C. R. Acad. Sci. Paris, Ser. I 353 (2015) 551–555

Contents lists available at ScienceDirect

C. R. Acad. Sci. Paris, Ser. I

www.sciencedirect.com

Dynamical systems/Probability theory

Equivalence of Palm measures for determinantal point processes associated with Hilbert spaces of holomorphic functions

Équivalence de measures de Palm pour les processus déterminantaux associés aux espaces de Hilbert des fonctions holomorphes

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ARTICLE INFO

Article history:
Received 29 January 2015
Accepted after revision 4 March 2015
Available online 27 April 2015

Presented by Michel Talagrand

ABSTRACT

We obtain explicit formulae, in the form of *regularized multiplicative functionals* related to certain Blaschke products, of the Radon–Nikodym derivatives between reduced Palm measures of all orders for determinantal point processes associated with a large class of weighted Bergman spaces on the disk. Our method also applies to determinantal point processes associated with weighted Fock spaces.

RÉSUMÉ

On obtient des formules explicites, sous forme des fonctionnelles multiplicatives régularisées liées à certains produits de Blaschke, des dérivées de Radon–Nikodym entre toutes les mesures de Palm pour les processus déterminantaux associés aux espaces de Bergman pondérés sur le disque. Notre méthode s'applique également aux processus déterminantaux associés aux espaces de Fock pondérés.

Version française abrégée

Soit $\omega : \mathbb{D} \to \mathbb{R}^+$ un poids sur le disque unité $\mathbb{D}$. Soit $L^2_{\text{hol}}(\mathbb{D}, \omega(z)dz)$ l'espace de Bergman associé et soit K_ω son noyau reproduisant. Notons par $\mathbb{P}_{K_\omega}$ le processus déterminantal induit par K_ω (cf. [6,12]). Étant donné un l-uplet $\mathfrak{p} = (p_1, \cdots, p_l)$ de points distincts dans $\mathbb{D}$, sous certaines conditions sur le poids ω, la limite suivante

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http://dx.doi.org/10.1016/j.crma.2015.03.018

$$S_{\mathfrak{p}}(\mathcal{Z}) := \lim_{r\to 1^-}\left(\sum_{z\in\mathcal{Z}:|z|\le r}\log\prod_{j=1}^{l}\left|\frac{p_j-z}{1-\bar{p}_j z}\right| - \mathbb{E}_{\mathbb{P}_{K_w}}\sum_{z\in\mathcal{Z}:|z|\le r}\log\prod_{j=1}^{l}\left|\frac{p_j-z}{1-\bar{p}_j z}\right|\right)$$

existe presque sûrement et la fonction $\mathcal{Z}\to e^{2S_{\mathfrak{p}}(\mathcal{Z})}$ est intégrable par rapport à $\mathbb{P}_{K_\omega}$. En revanche, on remarque que le produit $\prod_{z\in\mathcal{Z}}|\frac{p-z}{1-\bar{p}z}|$ diverge presque sûrement pour tout $p\in\mathbb{D}$.

Notre premier résultat (voir Theorem 3.1) est : soit ω un poids sur $\mathbb{D}$ vérifiant certaines conditions (voir inegalité (1)). Pour tout l-uplet $\mathfrak{p}=(p_1,\dots,p_l)\in\mathbb{D}^l$ de points distincts, notons par $\mathbb{P}_{K_\omega}^{\mathfrak{p}}$ la mesure de Palm réduite du processus $\mathbb{P}_{K_\omega}$ correspondant aux points $p_1,\dots,p_l$. Alors, la dérivée de Radon–Nikodym de $\mathbb{P}_{K_\omega}^{\mathfrak{p}}$ par rapport à $\mathbb{P}_{K_\omega}$ est donnée par

$$\frac{d\mathbb{P}_{K_\omega}^{\mathfrak{p}}}{d\mathbb{P}_{K_\omega}}(\mathcal{Z}) = \frac{e^{2S_{\mathfrak{p}}(\mathcal{Z})}}{\mathbb{E}_{\mathbb{P}_{K_\omega}}(e^{2S_{\mathfrak{p}}})}.$$

Il en résulte immédiatement l'équivalence de toutes les mesures de Palm de $\mathbb{P}_{K_\omega}$. Notons que l'équivalence des measures de Palm au cas uniforme $\omega\equiv 1$ est due à Holroyd–Soo [3]. Un corollaire du résultat précédent est la quasi-invariance de la mesure $\mathbb{P}_{K_\omega}$ sous l'action naturelle du groupe $\mathrm{Diff}_c(\mathbb{D})$ de difféomorphismes ayant des supports compacts dans $\mathbb{D}$ sur l'espace de configuration $\mathrm{Conf}(\mathbb{D})$.

