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Groups Preserving the Cardinality of Subsets Product under Permutations.

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ABSTRACT - A group G is said to preserve the cardinality of 2-element subsets product under permutations, or G is a $PC(2, n)$-group if either $G = 1$ or for each n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G, there is a non-identity permutation σ in Σ_n such that $|S_1 S_2 \ldots S_n| = |S_{\sigma(1)} S_{\sigma(2)} \ldots S_{\sigma(n)}|$, where $|S|$ means the cardinality of a set S. Some characterizations of $PC(2, n)$-groups are presented here.

1. - Introduction.

Recently there has been much interest in the study of groups satisfying «finiteness conditions», for example, groups with various permutability conditions (see, for instance, [1,2] and [3]). A group G is called a PSP-group if there exists an integer $n > 1$ such that for each n-tuple $(H_1, \ldots, H_n)$ of subgroups of G, there is $\sigma(\neq 1) \in \Sigma_n$ such that the two complexes $H_1 H_2 \ldots H_n$ and $H_{\sigma(1)} H_{\sigma(2)} \ldots H_{\sigma(n)}$ are equal. It was shown in [5] that a finitely generated soluble PSP-group is finite-by-abelian. In this note, we consider a similar notion of permutable products, for 2-element subsets of G instead of subgroups of G.

NOTATIONS. For subsets $S, S_1, \ldots, S_n$ of a group G and an element g in G, $S_1 S_2 \ldots S_n = \{s_1 \ldots s_n;\ s_i \in S_i\}$, $S \cdot g = \{sg;\ s \in S\}$ and $g \cdot S = \{gs;\ s \in S\}$. Furthermore $|S|$ means the cardinality of a set S.

DEFINITION. For an integer $n > 1$, a group G is said to preserve the cardinality of 2-element subsets product under permutations, or G is a

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$PC(2, n)$-group if either $G = 1$ or for each n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G, there is a permutation $\sigma(\neq 1)$ in Σ_n such that

$$(1.1)\qquad |S_1 S_2 \ldots S_n| = |S_{\sigma(1)} S_{\sigma(2)} \ldots S_{\sigma(n)}| .$$

Let $PC(2)$ be the class $\bigcup_{n>1} PC(2, n)$. We give a complete description of $PC(2, 2)$ and $PC(2, 3)$-groups and show that $PC(2)$-groups are center-by-finite exponent. As an immediate corollary, we note that $PC(2)$-groups are collapsing in the following sense. In [8], Semple and Shalev called a group G n-collapsing if for any set S of n-element in G, $|S^n| < n^n$ and G is collapsing if it is n-collapsing for some $n > 0$. They proved that for a finitely generated residually finite group G, it is collapsing if and only if it is nilpotent-by-finite.

As we see in the following remark, it makes sense to fix one side of 1.1.

2. - Remark.

A non-trivial group G has the following property. Let $n \geq 3$. For each n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G, there exist distinct permutations $\sigma, \tau \in \Sigma_n$ such that the cardinalities of $S_{\sigma(1)} \ldots S_{\sigma(n)}$ and $S_{\tau(1)} \ldots S_{\tau(n)}$ are the same. Note $|S_1 S_2 \ldots S_n| \leq 2^n$. If $n \geq 4$, then $n! > 2^n$. So the number of permutations is strictly greater than the number of possible cardinalities of all permutable products. Hence there are two distinct permutations with the above property. Suppose $n = 3$. Let S_1, S_2 and S_3 be three given 2-element subsets of G. If $|S_{\sigma(1)} S_{\sigma(2)} S_{\sigma(3)}| \neq 2, 3$ for all $\sigma \in \Sigma_3$, we are already done. So we can assume $|S_{\sigma(1)} S_{\sigma(2)} S_{\sigma(3)}| = 2$ or 3 for some $\sigma \in \Sigma_3$. Write $S_1 = \{x_1, x_1 x\}$, $S_2 = \{y, y_1\}$ and $S_3 = \{z_1, z z_1\}$. Suppose $|S_1 S_2 S_3| = 2$. Then $|S_1 S_2| = |S_2 S_3| = 2$. Now by a simple calculation, we get that $|S_3 S_1 S_2|$ and $|S_2 S_3 S_1|$ are 2 or 4. Assume $|S_1 S_2 S_3| = 3$. Write $S_1' = \{1, x\}$ and $S_3' = \{1, z\}$. If $|S_1 S_2| = |S_1' S_2| = 2$, then we have $y = x y_1$ and $y_1 = xy$. Moreover $S_1' S_2 S_3' = \{y, y_1, yz, y_1 z\}$. Since $|S_1' S_2 S_3'| = 3$, we have $y = y_1 z$ or $y_1 = yz$. Notice that $y = y_1 z \Leftrightarrow x y_1 = y_1 z = xyz \Leftrightarrow y_1 = yz$. Hence $|S_1 S_2 S_3| = 2$, a contradiction. So $|S_1' S_2| = |\{y, y_1, xy, xy_1\}| = 3$. Without loss of generality, we can assume $y = xy_1$. Since $S_1' S_2 S_3' = S_1' S_2 \cup S_1' S_2 \cdot z$, there are two cases to examine.

