuel consumption rate (FR) (g/s). The calculation of FR is complex as it depends on a number of factors. The calculation of FR is explained in relation (3.1):

$$FR(t) = (k.N.V + (P_t/\epsilon + P_a)/\eta)\gamma \tag{3.1}$$

where k is the engine friction factor, N is the engine speed (radian per second (rps)), V is the engine displacement (liter), P_t is the total tractive power demand in watts (or kg m^2/s ^{3}), ϵ is vehicle drive train efficiency, P_a is the engine power demand associated with additional vehicle accessories (Watt) such as an air conditioner,η is a measure of efficiency for diesel engines and γ is a constant. Moreover P_t (kilowatt) is calculated as follow:

$$P_t = (Mav + Mgvsin\theta + 0.5C_d\rho Av^3 + MgvC_r\cos\theta)/1000 \tag{3.2}$$

where M is the mass of the vehicle (kilogram),v is speed (meter/second),a is the acceleration (meter/second ^{2}),g is the gravitational constant (9.81 meter/second ^{2}), θ is the road angle (radian), A is the surface area in front of the vehicle (meter^2), ρ is the air density (kilogram/meter ^{3}), and C_r and C_d are the coefficients of rolling resistance and drag, respectively. The engine speed N (rps) can be calculated as:

$$N = \frac{n_d n_g v}{R} \tag{3.3}$$

Where n_d is the differential ratio, n_g is the gear ratio and R is radius of the wheel.

3.2. Deterministic model

The proposed problem can be defined as a complete directed graph $G = (N, A)$, in which
$N = \{0, 1, \ldots, n\}$ is the set of nodes, the node 0 represents the depot, and $N_0 = N/\{0\}$ is the set of customers. In addition, $A = \{(i, j)|i, j \in N_0, i \neq j\}$ is the set of arcs between the customers. Each customer has both the pickup and delivery demands denoted by pd_i and dd_i, respectively. Moreover, a time window $[a_i, b_i]$ is considered for each customer. The early arrivals of the vehicles are allowed but the vehicle should wait until a certain time a_i to start servicing the customer. S_i denotes the service time at customer i.

The CMEM, which is a function of the speed, acceleration, vehicle specification and road slope is used in the objective function. For a given arc (i, j) with the length of d_{ij}, it is assumed that v_{ij} be the speed of the vehicle on the arc. The fuel consumption F (in Liter) for the given arc can be calculated as:

$$F_{ij} = FR(t)d_{ij}/v_{ij} = \gamma\alpha_{ij}(f_{ij} + w)d_{ij} + \gamma\beta v_{ij}^2 d_{ij} + \gamma kNV d_{ij}/v_{ij} \tag{3.4}$$

where $\alpha_{ij} = a+g\sin\theta_{ij}+gC_r\cos\theta_{ij}$ and $\beta = 0.5C_d\rho A$. The first term of relation (3.4) is related to the weighted load carried by vehicles which is derived by multiplying the load and distance. Therefore, the customers with small (large) delivery demands and large (small) pick-up demands will be visited, later (earlier). Moreover, the two last terms are related to speed variable. It is worth mentioning that both the low and high speed can result in higher fuel consumption.

Parameters

- C Capacity of vehicle.
- d_{ij} Distance between node i and j.
- U_{ij} Picked up demand up to node i by vehicle and carried in arc (i, j).
- W_{ij} Delivered demand up to node i by vehicle and carried in arc (i, j).
- C_f Cost of each gram of fuel.
- C_e Cost of emission.
- M Mass of vehicle.
- A Frontal surface area of the vehicle.

v_l Minimum amount of speed.
v_m Maximum amount of speed.

Decision variables

$$X_{ij} = \begin{cases} 1 \\ 0 \end{cases} \quad \text{A binary variable that takes 1 if vehicle travelled from node } i \text{ to } j\text{, otherwise 0}$$

f_{ij} amount of load travelled from i to j.
y_i beginning time of the service of node j.
v_{ij} speed of vehicle from node i to j.

$$\begin{aligned} \min Z = &\sum_{j\in N}\sum_{i\in N}(C_f+C_e)\gamma\alpha_{ij}(w+f_{ij})d_{ij}X_{ij} \\ &+\sum_{j\in N}\sum_{i\in N}(C_f+C_e)\gamma\beta(v_{ij}^2)d_{ij}X_{ij}+\sum_{j\in N}\sum_{i\in N}(C_f+C_e)kNVd_{ij}X_{ij}/(v_{ij})+\sum_{j\in N_0}pS_j \end{aligned} \tag{3.5}$$

$$\text{S.t}: \sum_{i\in N} X_{ij} = 1 \quad \forall j\in N_0 \tag{3.6}$$

$$\sum_{j\in N_0} X_{0j} = m \tag{3.7}$$

$$\sum_{i\in N_0} X_{ij} - \sum_{i\in N_0} X_{ji} = 0 \quad \forall j\in N, \quad i\neq j \tag{3.8}$$

$$\sum_{i\in N} U_{ji} - \sum_{i\in N} U_{ij} = pd_j \quad \forall j\in N_0 \tag{3.9}$$

$$\sum_{i\in N} W_{ij} - \sum_{i\in N} W_{ji} = dd_j \quad \forall j\in N_0 \tag{3.10}$$

$$U_{ij} + W_{ij} \leq CX_{ij} \quad \forall i, \quad j\in N, \quad i\neq j \tag{3.11}$$

$$f_{ij} = U_{ij} + W_{ij} \quad \forall i, \quad j\in N \tag{3.12}$$

$$y_i + \frac{d_{ij}}{v_{ij}} + S_i - y_j \leq M(1-X_{ij}) \quad \forall i\in N \tag{3.13}$$

$$a_i \leq y_i \leq b_i \quad \forall i\in N_0 \tag{3.14}$$

$$\sum_j U_{0j} = 0, \tag{3.15}$$

$$\sum_i W_{i0} = 0, \tag{3.16}$$

$$U_{ij} \leq (C - pd_j)X_{ij} \quad \forall i\in N, \quad j\in N_0 \tag{3.17}$$

$$W_{ij} \leq (C - dd_i)X_{ij} \quad \forall i\in N_0, \quad j\in N \tag{3.18}$$

$$U_{ij} \geq pd_i X_{ij} \quad \forall i \in N_0, \quad j \in N \tag{3.19}$$

$$W_{ij} \geq dd_j X_{ij} \quad \forall i \in N, \quad j \in N_0 \tag{3.20}$$

$$v_l \leq v_{ij} \leq v_m \quad \forall i, \quad j \in N \tag{3.21}$$

$$X_{ij} \in \{0,1\} \quad \forall i, \quad j \in N \tag{3.22}$$

$$f_{ij} \geq 0 \quad \forall i, \quad j \in N \tag{3.23}$$

$$y_i \geq 0 \quad \forall i \in N \tag{3.24}$$

The objective function (3.5) is derived from the relation (3.4). The first three terms are related to the total fuel consumption and emissions. The last part is associated with the driver payments. Constraint (3.6) guarantees that each customer must be served by only one vehicle. The number of vehicles leaving the depot is shown by the constraint (3.7). Moreover, the constraint (3.8) represents the flow conservation, while the constraints (3.9) and (3.10) are about simultaneous pickup and delivery of the demand. Constraint (3.11) represents that the carried load between nodes i and j (f_{ij})cannot be exceed the capacity of the vehicle. Constraint (3.12) guarantees that total amount of carried load from i to j is equal to sum of pickup and delivery amount carried in arc (i,j). The time windows for the customers are also imposed by the constraints (3.13) and (3.14). Constraints (3.15) and (3.16) ensure that the vehicle leaves depot without any pickup demand load and returns to depot after delivering all the demand. Constraints (3.17) to (3.20) are also related to the pick-up and delivery constraints. Finally, the range of speed is denoted by constraint (3.21).

