

## Erratum to the Finite Free Extension of Artinian $K$ -Algebras with the Strong Lefschetz Property.

TADAHITO HARIMA(\*) - JUNZO WATANABE(\*\*)

It has come to our attention that Proposition 21 in the paper cited in the heading is in error. It affects the proof of Main Theorem. The Main Theorem itself, however, can be proved as follows. With the notation of Main Theorem, write  $A$  and  $B$  as homomorphic images of polynomial rings:  $A = R/I$  and  $B = S/J$ , with  $R = K[v_1, \dots, v_n] \supset I$  and  $S = K[u_1, \dots, u_m, v_1, \dots, v_n] \supset J$ . Put  $U = (u_1, \dots, u_m)$  and  $V = (v_1, \dots, v_n)$ . Let  $>$  be a monomial order with the following properties:

- (i)  $\text{In}(J) \cap R = \text{In}(I)$ ,
- (ii)  $\text{In}(\alpha : v_n^k) = \text{In}(\alpha) : v_n^k$  for any  $k > 0$  and for any homogeneous ideal  $\alpha \subset R = K[V]$ ,
- (iii)  $\text{In}(\alpha : u_m^k) = \text{In}(\alpha) : u_m^k$  for any  $k > 0$  and for any homogeneous ideal  $\alpha \subset S/(V) = K[U]$ .

For example an elimination order with respect to  $V$  with the refinement such that it is the reverse lexicographic order both on  $V$  and on  $U$  has this property.

Now, since  $B$  is free over  $A$ , it is easy to see that the minimal generating set of  $\text{In}(J)$  consists of monomials only in  $U$  and in  $V$ . Hence we have  $S/\text{In}(J) \cong S/\text{In}(J) + (V) \otimes_K R/\text{In}(I)$ . It is possible to assume that  $U$  are generic variables of  $S/(V)$  and  $V$  are generic variables of  $R$ . Then by properties (ii) and (iii) we have that both  $S/\text{In}(J) + (V)$  and  $R/\text{In}(I)$  have the strong Lefschetz property. In fact  $u_m$  and  $v_n$  are Lefschetz elements in the respective algebras. Now by virtue of Conca [1] Theorem 1.1, we

(\*) Indirizzo dell'A.: Hokkaido University of Education, Shiroshima, Kushiro, Hokkaido 085-8580, Japan. E-mail: harima@kus.hokkyodai.ac.jp

(\*\*) Indirizzo dell'A.: Department of Mathematical Sciences, Tokai University, Hiratsuka 259-1292, Japan. E-mail: junzowat@ss.u-tokai.ac.jp

have that

$$\dim_K S/J + (g) \leq \dim_K S/\text{In}(J) + (g).$$

Here  $g$  is a generic element of  $S$ . The right hand side of the inequality is equal to the Sperner number of  $B$ , as the SLP is preserved by tensor product. This shows that the left hand side of this inequality is also equal to the Sperner number of  $B$ . Thus we have shown that  $B$  has the WLP. The same argument can be used to prove that  $B[t]/(t^k)$  has the WLP for any  $k > 0$ . Thus by Proposition 18 of the cited paper in the heading we conclude  $B$  has the SLP.

The correct statement of Proposition 21 in the intended formulation will be treated in our paper [2].

#### REFERENCES

- [1] A. CONCA, *Reduction numbers and Initial ideals*, Proceedings of the American Mathematical Society, **131** (2003), pp. 1015-1020.
- [2] T. HARIMA - J. WATANABE, *The commutator algebra of a nilpotent matrix and an application to the theory of Artinian  $K$ -algebras*, To appear.

Manoscritto pervenuto in redazione il 6 febbraio 2004.