Case (i). $y = xyz$, $y_1 = yz$ and $xy = y_1 z$.

Then $y = xy \cdot z = y_1 z \cdot z = yz^3$ and $y = xyz = xxy_1 z = x^3 y$. Thus $x^3 = z^3 = 1$. Note that $S_2 S_3 S_1 = S_2 S_3 \cdot x_1 \cup S_2 S_3 \cdot x_1 x$ and $S_2 S_3 =$

$= \{yz_1, yzz_1, yzzz_1\}$. Now suppose $|S_2S_3S_1| < 6$. Then at least one element in $S_2S_3 \cdot x_1$ lies in $S_2S_3 \cdot x_1x$. Note that $yzz_1x_1 = yz_1x_1x \Leftrightarrow$ $\Leftrightarrow zz_1x_1 = z_1x_1x \Leftrightarrow yzzz_1x_1 = yzz_1x_1x \Leftrightarrow yz_1x_1 = yzzzz_1x_1 = yzzz_1x_1x$ and $yzzz_1x_1 = yz_1x_1x \Leftrightarrow yz_1x_1 = yzz_1x_1x \Leftrightarrow yzz_1x_1 = yzzz_1x_1x$. So that one element in $S_2S_3 \cdot x_1$ lies in $S_2S_3 \cdot x_1x$ implies that the other two elements in $S_2S_3 \cdot x_1$ belong to $S_2S_3 \cdot x_1x$. Hence $|S_2S_3S_1| = 6$ or 3. Similarly we can show $|S_3S_1S_2| = 6$ or 3.

Case (ii). $y = y_1z$, $y_1 = xyz$ and $xy = yz$.

This case can be checked by the same argument as in case (i).

3. - Results.

Clearly $PC(2)$ contains all finite groups. So for a given n, it seems hard to characterize $PC(2, n)$-group. However in a very particular case, we have a complete result.

LEMMA 3.1. *Let G be a $PC(2, 2)$ or $PC(2, 3)$-group and $x, y \in G$. Then*

(i) *if $x^2 = 1$, then $x \in Z(G)$, the center of G;*

(ii) *if $[x, y] \neq 1$, then $x^y = x^{-1}$.*

PROOF. (i) If x has order 2 and $[x, y] \neq 1$, take $S_1 = \{1, x\}$, $S_2 = \{xy, y\}$ and $S_3 = \{1, y^{-1}xy\}$. Then $|S_1S_2S_3| \neq |S_{\sigma(1)}S_{\sigma(2)}S_{\sigma(3)}|$ for all $\sigma(\neq 1) \in \Sigma_3$ and $|S_1S_2| \neq |S_2S_1|$.

(ii) Let G be a $PC(2, 3)$-group. For $S_1 = \{1, x\}$, $S_2 = \{y, x^{-1}y\}$ and $S_3 = \{1, y^{-1}xy\}$, there is a non-trivial $\sigma \in \Sigma_3$ such that $|S_1S_2S_3| = |S_{\sigma(1)}S_{\sigma(2)}S_{\sigma(3)}|$.

There are five cases to check. We consider one of them (the others are similar). Suppose $|S_1S_2S_3| = |S_3S_1S_2| \leq 4$. If $|S_1S_2| = 2$, $x^2 = 1$ and so $x \in Z(G)$, a contradiction. Hence $|S_1S_2| = |\{y, xy, x^{-1}y\}| = 3$. Note that $S_3S_1S_2 = S_1S_2 \cup y^{-1}xy \cdot S_1S_2$. So at least two elements in $y^{-1}xy \cdot S_1S_2$ are in S_1S_2. The non-trivial possible cases are (i) $y = y^{-1}xyxy$, (ii) $xy = y^{-1}xyx^{-1}y$, (iii) $x^{-1}y = y^{-1}xyy$ and (iv) $x^{-1}y = y^{-1}xyxy$. Moreover two of these relations should hold. Note that (i) or (iii) is equivalent to the relation we want. If (ii) and (iv) are true, then $y^{-1}xy = x^{-2} = x^2$. Since x^2 lies in the center of G, $y^{-1}xy = x^2$ gives a contradiction. If G is a $PC(2, 2)$-group, take $S_1 = \{1, x\}$ and $S_2 = \{xy, y\}$. We then get the same result by a simple calculation. ■