On étudie également les espaces de Fock généralisés. Soit $\psi:\mathbb{C}\to\mathbb{R}^+$ et soit G_ψ le noyau reproduisant d'espace de Fock $\mathcal{F}_\psi = L^2_{\mathrm{hol}}(\mathbb{C}, e^{-\psi(z)}\,dz)$. La fonction ψ est supposée de vérifier certaines conditions (voir Theorème 3.2). Alors, pour tout couple $\mathfrak{p}=(p_1,\dots,p_l), \mathfrak{q}=(q_1,\dots,q_l)\in\mathbb{C}^l$ de points distincts, les mesures de Palm réduites $\mathbb{P}_{G_\psi}^{\mathfrak{p}}$ et $\mathbb{P}_{G_\psi}^{\mathfrak{q}}$ sont équivalentes et la dérivée de Radon–Nikodym de $\mathbb{P}_{G_\psi}^{\mathfrak{p}}$ par rapport à $\mathbb{P}_{K_G}^{\mathfrak{q}}$ est calculée explicitement ; notons que le cas gaussien $\psi(z)=|z|^2$ est dû à Osada–Shirai [8].

1. Outline of results

The main purpose of this paper is to announce the equivalence between reduced Palm measures of determinantal point processes associated with a large class of Hilbert spaces of holomorphic functions on a domain or the whole plane $\mathbb{C}$. Two main cases considered in this paper are weighted Bergman spaces on the unit disk $\mathbb{D}$ and weighted Fock spaces on the whole plane $\mathbb{C}$. In the case of Bergman spaces, we obtain explicit formulae of Radon–Nikodym derivatives between reduced Palm measures *of all orders* in the form of certain Blaschke products.

We start with the Bergman case. Let $\omega:\mathbb{D}\to\mathbb{R}^+$ be a weight on $\mathbb{D}$ and let $L^2_{\mathrm{hol}}(\mathbb{D},\omega(z)\,dz)$ be the associated Bergman space. Denote by K_ω the corresponding reproducing kernel. Let $\mathbb{P}_{K_\omega}$ denote the determinantal point process induced by K_ω (cf. [6,12]). Our notation follows that of [1]. Under appropriate conditions on ω, all the reduced Palm measures of $\mathbb{P}_{K_\omega}$ are proved to be equivalent. We obtain explicit formulae for the corresponding Radon–Nikodym derivatives in the form of certain Blaschke products. To have a quick view of the formulae, suppose that $\mathfrak{p}=(p_1,\dots,p_l)\in\mathbb{D}^l$ is any l-tuple of distinct points, then under certain conditions on the weight ω, the limit

$$S_{\mathfrak{p}}(\mathcal{Z}) := \lim_{r\to 1^-}\left(\sum_{z\in\mathcal{Z}:|z|\le r}\log\prod_{j=1}^{l}\left|\frac{p_j-z}{1-\bar{p}_j z}\right| - \mathbb{E}_{\mathbb{P}_{K_w}}\sum_{z\in\mathcal{Z}:|z|\le r}\log\prod_{j=1}^{l}\left|\frac{p_j-z}{1-\bar{p}_j z}\right|\right)$$

exists almost surely and the function $\mathcal{Z}\to e^{2S_{\mathfrak{p}}(\mathcal{Z})}$ is integrable with respect to $\mathbb{P}_{K_\omega}$. Note however that for any $p\in\mathbb{D}$, the product $\prod_{z\in\mathcal{Z}}|\frac{p-z}{1-\bar{p}z}|$ diverges for $\mathbb{P}_{K_w}$-almost every configuration $\mathcal{Z}$.

Our first result says that

$$\frac{d\mathbb{P}_{K_\omega}^{\mathfrak{p}}}{d\mathbb{P}_{K_\omega}}(\mathcal{Z}) = \frac{e^{2S_{\mathfrak{p}}(\mathcal{Z})}}{\mathbb{E}_{\mathbb{P}_{K_\omega}}(e^{2S_{\mathfrak{p}}})}.$$

The Radon–Nikodym derivatives between $\mathbb{P}_{K_\omega}^{\mathfrak{p}}$ and $\mathbb{P}_{K_\omega}^{\mathfrak{q}}$ for tuples $\mathfrak{p}\in\mathbb{D}^l$ and $\mathfrak{q}\in\mathbb{D}^k$ follow immediately. Note that the equivalence of Palm measures in the uniform case $\omega\equiv 1$ is due to Holroyd and Soo [3].