In Table 2 the values of the parameters and coefficients are listed.

TABLE 2. Parameters used in the model.

Notation	Typical Values
w (kilogram)	6350
k (kilogram/rev/second)	0.2
N (rev/second)	33
V (liters)	5
η_{tf}	0.4
η	0.9
ρ(kilogram/meter^3)	1.2041
g(meter/second ^{2})	9.81
A(meter^2)	3.912
C_d	0.7
C_r	0.01
C_f (\$)	1
C_e(\$)	0.4
v_l (meter/second)	5
v_m(meter/second)	25

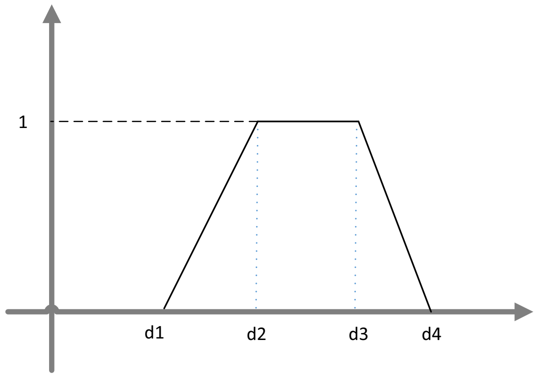

FIGURE 1. A trapezoidal fuzzy variable.

3.3. Fuzzy credibility measure

The Fuzzy set concept and membership functions were presented by Zadeh [40]. This approach is applied in many real world problems. The term of fuzzy variable was presented by Kaufman [19] for measuring a fuzzy event and then possibility measure theory for fuzzy variable was proposed by Zadeh [41]. Afterward, credibility theory was developed by Liu [19]. This section deals with some basic concepts of fuzzy variables. First, the concepts of possibility will be introduced, necessity and credibility of a fuzzy event, then our deterministic model is developed based on this concept.

Definition 3.1 (Nahmias [27])**.** Let Θ be a nonempty set,$\emptyset$ be an empty set, and $\rho(\Theta)$ is the power set of Θ. For each set $A \in \rho(\Theta)$, the possibility is denoted by a nonnegative number$Pos(A)$, there are three principles for it as follows:

(1) $Pos\{\emptyset\} = 0$;
(2) $Pos\{H\} = 1$;
(3) For any optional collection A_k in $\rho(\Theta) Pos \cup_k A_k = sup_k Pos(A_k)$

The possibility space is defined by $(\Theta, \rho(\Theta), Pos)$, and the possibility measure is referred by the function Pos.

Definition 3.2. The necessity measure for $A \in \rho(\Theta)$, is defined as $Nec\{A\} = 1 - PosA$.

Definition 3.3. Credibility measure for $A \in \rho(\Theta)$, is given by,

$$\mathrm{Cr}\{A\} = \frac{1}{2}[Pos\{A\} + Nec\{A\}] \tag{3.25}$$

In this paper both types of demands at each node are assumed as fuzzy variables and they are represented by a trapezoidal fuzzy number as illustrated in Figure 1.

Assume ξ is a fuzzy variable of customer demand with membership function of $\mu(x)$ and r is a real number. For ξ in type of trapezoidal fuzzy number $\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$, the credibility measure is calculated as follows:

$$\mathrm{Cr}\{\xi \leq r\} = \begin{cases} 0, & r \in (-\infty, \xi_1] \\ \dfrac{r-\xi_1}{2(\xi_2-\xi_1)}, & r \in (\xi_1, \xi_2] \\ \dfrac{1}{2}, & r \in (\xi_2, \xi_3] \\ \dfrac{r-2\xi_3+\xi_4}{2(\xi_4-\xi_3)}, & r \in (\xi_3, \xi_4] \\ 1, & r \in (\xi_4, \infty] \end{cases} \tag{3.26}$$

$$\mathrm{Cr}\{\xi \geq r\} = \begin{cases} 1, & r \in (-\infty, \xi_1] \\ \dfrac{2\xi_2 - \xi_1 - r}{2(\xi_2 - \xi_1)}, & r \in (\xi_1, \xi_2] \\ \dfrac{1}{2}, & r \in (\xi_2, \xi_3] \\ \dfrac{\xi_4 - r}{2(\xi_4 - \xi_3)}, & r \in (\xi_3, \xi_4] \\ 0, & r \in (\xi_4, \infty] \end{cases} \tag{3.27}$$

For service level $\alpha > 0.5$ credibility gives:

$$\mathrm{Cr}\{\xi \leq r\} \geq \alpha \to r \geq (2 - 2\alpha)\xi_3 + (2\alpha - 1)\xi_4 \tag{3.28}$$

$$\mathrm{Cr}\{\xi \geq r\} \geq \alpha \to r \leq (2\alpha - 1)\xi_1 + (2 - 2\alpha)\xi_2 \tag{3.29}$$

By applying the above equations in the deterministic model; the constraints (3.9), (3.10), (3.17)−(3.20) should change as follows.

$$CX_{ij} - U_{ij} \geq (2 - 2\alpha)pd3_j + (2\alpha - 1)pd4_j \quad \forall i \in v, j \in v_c \tag{3.30}$$

$$CX_{ij} - W_{ij} \geq (2 - 2\alpha)pd3_j + (2\alpha - 1)pd4_j \quad \forall i \in v, j \in v_c \tag{3.31}$$

$$U_{ij} \geq (2 - 2\alpha)pd3_j + (2\alpha - 1)pd4_j \quad \forall i \in v_c, j \in v \tag{3.32}$$

$$W_{ij} \geq (2 - 2\alpha)pd3_j + (2\alpha - 1)pd4_j \quad \forall i \in v_c, j \in v \tag{3.33}$$

$$\sum_{i \in v} U_{ji} - \sum_{i \in v} U_{ij} \leq pd3_j \sum_{i \in v} U_{ji} - \sum_{i \in v} U_{ij} \geq pd2_j \quad \forall j \in v_c \tag{3.34}$$

$$\sum_{i \in v} W_{ji} - \sum_{i \in v} W_{ij} \leq pd3_j \sum_{i \in v} W_{ji} - \sum_{i \in v} W_{ij} \geq pd2_j \quad \forall j \in v_c \tag{3.35}$$

4. Heuristic approach based on the adaptive large neighborhood search algorithm

The Adaptive Large Neighborhood Search (ALNS) algorithm which is an effective heuristic method was presented by Pisinger and Ropke [28] to solve different classes of the routing problems. This method is an extension of Large Neighborhood Search (LNS). In the LNS, an initial solution is improved by destroying and repairing the solution, but in the ALNS, several removal and insertion operators are applied to improve a solution. To obtain a new solution, a number of customers are removed from the current solution and reinserted into the routes. To determine the frequency of using operators, a weight is assigned according to the performance of each destroy or repair. The weights will be adjusted dynamically after definite iterations [25].

4.1. Structure of heuristic algorithm

In the proposed algorithm, firstly, an initial solution (X_{initial}) is constructed by initialization algorithm described in Section 4.2, and initial temperature is adjusted ($T = T_0$). In each iteration, the algorithm aims to find a new solution by using a series of removal and insertion operators. The current solution, which is obtained at the beginning of an iteration, is denoted by (X_{current}). The new solution (X_{new}) is a temporary solution that will be evaluated for making a decision on rejecting or accepting it. The cost of solution X is denoted by $Ob(X)$. If $Ob(X_{\text{new}}) < Ob(X_{\text{current}})$, the new solution is accepted, otherwise, it could be accepted by the probability

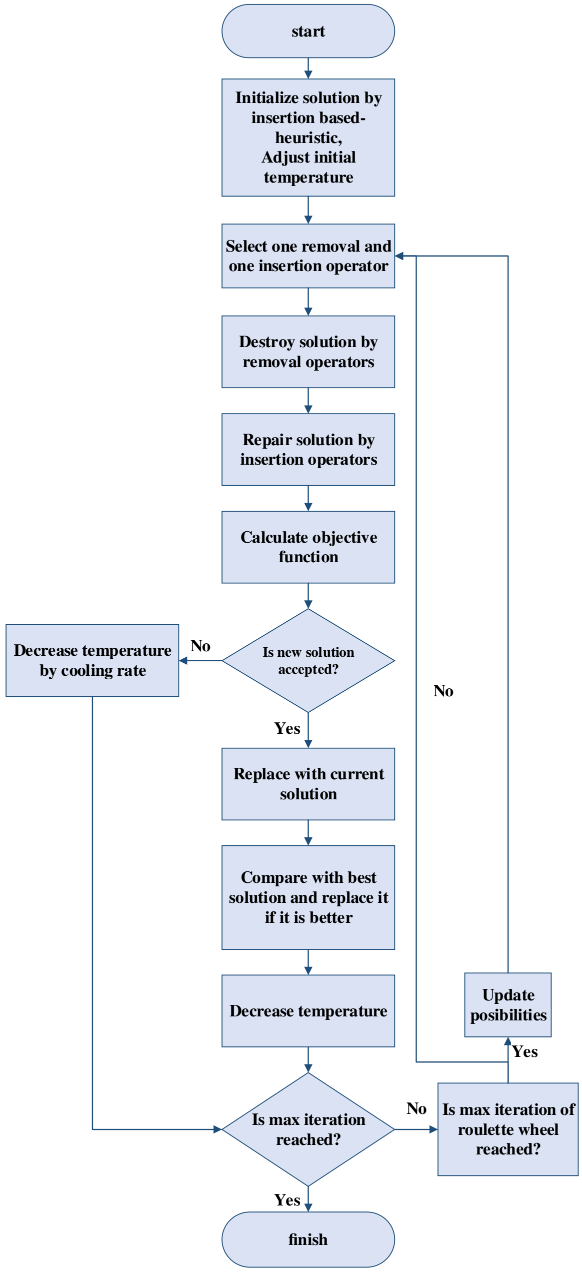

FIGURE 2. Flowchart of ALNS algorithm embedded by SA.

obtained from the equation $e^{-(Ob(X_{\text{new}})-Ob(X_{\text{current}}))/T}$. If the new solution is accepted, it will be set as the current solution. At the end of each iteration, it is compared with the best solution found during the search mechanism (X_{best}). The ALNS method should be embedded within any local search heuristic such as Simulated Annealing (SA). At the end of each iteration, the temperature is gradually decreased by a definite rate to limit the neighborhood space. The algorithm is run until the maximum iteration Figure 2. illustrates the flowchart of the proposed algorithm [25].

4.2. Initialization

There are two main algorithms for constructing routs in the VRP: (3.1) sequential and, (3.2) parallel methods. The sequential procedures construct only one route at a time, while the parallel procedures create simultaneously more than one route [32]. In this paper, a parallel insertion-based construction heuristic method is proposed for finding the initial solution. It is noteworthy that finding a good initial solution is very important, as it can impressively decrease the number of iterations to reach the optimal solution.

Regarding the objective function (3.4), for reducing the fuel consumption, minimizing the weighted load carrying by vehicles is of major importance. Consequently, it is considered in the criterion of insertion algorithm.

To describe the algorithm, suppose nodes $(i_1, i_2, i_3, \ldots, i_m)$ are in a route. For inserting node u in the position between (i_p) and (i_{p+1}), cost of insertion $C(u)$ is calculated as follows:

$$(i_1, i_2, i_3, \ldots, i_m) \leftarrow u \quad C(u) = \min \sum_{n=1}^{p-1} f_{i_n i_{n+1}} d_{i_n i_{n+1}} + f_{i_p u} d_{i_p u} + \sum_{n=p+1}^{m} f_{i_n i_{n+1}} d_{i_n i_{n+1}} \forall p = 1, \ldots, m \tag{4.1}$$

The cost of insertion is calculated for all the available feasible positions for node u in the solution. After calculating the costs for all the unrouted nodes, then the minimum cost will be selected as follows:

$$u = \min C(u) \forall u \text{ that is unrouted yet} \tag{4.2}$$

To obtain a feasible solution, capacity constraints should be satisfied. The equations which were reported by Dethloff [7] are considered for capacity constraint and two functions are defined as Residual Delivery (RD) and Residual Pickup (RP). $RD(i)$, for customer i represents the amount of load that can be transported additionally from the depot to an inserted customer after customer i. Like $RD(i)$, $RP(i)$ is defined as residual pick-up capacity, representing the largest possible amount to pick-up from a customer inserted after node i.

$$RD(0) = C - \sum_{s \in T} pd_s \tag{4.3}$$

$$RD(q) = \min\{RD(PRI(q)), C - l_q\}(q \in T) \tag{4.4}$$

$$RP(0) = C - \sum_{s \in T} dd_s \tag{4.5}$$

$$RP(q) = \min\{RP(SUI(q)), C - l_q\}(q \in T) \tag{4.6}$$

One of the important constraints is time windows, as it affects the speed. Therefore, in the following, the best possible arrival time of the vehicles will occur considering the maximum speed. In this case, y_i is defined as arrival time of a vehicle at node i and w_i is also the waiting time if vehicle arrives before lower bound of time windows. To insert node u between (i_p) and (i_{p+1}) in route $(i_1, i_2, i_3, \ldots, i_m)$, the relations (4.7) and (4.8) should be considered,

$$w_{i_p} = \begin{cases} a_{i_p} - y_{i_p} if a_{i_p} > y_{i_p} \\ 0 if a_{i_p} > y_{i_p} \end{cases} \tag{4.7}$$

$$y_{i_p} = y_{i_{p-1}} + w_{i_{p-1}} + s_{i_{p-1}} + d_{i_{p-1}, i_p} / v_m \frac{d_{i_p u}}{b_u - (y_{i_p} + w_{i_p} + s_{i_p})} < v_m y_{i_n} < b_{i_n} \; for \quad n = p+1, \ldots, m \tag{4.8}$$

Pseudo code for construction algorithm is presented as Algorithm 1,

Algorithm 1. Pseudo code for the proposed initial solution algorithm.

```
Iteration=1;
 While (iteration until all the nodes routed)
  Best cost=big number;
   For (all unrouted nodes)
   Cost=big number;
      For (all routes)
        For (all positions)
           If (the vehicle capacity and time windows
           are satisfied)
          COI=Calculate the cost of insertion;
            If (COI < Cost)
              Cost=COI;
              End
           End       End
     End
     If Cost< Best cost
     Best cost=Cost;
     End
    End
Return Best cost;
Insert selected node in the best position in best route;
Iteration+1;
End
```