We also consider the generalized Fock spaces. Let $\psi:\mathbb{C}\to\mathbb{R}^+$ and denote $d\nu_\psi(z)=e^{-\psi(z)}dz$. Denote by $\mathcal{F}_\psi = L^2_{\mathrm{hol}}(\mathbb{C}, d\nu_\psi)$ the associated Fock space and let G_ψ be its reproducing kernel. Consider $\mathbb{P}_{G_\psi}$ the determinantal point process induced by G_ψ with respect to measure $d\nu_\psi$. Then under appropriate conditions, we get the equivalence between the reduced Palm measures *of the same order*; the corresponding Radon–Nikodym derivatives are also derived, which in this time have similar formulae as those for the Ginibre point process (corresponding to ψ_2) obtained in [8].

Remark 1. The Radon–Nikodym derivatives between reduced Palm measures of the same orders in the case of Bergman kernel processes are quite different from the formula in Theorem 3.2. Note that the formulae in [7] in the case of Gamma kernel processes, formulae in [1] in the case of determinantal point processes on $\mathbb{R}$ with integrable kernels and formulae in [8] in the case of Ginibre point process are all similar to the formula in Theorem 3.2.

2. Notation and preliminaries

2.1. Palm measures of determinantal point processes

We refer to [4,5,12,6] for the background on determinantal point processes and [10,11] for the characterization of the reduced Palm measures of determinantal point processes. We follow the notation of [1].

Let E be a locally compact Polish space, considered as the phase space. Let μ be a Radon measure on E. In our case (E, μ) will be the open unit disk $\mathbb{D}$ or the whole complex plane $\mathbb{C}$ equipped with measures given by appropriate weights. Denote by $\mathrm{Conf}(E)$ the space of configurations on E. All determinantal point processes considered in this paper are induced by a locally trace class *orthogonal projection*. Let Π be such a projection on $L^2(E,\mu)$ with range $L \subset L^2(E,\mu)$, and denote by $\mathbb{P}_\Pi$ the induced determinantal point process.

Let $\mathfrak{p} = (p_1, \dots, p_k) \in E^k$ be any tuple of distinct points in the phase space E. Define $L(\mathfrak{p}) = \{\varphi \in L : \varphi(p_1) = \cdots \varphi(p_k) = 0\}$. Let $\Pi^{\mathfrak{p}}$ be the orthogonal projection onto $L(\mathfrak{p})$ and let $\mathbb{P}_\Pi^{\mathfrak{p}}$ denote the corresponding reduced Palm measures. The theorem of Shirai–Takahashi says that for almost all tuples $\mathfrak{p}$, we have $\mathbb{P}_\Pi^{\mathfrak{p}} = \mathbb{P}_{\Pi^{\mathfrak{p}}}$.

Problem. *Let $\mathfrak{p} = (p_1, \dots, p_l) \in E^l$ and $\mathfrak{q} = (q_1, \dots, q_k) \in E^k$ be two tuples of distinct points in E. What is the relation between $\mathbb{P}_\Pi^{\mathfrak{p}}$ and $\mathbb{P}_\Pi^{\mathfrak{q}}$? For which pairs $\mathfrak{p}$ and $\mathfrak{q}$ are the measures $\mathbb{P}_\Pi^{\mathfrak{p}}$ and $\mathbb{P}_\Pi^{\mathfrak{q}}$ equivalent? When they are equivalent, what are the corresponding Radon–Nikodym derivatives?*

Equivalence of Palm measures of the same order implies the quasi-invariance of the associated determinantal point process under a certain natural group action (cf [1, Prop. 2.9]). We refer to G. Olshanski's paper [7] for the quasi-invariance of the Gamma kernel determinantal point processes and refer to [1] for the quasi-invariance of determinantal point processes on $\mathbb{R}$ with integrable kernels. We mention that the rigidity phenomenon in [2] and insertion–deletion tolerance properties in [3] for determinantal point processes are also closely related to the above problem.