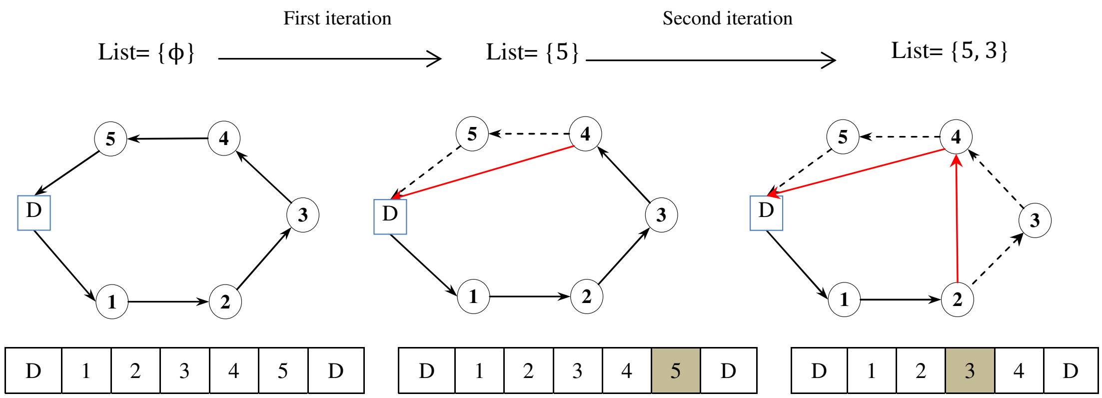

Figure 3. Depiction of a Removal Opetation.

4.3. Removal operators

After obtaining the initial solution, operators are defined to improve the current solution by searching the neighborhood space. There are two types of operators: (1) removal and, (2) insertion.

In this section, four removal operators are defined. A removal operator starts with an empty removal list, and selects s nodes by a specific strategy during ϕ iterations. In each iteration of removal operator, one node is selected and removed from the solution and consequently a reduced solution will be obtained. The nodes which are selected by the operator are placed in the removal list. The strategy of removal operator for selecting nodes is important in a search mechanism and it depends on the nature of the problem. The number of removed nodes is determined based on the percentage of customers.

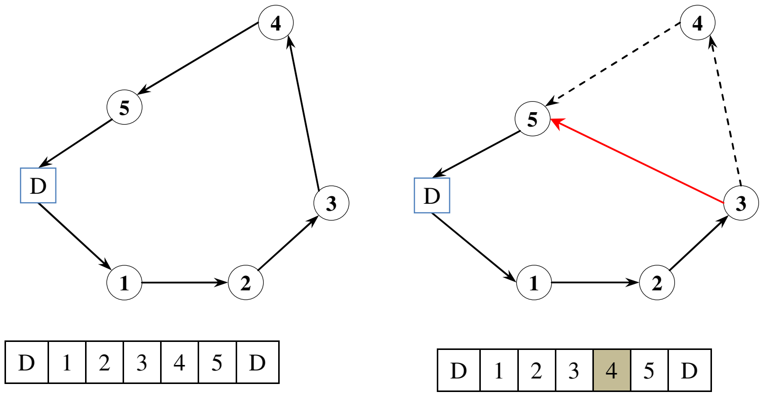

FIGURE 4. Worst Distance Removal Operator.

Figure 3 illustrates the mechanism of a removal operator. In Algorithm 2, the pseudo code of the removal operator is presented.

Algorithm 2. Pseudo code for a removal operator algorithm.

Input: feasible solution X, ϕ maximal number of iterations;
Output: a partial solution Xp;
Initialize removal list ($L \leftarrow \emptyset$)
 FOR ϕ do
  Apply remove operator to find set of S of nodes
  $L \leftarrow L \cup S$
  Remove subset of S from X
 END

---

1) Random Removal (RR)

In each iteration of the Random Removal operator, firstly, one route is randomly selected and then one node is randomly chosen and removed on the selected route. Afterward, the selected node is placed on the removal list. This algorithm runs for ϕ iterations based on the random selection strategy for better diversification in the search mechanism.

2) Worst Distance Removal (WDR)

The Worst Distance Removal operator selects a customer with the highest cost in each iteration. Note that cost of each customer is calculated as the sum of distances from the previous and following customers on the route. In each iteration, this cost is calculated for all the nodes in all routes and the maximum amount of those is selected, *i.e.*, it removes node $j = \text{argmax}\{|d_{ij} + d_{jk}|\}$. Figure 4 shows the mechanism of this operator.

3) Worst Time Removal (WTR)

The Worst Time Removal operator calculates the deviation of arrival time of vehicle at node$j$, from start time of servicea_j, and then removes the node with the maximum deviation. The operator prevents from long waiting times or delayed services. For example the operator selects$j = \text{argmax}\{|y_j - a_j|\}$.

4) Minimum Route Removal (MRR)

The Minimum Route Removal operator aims to find the routes with a minimum number of customers and selects a number of nodes randomly. Indeed, the aim of this removal operator is reducing the total number of routes. The operator mechanism is illustrated graphically in Figure 5.

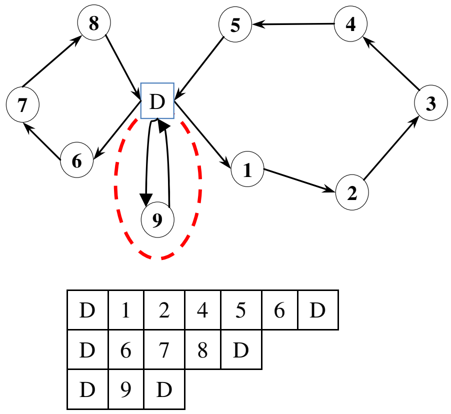

FIGURE 5. Minimum Route Removal Operator.

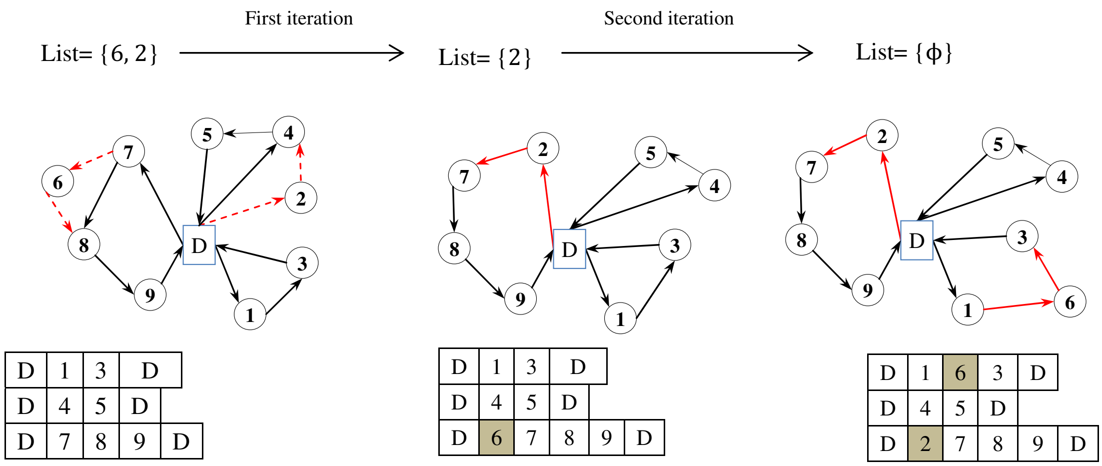

FIGURE 6. Depiction of an Insetion Operation.