2.2. Regularized multiplicative functionals

To write the Radon–Nikodym derivatives explicitly, we need some preparation. Let $f : E \to \mathbb{C}$ be a Borel function. Set

$$\mathrm{Var}(\Pi, f) = \frac{1}{2} \int_E \int_E |f(z) - f(w)|^2 |\Pi(z,w)|^2 \, \mathrm{d}\mu(z) \, \mathrm{d}\mu(w).$$

Definition 2.1. Define the Hilbert space $\mathcal{V}(\Pi)$ in the following way: elements of $\mathcal{V}(\Pi)$ are functions f on E satisfying $\mathrm{Var}(\Pi, f) < \infty$; functions that differ by a constant are identified and $\|f\|_{\mathcal{V}(\Pi)} := \sqrt{\mathrm{Var}(\Pi, f)}$.

If f is bounded and compactly supported, then we may define $S_f : \mathrm{Conf}(E) \to \mathbb{C}$ by $S_f(X) = \sum_{x \in X} f(x)$ and we set $\overline{S}_f = S_f - \mathbb{E}_{\mathbb{P}_\Pi} S_f$. Then it is easy to see that $\|\overline{S}_f\|_{L^2(\mathrm{Conf}(E), \mathbb{P}_\Pi)} = \|f\|_{\mathcal{V}(\Pi)}$. The correspondence $f \to \overline{S}_f$ extends uniquely to an isometric embedding $\overline{S} : \mathcal{V}(\Pi) \to L^2(\mathrm{Conf}(E), \mathbb{P}_\Pi)$.

Definition 2.2. Let $h : E \to \mathbb{R}^+$ be such that $\|\log h\|_{\mathcal{V}(\Pi)} < \infty$, then we set $\widetilde{\Psi}_h = \exp(\overline{S}_{\log h})$. If, moreover, $\widetilde{\Psi}_h \in L^1(\mathrm{Conf}(E), \mathbb{P}_\Pi)$, then we define $\overline{\Psi}_{h,\Pi} = \frac{\widetilde{\Psi}_h}{\mathbb{E}_{\mathbb{P}_\Pi} \widetilde{\Psi}_h}$.

The function $\overline{\Psi}_{h,\Pi}$ is called the *regularized multiplicative functional* associated with h and $\mathbb{P}_\Pi$.

The following result is an extension of Proposition 4.6 in [1].

Lemma 2.3. *Let L and Π be as above. Let $E_0 \subset E$ be a Borel subset satisfying $\mathrm{tr}(\chi_{E_0} \Pi \chi_{E_0}) < \infty$ and such that if $\varphi \in L$ satisfies $\chi_{E \setminus E_0} \varphi = 0$, then $\varphi = 0$ identically. Let g be a nonnegative Borel function on E, which is positive on the complement of E_0. Assume that there exists $0 < \varepsilon < \varepsilon_0$ such that the set $E_\varepsilon = \{x \in E : |g(x) - 1| \geq \varepsilon\}$ is precompact, where ε_0 is a small enough number (explicit). Assume that the space $\sqrt{g}L$ is closed and the corresponding operator of orthogonal projection Π^g satisfies, for sufficiently large R, the condition $\mathrm{tr}(\chi_{\{g>R\}} \Pi^g \chi_{\{g>R\}}) < \infty$. Assume further*

$$\int_{E_\varepsilon} |g(x) - 1| \Pi(x,x) \, \mathrm{d}\mu(x) < \infty;$$

$$\int_{E_\varepsilon^c} |g(x) - 1|^3 \Pi(x,x) \, \mathrm{d}\mu(x) < \infty; \qquad \int_{E_\varepsilon^c \times E_\varepsilon^c} |g(x) - g(y)|^2 |\Pi(x,y)|^2 \, \mathrm{d}\mu(x) \, \mathrm{d}\mu(y) < \infty.$$

Assume also that there exists a sequence of compact subsets $(E_n)_{n\geq 1}$ exhausting the whole phase space E, such that $\lim_{n\to\infty} \mathrm{tr}(\chi_{E_n}\Pi|g-1|^2\chi_{E_n^c}\Pi\chi_{E_n}) = 0$. Then we have

$$\mathbb{P}_{\Pi^g} = \overline{\Psi}_{g,\Pi} \cdot \mathbb{P}_\Pi.$$

Remark 2. The main point of this extension is to replace the convergence of $\int_{E_\varepsilon^c} |g(x)-1|^2\Pi(x,x)\,\mathrm{d}\mu(x)$ in Proposition 4.6 of [1] by the convergence of $\int_{E_\varepsilon^c} |g(x)-1|^3\Pi(x,x)\,\mathrm{d}\mu(x)$, this extension is crucial for treating the case of Fock space, since the former condition is violated even for the Ginibre point process.

3. Main results

3.1. Bergman case

Assume that $\omega : \mathbb{D} \to \mathbb{R}^+$ is a weight on $\mathbb{D}$ whose essential support is not contained in any compact subset of $\mathbb{D}$ and assume that the set of polynomials is dense in the corresponding weighted Bergman space $L^2_{\mathrm{hol}}(\mathbb{D}, \omega(z)\,\mathrm{d}z)$. Let K_ω be the reproducing kernel. Assume further that

$$\int_{\mathbb{D}} (1-|z|)^2 K_\omega(z,z)\omega(z)\,\mathrm{d}z < \infty. \tag{1}$$

Given any $\mathfrak{p} = (p_1, \dots, p_l) \in \mathbb{D}^l$ and $\mathfrak{q} = (q_1 \dots, q_k) \in \mathbb{D}^k$ two tuples of distinct points, we define

$$f_{\mathfrak{p},\mathfrak{q}} = \frac{\prod_{i=1}^{l} \frac{p_i - z}{1-\bar{p}_i z}}{\prod_{j=1}^{k} \frac{q_j - z}{1-\bar{q}_j z}}.$$

Recall that $K_\omega^{\mathfrak{q}}$ is the orthogonal projection onto $\{\varphi \in L^2_{\mathrm{hol}}(\mathbb{D}, \omega(z)\mathrm{d}z) : \varphi(q_1) = \cdots = \varphi(q_k) = 0\}$. Then Lemma 2.3 implies that the regularized multiplicative functional $\overline{\Psi}_{|f_{\mathfrak{p},\mathfrak{q}}|^2, K_\omega^{\mathfrak{q}}}$, corresponding to the function $f_{\mathfrak{p},\mathfrak{q}}$ and the kernel $K_\omega^{\mathfrak{q}}$, is well-defined.

Our main result for weighted Bergman spaces is the following.

Theorem 3.1. *Under the above assumptions on the weight ω, for any $\mathfrak{p} \in \mathbb{D}^l$ and $\mathfrak{q} \in \mathbb{D}^k$ of tuples of distinct points of orders l and k, the reduced Palm measures $\mathbb{P}^{\mathfrak{p}}_{K_w}$ and $\mathbb{P}^{\mathfrak{q}}_{K_w}$ are equivalent and we have*

$$\frac{\mathrm{d}\mathbb{P}^{\mathfrak{p}}_{K_\omega}}{\mathrm{d}\mathbb{P}^{\mathfrak{q}}_{K_\omega}} = \overline{\Psi}_{|f_{\mathfrak{p},\mathfrak{q}}|^2, K_\omega^{\mathfrak{q}}}.$$

Remark 3. The power series $\mathbf{f}_{\mathbb{D}}(z) := \sum_{n=0}^\infty g_n z^n$, where the g_n's are independent standard complex Gaussian random variables with density $\pi^{-1}\mathrm{e}^{-|z|^2}$, defines a random analytic function in $\mathbb{D}$. Peres and Virág [9] showed that the zero set $Z_{\mathbb{D}} = \{z \in \mathbb{D} : \mathbf{f}_{\mathbb{D}}(z) = 0\}$ of the random analytic function $\mathbf{f}_{\mathbb{D}}$ is a determinantal point process in $\mathbb{D}$ induced by the standard Bergman kernel $K_\omega(z,w) = \frac{1}{\pi(1-z\bar{w})^2}$ with $\omega \equiv 1$.

Remark 4. When $\omega \equiv 1$, the equivalence of $\mathbb{P}^{\mathfrak{p}}_{K_\omega}$ and $\mathbb{P}^{\mathfrak{q}}_{K_\omega}$ is due to Holroyd and Soo [3].

Let $\mathrm{Diff}_c(\mathbb{D})$ be the group of diffeomorphisms F of $\mathbb{D}$, such that the subset $\{z \in \mathbb{D} : F(z) \neq z\}$ is precompact in $\mathbb{D}$. Any diffeomorphism F in $\mathrm{Diff}_c(\mathbb{D})$ acts naturally on $\mathrm{Conf}(\mathbb{D})$ by sending a point configuration $\mathcal{Z} \in \mathrm{Conf}(\mathbb{D})$ to the point configuration

$$F(\mathcal{Z}) = \{F(z) : z \in \mathcal{Z}\};$$

we keep the same symbol F for this induced action on $\mathrm{Conf}(\mathbb{D})$.