4.4. Insertion operators

After destroying the current solution, the nodes of the removal list, which were obtained from removal operator, should reinsert in the reduced solution. In order to improve the current solution and find a new one, specific insertion strategies should be used. In the following, two insertion operators are described. The depiction of an insertion operator is shown in Figure 6. Pseudo code for an insertion operator algorithm is provided in Algorithm 3.

1) Greedy Insertion

The Greedy Insertion operator repeatedly inserts each removed node in the best possible position of a solution. The insertion cost is calculated as follow:

$$C(i) = d_{ji} + d_{ik} - d_{jk} i \in S \quad i^{*} = \operatorname{argmin}\{C(i)\} \tag{4.9}$$

2) Random Insertion

The Random Insertion operator firstly finds feasible positions for each removed node and then randomly selects one of the positions for inserting the node. Figure 7 shows the mechanism of this operator.

Algorithm 3. Pseudo code for an insertion operator algorithm.

Input: a partially solution Xp; $\emptyset$ maximal number of iterations; removal list L
Output: a new solution X;
Initialize removal list $(L \leftarrow \emptyset)$
FOR $i \in L$ do
  Apply insert operator to find set of S of nodes
  $L \leftarrow L - S$
  insert i in Xp
END

TABLE 3. Definition of scores.

Score	Description
σ_1	If the solution obtained by the operator be the best solution
σ_2	If the solution obtained by the operator be better than the current solution
σ_3	if the solution obtained by the operator be worse than the current solution but it is accepted

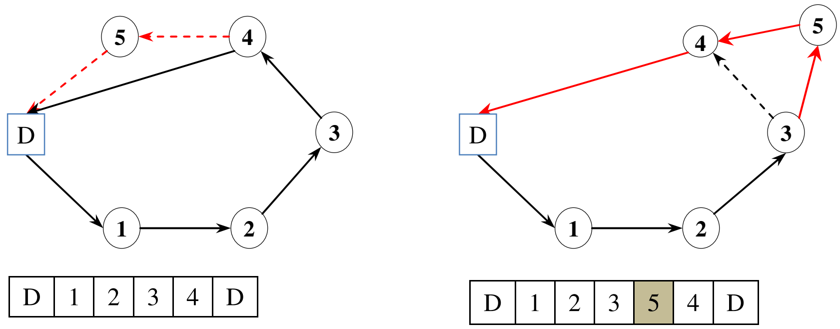

FIGURE 7. Randon insection operation.

4.5. Roulette wheel mechanism

The selection of both the removal and insertion operators in each iteration is based on their recorded performances. To measure the performance of the operators, three different scores are defined as shown in Table 3. At first, all the probabilities for each removal and insertion operator are the same, after running definite iterations (N) of ALNS algorithm, probabilities are updated as follow,

$$p_d^{t+1} = p_d^t(1 - r_p) + r_p \pi_i / \omega_i \tag{4.10}$$

where r_p is a parameter that is defined for the roulette wheel, π_i is the score of the operator i, ω_i is the number of using the operator i during N iterations. The scores of operators should be set to zero after each iteration of roulette wheel.

5. Computational results

The proposed algorithm is coded in MATLAB on a server with Intel core i7, 2.4 GHz processor and 3 GB memory.

We use a data set from the literature which was proposed by Demir *et al.* [9], According to the mentioned article, there are nine data sets, each of which consists of 20 instances including 10 to 200 nodes. The best known solutions for two sets of UK10 and UK100 are reported by Demir *et al.* [9]. We compare the results

TABLE 4. Parameters of the proposed algorithm.

Parameter	Values
Max iteration of ALNS	10 000
r_p, Roulette wheel mechanism parameter	0.1
T_0, Initial temperature	100
δ, Cooling rate	0.9
S Number of removal nodes	10% of n

TABLE 5. Assumed parameters for parameter setting.

Parameter	Values	
Max iteration of roulette wheel	400	600
σ_1	0	5
σ_2	0	4
σ_3	0	2

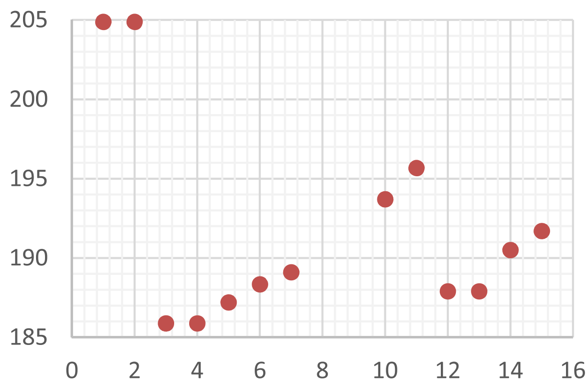

FIGURE 8. Cost comparison for different values of parameters.

of the proposed algorithm in this paper with those obtained in the mentioned study. As new assumptions are considered in the current study, new data sets are generated in order to meet the constraints of the proposed model. In the following, new experiments are performed to evaluate the proposed algorithm by these data sets.

In the following subsections, parameter setting for the proposed algorithm is performed, and then, experimental results are reported and analyzed. Finally, numerical experiments for the fuzzy model are performed and the results are provided.

5.1. Parameter setting

The parameters related to the ALNS and SA algorithms are shown in Table 4. In this section, numerical experiments are performed to tune four parameters, including number of iteration for roulette wheel and three scores $(\sigma_1\sigma_2, \sigma_3)$. To determine the best combination of these parameters, different values of them are experimented and the best one is reported.

In Table 5 assumed values for the parameters are listed. As shown in Figure 7, minimum cost occurs under parameter values (600, 5, 4, and 2).

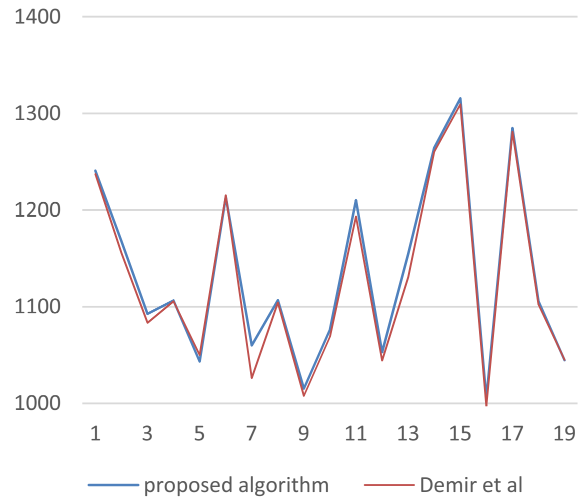

FIGURE 9. Comparison of results of total cost for data set UK100.

5.2. Comparison with results of Demir *et al.* [Deterministic model]

The experiments are performed for the instances UK10 and UK100 and results are compared with the reported results in (Demir *et al.* [9]). Two criterions, *i.e.*, the total distance and total cost (equal to the fuel and emission costs in addition to driver wage) are calculated. As mentioned previously, fuel consumption depends on a lot of factors which among them distance, load and speed are the most important ones. Although minimizing the total distance results in minimizing the fuel consumption, it is possible to generate tours with higher distance and lower fuel consumption due to the role of other factors.

The results of numerical experiments are reported in Tables 7 and 8. As the results show, for instances with 10 customers, our proposed algorithm finds the same results as the results of Demir *et al.* [9]. While for instances including 100 customers, the results show improvements compared to the results of literature. There are improvements in the most of the instances and consequently, it can be concluded that the proposed model performs better than the algorithm by Demir *et al.* [9]. The improvement of solution is shown by $DEV(D)\%$ for distance and $DEV(C)\%$ for cost that are calculated as follows:

$$DEV(C)\% = (Demirresult - ourresult/Demirresult)100$$

Figure 9 shows the improvement trend of the total costs for instances with 100 customers. The convergence of the algorithm is investigated to validate the performance of the algorithm in finding the best solution which is demonstrated in Figure 10.

5.3. Comparison between removal operators [Deterministic model]

In this section, new numerical experiments are performed to compare the performance of the removal operators in the proposed algorithm. First of all, new test problems are generated similar to the data sets which exist in the literature with the same amounts of distances, time windows, service time and delivery demand. Moreover, pick up demand is added to the data set to fit our model (see Tab. 8). Four data sets including 10 instances are generated. Experiments are conducted for ALNS-RR (algorithm with just random removal), ALNS-WTR (algorithm with just worst time removal), ALNS-WDR (algorithm with just worst distance removal), and ALNS-MRR (algorithm with just minimum route removal). The experiments are carried out for 10 000 iterations with the same values of the parameters. The results are reported in Table 9. As the results show, each removal operator cannot individually result in the best solution as it cannot search the whole neighborhood space

Table 6. Results of instances with 100 customers, comparison of proposed heuristic with CPLEX and Demir *et al.* 2012.

Instances	CPLEX & Demir *et al.* 2012	Proposed heuristic		
	TC	TD (km)	TC	NV
UK10-01	170.66	409	170.64	2
UK10-02	204.87	529.8	204.88	2
UK10-03	200.33	507.3	200.42	2
UK10-04	189.94	480.1	189.99	2
UK10-05	175.61	447	175.59	2
UK10-06	214.56	548	214.48	2
UK10-07	190.14	494.7	190.14	2
UK10-08	222.16	567.8	222.17	2
UK10-09	174.53	457	174.54	2
UK10-10	189.83	486.7	190.04	2
UK10-11	262.07	697.2	262.08	2
UK10-12	183.18	460.3	183.19	2
UK10-13	195.97	510.5	195.97	2
UK10-14	163.17	397.8	163.28	2
UK10-15	127.15	291.4	127.24	2
UK10-16	186.63	451.1	186.73	2
UK10-17	159.07	387.5	159.03	2
UK10-18	162.09	401.5	152.09	2
UK10-19	169.46	414.5	169.59	2
UK10-20	168.8	412.8	168.8	2

Table 7. Results of instances with 100 customers, comparison of proposed heuristic with CPLEX and Demir *et al.* 2012.

Instances	CPLEX	Demir *et al.* 2012			Proposed heuristic			DEV(D)%	DEV(C)%
	TC	TD (Km)	TC	NV	TD (Km)	TC	NV		
UK100−01	1389.05	2914.4	1240.79	14	2881	1237.2	14	1.1	0. 2
UK100−02	1302.16	2690.7	1168.17	13	2706.5	1155.8	13	−0. 5	0.01
UK100−03	1231.44	2531.8	1092.73	13	2510.1	1083.4	13	0. 8	0. 8
UK100−04	1174.75	2438.5	1106.48	14	2413.7	1105.67	14	1.2	0.07
UK100−05	1121.71	2328.5	1043.41	14	2355	1050.12	14	−1	−0. 6
UK100−06	1320.4	2782.4	1213.61	14	2790.3	1215.4	14	−2	−0. 1
UK100−07	1177.8	2463.9	1060.08	12	2438.2	1026.3	12	1	0.03
UK100−08	1230.92	2597.4	1106.78	13	2683.1	1104.4	12	−3	0. 2
UK100−09	1092.2	2219.2	1015.46	13	2431	1007.9	12	−9.55	0. 7
UK100−10	1163.95	2510.1	1076.56	12	2508.6	1069.2	12	0.07	0. 6
UK100−11	1343.18	2792.1	1210.25	15	2685.3	1193.4	14	3.83	0.01
UK100−12	1227.01	2427.3	1053.02	12	2417.01	1044.28	12	0. 4	0. 8
UK100−13	1333.1	2693.1	1154.83	13	2683	1130.6	13	0. 3	0.02
UK100−14	1410.18	2975.3	1264.5	14	2967.4	1260.3	14	0. 2	0. 3
UK100−15	1453.81	3072.1	1315.5	15	3060.7	1309.43	15	0. 3	0. 4
UK100−16	1105.58	2219.7	1005.03	12	2207.13	997.08	12	0. 5	0. 7
UK100−17	1389.99	2960.4	1284.81	15	2947.2	1280.92	15	0. 4	0. 3
UK100−18	1219.45	2525.2	1106	13	2510	1102.45	13	0. 6	0. 3
UK100−19	1115.82	2332.6	1044.71	13	2330.7	1045.1	13	0. 08	−0. 3
UK100−20	1396.97	2957.8	1263.06	14	2940.13	1259.28	14	0. 5	0. 2

TABLE 8. Specification of generated test problems.

Data Set	No. of Customers	NV	Vehicle Capacity	Range of Speed (m/s)
SM10	10	2	3650	5 to 25
SM20	20	5	3650	5 to 25
SM50	50	15	3650	5 to 25
SM100	100	20	3650	5 to 25

TABLE 9. Results of comparing the performance of four removal operators with data set for 10, 20, 50 and 100 customers.

Instances	ALNS-RR	ALNS-WTR	ALNS-WDR	ALNS-MRR	ALNS (proposed algorithm)
10 customer					
SM10-1	120.45	115.45	107.8	109.2	100.43
SM10-2	128.97	130.4	129.19	129.4	128.37
SM10-3	132.44	134.06	132.97	134.67	132.44
SM10-4	119.4	117.9	119.4	120.45	117.7
SM10-5	109.6	109.28	110.32	108.9	108.73
20 customer					
SM20-1	220.5	217.95	219.03	220.37	216.99
SM20-2	254.21	257.8	255.4	255	254.21
SM20-3	345.6	337.5	348	342.06	325.9
SM20-4	398.1	401.5	401.5	390.17	378.9
SM20-5	375.4	390.5	381.7	394	375.4
50 customer					
SM50-1	462.34	462.41	457.86	450.3	419.34
SM50-2	456.7	457.3	473.85	445.09	443.12
SM50-3	514.78	506.66	501.6	500.43	491.61
SM50-4	466.8	490.3	481.23	476.2	460.4
SM50-5	480.9	403.16	480.1	456.3	440.6
100 customer					
SM100-1	900.3	890.4	912.5	907.2	868.67
SM100-2	958.6	976.4	968.9	970.3	949.26
SM100-3	1025.6	1124.5	1026.7	1026	1025.3
SM100-4	1165.3	1126.67	1188.9	1201	1145.8
SM100-5	1256.3	1184	1249.4	1184	1184

and consequently obtains a local optimum solution. While the combination of these operators by making the diversification in the search mechanism results in a better solution.

5.4. Experimental results [Fuzzy model]

In this section, the trapezoidal fuzzy variable is considered for both pickup and delivery demands. For each set of tests, the proposed algorithm is implemented according to the presented fuzzy model in Section 3−3.