Proposition 3.1. *The determinantal measure $\mathbb{P}_{K_\omega}$ is quasi-invariant under the induced action of the group $\mathrm{Diff}_c(\mathbb{D})$ on $\mathrm{Conf}(\mathbb{D})$.*

3.2. Fock case

Let $\psi : \mathbb{C} \to \mathbb{R}^+$ and denote $\mathrm{d}\nu_\psi(z) = \mathrm{e}^{-\psi(z)}\mathrm{d}z$. Recall that we denote by $\mathcal{F}_\psi = L^2_{\mathrm{hol}}(\mathbb{C}, \mathrm{d}\nu_\psi)$ the associate Fock space and by G_ψ its reproducing kernel. Consider $\mathbb{P}_{G_\psi}$ the determinantal point process induced by the reproducing kernel G_ψ. Assume that ψ is such that all conditions in Lemma 2.3 are verified for the pair (g, G_ψ) with $g(z) = 1 + z^{-1}\chi_{\{|z|\geq 1\}}$.

Recall that the Ginibre point process, as weak limit of eigenvalue distributions of non-Hermitian Gaussian random matrices, when the size of matrices goes to infinity, corresponds to $\mathbb{P}_{G_{\psi_2}}$, with $\psi_2(z) = |z|^2$; note that ψ_2 satisfies the above assumptions.

Let $\mathfrak{p} = (p_1, \dots, p_l), \mathfrak{q} = (q_1, \dots, q_l) \in \mathbb{C}^l$ be any two l-tuples of distinct points in $\mathbb{C}$, denote $\mathcal{F}_\psi(\mathfrak{p}) = \{\varphi \in \mathcal{F}_\psi : \varphi(p_1) = \dots = \varphi(p_l) = 0\}$, and similarly for $\mathcal{F}_\psi(\mathfrak{q})$. Define

$$g_{\mathfrak{p},\mathfrak{q}}(z) = \frac{(z - p_1)\cdots(z - p_l)}{(z - q_1)\cdots(z - q_l)}.$$

It is easy to verify that $\mathcal{F}_\psi(\mathfrak{p}) = g_{\mathfrak{p},\mathfrak{q}}(z)\mathcal{F}_\psi(\mathfrak{q})$. Let $G_\psi^{\mathfrak{q}}$ be the orthogonal projection onto $\mathcal{F}_\psi(\mathfrak{q})$. Then Lemma 2.3 implies that the regularized multiplicative functional $\overline{\Psi}_{|g_{\mathfrak{p},\mathfrak{q}}|^2, G_\psi^{\mathfrak{q}}}$ is well defined.

Theorem 3.2. *Let $\mathfrak{p}, \mathfrak{q} \in \mathbb{C}^l$ be any pair of l-tuples of distinct points in $\mathbb{C}$, under the above assumption on ψ, the corresponding reduced Palm distributions $\mathbb{P}_{G_\psi}^{\mathfrak{p}}$ and $\mathbb{P}_{G_\psi}^{\mathfrak{q}}$ are equivalent, and we have* $\frac{d\mathbb{P}_{G_\psi}^{\mathfrak{p}}}{d\mathbb{P}_{G_\psi}^{\mathfrak{q}}} = \overline{\Psi}_{|g_{\mathfrak{p},\mathfrak{q}}|^2, G_\psi^{\mathfrak{q}}}$.

Proposition 3.2. *The determinantal measure $\mathbb{P}_{G_\psi}$ is quasi-invariant under the induced action of the group* $\mathrm{Diff}_c(\mathbb{C})$ *on* $\mathrm{Conf}(\mathbb{C})$.

Acknowledgements

The authors would like to thank Professor Alexander Borichev for the useful discussion on the Bergman spaces. The authors are supported by A*MIDEX project (No. ANR-11-IDEX-0001-02), financed by Programme "Investissements d'avenir" of the French Government managed by the French National Research Agency (ANR). A.B. is also supported in part by the Grant MD-2859.2014.1 of the President of the Russian Federation, by the Programme "Dynamical systems and mathematical control theory" of the Presidium of the Russian Academy of Sciences, by the ANR-13-JS01-0010 under the project "VALET" of the Programme JCJC SIMI 1, and by the RFBR grants 11-01-00654, 12-01-31284, 12-01-33020, 13-01-12449. Y.Q. is supported in part by the ANR grant 2011-BS01-00801.

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