The results are presented for different service levels $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$, and instances with 10, 20, 50 and 100 customers. The numerical results for the costs of fuel and driver and total cost are considered and reported in Table 10. According to the results, in the fuzzy model, by increasing the uncertainty of demand (higher α_{de}), fuel and driver costs increase. Overall, the amount of total cost increases. In addition, fuzzy approach causes the increase of the number of vehicles to satisfy the customer's demands, comparing with the deterministic condition. This results in the significant increase of total cost. While for instances of 10 customers, it has no significant effect on the number of vehicles. Figure 11 illustrates the effect of different service levels

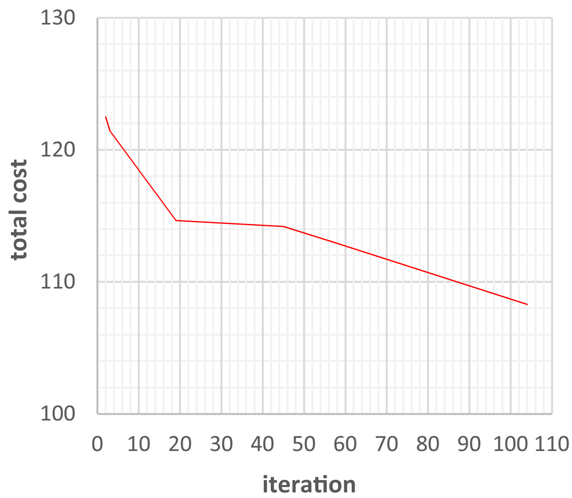

FIGURE 10. Convergence procedure in proposed algorithm for instance with 10 customer.

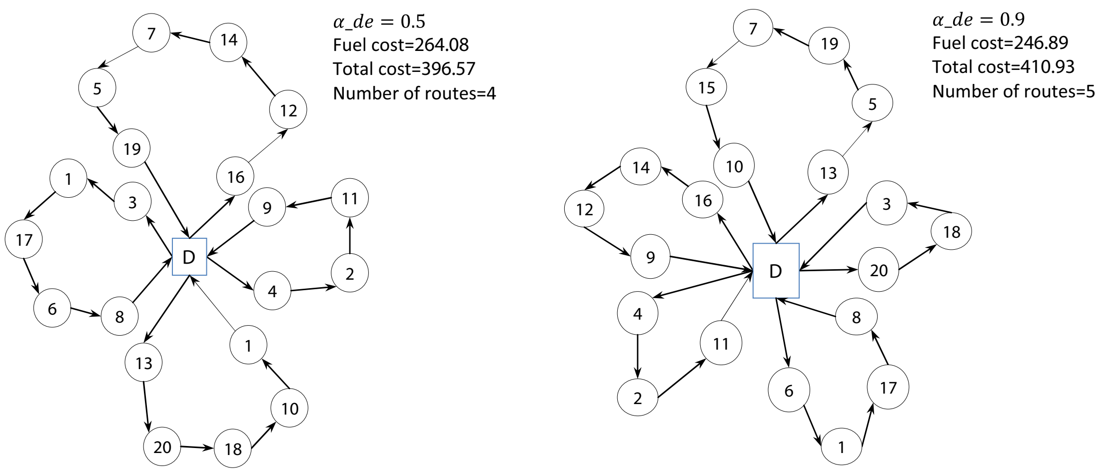

FIGURE 11. Schematic configuration of an instance with fuzzy demands.

on the total costs and the number of routes. So, for the $\alpha_{de} = 0.50.9$, schematic configurations of an instance with 20 customers are shown. The trends of results for two instances with 10 and 100 customers are also shown in Figures 12 and 13.

In another experiment, cost trends are investigated when both the pickup and delivery demands are assumed to be fuzzy numbers. In this experiment, the effects of fuzzy approach on different types of demands: delivery and pickup are explored. Therefore, different combinations of service levels for delivery demand (α_{de}) and for pick up demand (α_{pi}) are considered and the results are presented in Table 11. According to the results, at each α_{pi}, by increasing the α_{de}, the fuel cost and consequently the total cost will increase. Moreover, the minimum total cost is occurred when $\alpha_{pi} = 0.6$ and $\alpha_{de} = 0.5$. In Figures 13–18, costs change tendencies for different α_{de} are demonstrated.

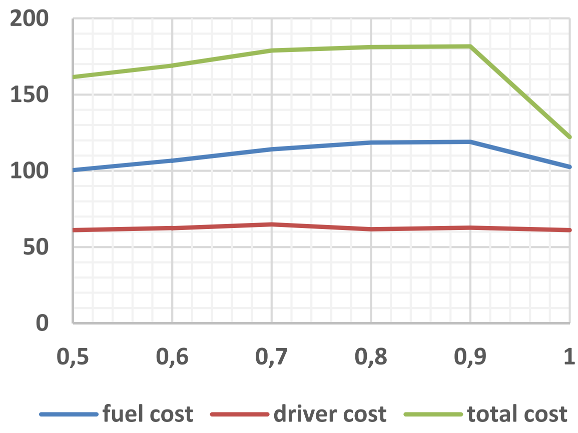

FIGURE 12. Costs trend analysis with different α_de for instance SM10.

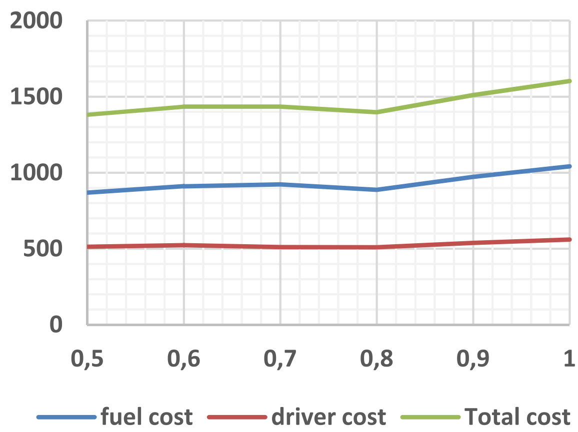

FIGURE 13. Costs trend analysis with different α_de for instance SM10.

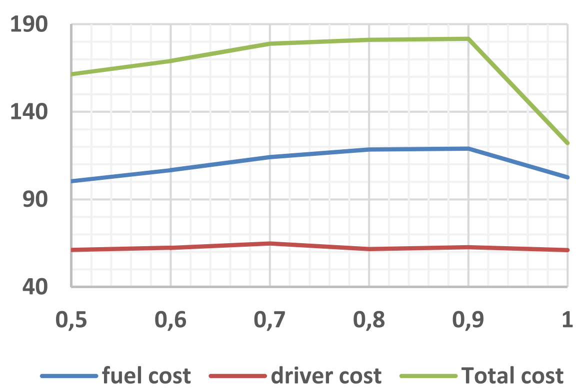

FIGURE 14. Costs change tendencies with different α_{de} for $\alpha_{pi} = 0.5$.

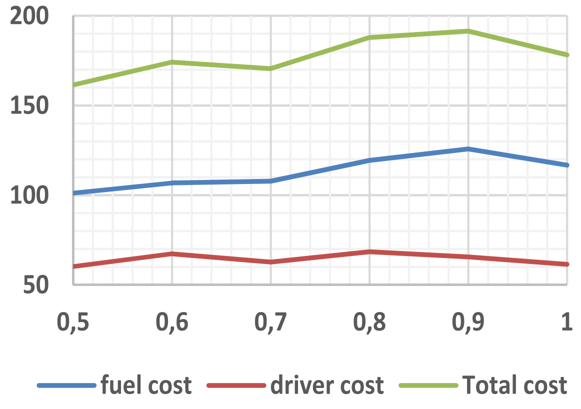

FIGURE 15. Costs change tendencies with different α_{de} for $\alpha_{pi} = 0.6$.

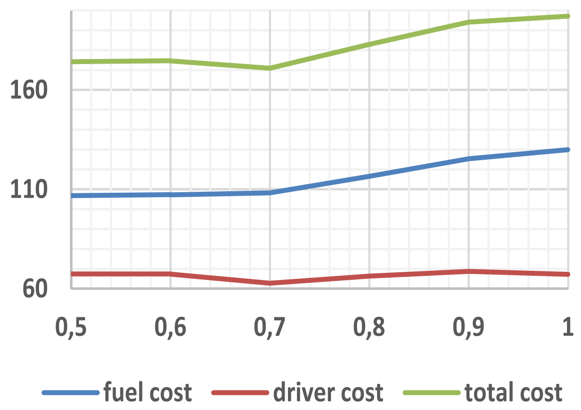

FIGURE 16. Costs change tendencies with different α_{de} for $\alpha_{pi} = 0.7$.

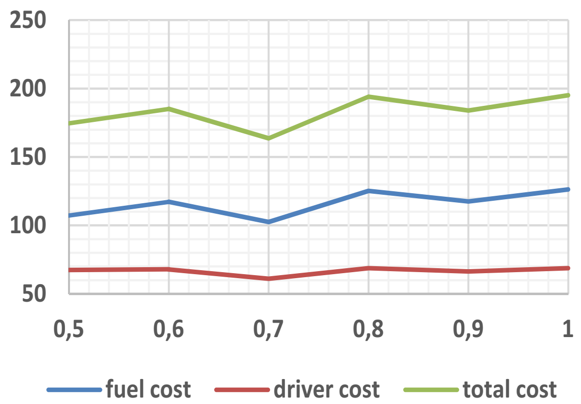

FIGURE 17. Costs change tendencies with different α_{de} for $\alpha_{pi} = 0.8$.

TABLE 10. Results of cost changes with different α for delivery demands.

Instances	NV	α_{de}	Fuel Cost	Driver cost	Total Cost
SM10-1(10 customers)	2	0.5	100.43	61.06	161.49
		0.6	106.67	62.36	169.03
		0.7	114.11	64.81	178.92
		0.8	118.48	61.67	181.15
		0.9	118.98	62.67	181.66
		1	102.52	61.06	163.58
SM20-1(20 customers)	4	0.5	264.08	150.49	396.57
	4	0.6	245.58	152.64	398.22
	5	0.7	246.83	152.75	399.58
	5	0.8	248.86	154.04	402.9
	5	0.9	246.89	160.04	410.93
	5	1	263.28	161.96	425.24
SM50-1(50 customers)	9	0.5	419.34	254.99	674.33
	10	0.6	490.52	282.27	776.79
	9	0.7	433.84	260.36	694.2
	9	0.8	446.29	267.09	713.38
	9	0.9	434.44	257.66	692.11
	9	1	470.85	261.4	732.25
SM100-1(100 customers)	16	0.5	868.67	512.69	1381
	17	0.6	910.7	523.47	1434
	17	0.7	923.61	510.41	1434
	17	0.8	887.32	509.89	1397
	18	0.9	972.26	538.41	1510
	18	1	1042	559.97	1602

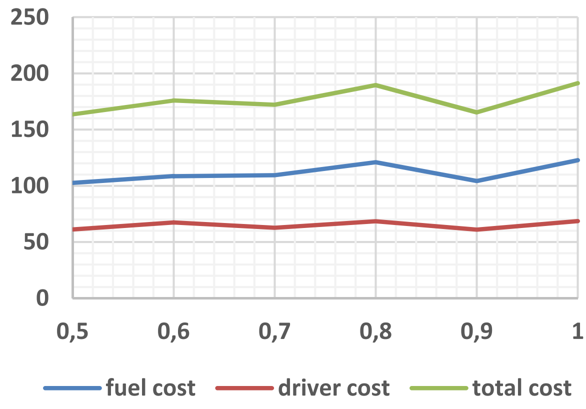

FIGURE 18. Costs change tendencies with different α_{de} for $\alpha_{pi} = 0.9$.

Table 11. Results of fuzzy model for input SM10.

α_{pi}	α_{de}	Fuel Cost	Driver Cost	Total Cost
0.5	0.5	100.43	61.06	161.49
	0.6	106.67	62.36	169.03
	0.7	114.11	64.81	178.92
	0.8	118.48	61.67	181.15
	0.9	118.98	62.67	181.66
	1	102.52	61.06	163.58
0.6	0.5	101.16	60.27	161.44*
	0.6	106.84	67.34	174.18
	0.7	107.78	62.72	170.50
	0.8	119.4	68.46	187.86
	0.9	125.8	65.63	191.44
	1	116.71	61.5	178.22
0.7	0.5	106.8	67.34	174.14
	0.6	107.26	67.34	174.6
	0.7	108.2	62.72	170.92
	0.8	116.57	66.32	182.9
	0.9	125.36	68.76	194.12
	1	129.9	67.21	197.12
0.8	0.5	107.23	67.34	174.57
	0.6	117.23	67.88	185.12
	0.7	102.53	61.06	163.59
	0.8	125.27	68.76	194.03
	0.9	117.49	66.42	183.94
	1	126.3	68.76	195.06
0.9	0.5	102.11	61.06	163.17
	0.6	108.28	57.85	166.13
	0.7	102.95	61.06	164.01
	0.8	125.69	68.76	194.45
	0.9	117.91	66.45	184.36
	1	127.08	70.17	197.26
1	0.5	102.53	61.06	163.59
	0.6	108.53	67.34	175.87
	0.7	109.44	62.72	172.16
	0.8	121.04	68.46	189.51
	0.9	104.2	61.06	165.26
	1	122.72	68.57	191.3

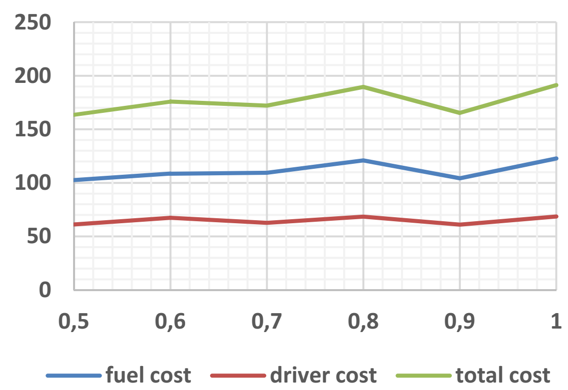

Figure 19. Costs change tendencies with different α_{de} for $\alpha_{pi} = 1$.

6. Conclusion

In this paper, the green vehicle routing and scheduling problem was proposed by considering the constraints of simultaneous pickup and delivery and time windows. A green objective function is considered in the model to minimize the fuel consumption and emissions. To solve the model, a heuristic method based on the ALNS including new construction algorithm for an initial solution and some new removal and insertion operators was proposed. The efficiency of the solution algorithm is evaluated by conducting some experiments to compare the algorithm results with the best known results of literature. Experimental results confirm the efficiency and better performance of the proposed algorithm. Moreover, a new data set was generated for investigating the presented model and also a set of analyses was performed to compare the performance of different removal operators to prove that combination of these operators in the algorithm result in better solutions. Afterward, a fuzzy model was presented by using credibility measure theory. In this model, demand of nodes were considered as fuzzy variables. The Fuzzy model was examined with different amounts of service levels (α) for the delivery and pickup demands. The results, which are reported in the tables and charts, represent that rising the service levels of fuzzy demands cause the rise of total costs and the number of vehicles. In addition, the best combinations of service levels for delivery and pickup demands is derived from the results.

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