THEOREM 3.2. *G is a $PC(2, 2)$ or $PC(2, 3)$-group if and only if either G is abelian or the direct product of a quaternion group of order 8 and an elementary abelian 2-group.*

PROOF. Let G be a $PC(2, 2)$ or $PC(2, 3)$-group. Then by Lemma 3.1(ii), $x^y = x^{\pm 1}$, any x, y in G. So G is a Dedekind group and every element of odd order is in the centre of G. If G is not abelian, then G has no elements of odd order, otherwise, with x, y, z in G, $[x, y] \neq 1$, z of odd order, we get $(xz)^y = x^{-1}z \neq (xz)^{\pm 1}$. Now the result follows from the structure of Dedikind groups (see [6], p. 139).

For the converse, let $G = Q \times D$ where D is an elementary abelian 2-group and Q a quaternion group of order 8. First we show that G is in $PC(2, 3)$. Let A, B and C be three given 2-element subsets of G. Write $A = \{g_1, g_1 ax\}$, $B = \{by, cz\}$ and $C = \{g_2, dwg_2\}$, where $a, b, c, d \in Q$, $x, y, z, w \in D$ and $g_1, g_2 \in G$. Then $|ABC| = |A'BC'|$ and $|CAB| = |C''A'B|$, where $A' = \{1, ax\}$, $C' = \{1, dw\}$ and $C'' = \{1, d^{\varepsilon}w\}$. Note that in C'', $\varepsilon = 1$ if $g_2 g_1$ lies in the centeralizer of d, and $\varepsilon = -1$ if not.

Case (i). $|AB| = 4$.

Since $C' = \{1, dw\}$ and $C'' = \{1, d^{\varepsilon}w\}$, $A'BC' = A'B \cup A'B \cdot dw$ and $C''A'B = A'B \cup d^{\varepsilon}w \cdot A'B$. Note that if there is one element in $A'B \cdot dw$ which is in $A'B$, then there is one element in $d^{\varepsilon}w \cdot A'B$ which is in $A'B$. The converse is also true. For example, suppose that $by = abdxyw$. Then $by = abdxyw = d^{\eta}abxyw \Leftrightarrow by = d^{\varepsilon}abxyw$ if $\varepsilon = \eta$, and $d^{\varepsilon}by = abxyw \Leftrightarrow d^{\varepsilon}byw = abxy$ if not. This means $|A'BC'| = |C''A'B|$ and so $|ABC| = |CAB|$.

Case (ii). $|AB| = 3$.

This case can be checked by the same argument as in case (i).

Case (iii). $|AB| = 2$.

Since $|A'B| = |\{1, ax\}\{by, cz\}| = 2$, we have $b = ac$ and $c = ab$. So $c = ab = aac$ and $a^2 = 1$. Hence A' lies in the center of G. Thus $|A'BC'| = |BC'A'|$. Clearly $|BC'A'| = |BCA|$.

Similar argument can be applied to show that G is in $PC(2, 2)$. ∎

THEOREM 3.3. *A $PC(2, n)$-group is center-by-(finite exponent $f(n)$).*

PROOF. We claim that there exists an integer k such that $[y^k, x] = 1$ for all $x, y \in G$. Let $x, y \in G$. We consider the n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G where $S_i = \{y, y^{1-i}xy^i\}$. Then $S_1 S_2 \ldots S_n =$

$= \{ y^n, xy^n, x^2 y^n, \ldots, x^n y^n \}$, and $|S_1 S_2 \ldots S_n| = \min(|x|, n+1)$. Since G is a $PC(2, n)$-group, there is a permutation $\sigma(\neq 1) \in \Sigma_n$ such that $|S_1 S_2 \ldots S_n| = |S_{\sigma(1)} S_{\sigma(2)} \ldots S_{\sigma(n)}|$. Write $g(i, j) = S_{\sigma(i)} S_{\sigma(i+1)} \ldots S_{\sigma(j)}$ for $i \leq j$.

If $|g(n-i, l)|$ and $|g(l, j)|$ are strictly increasing functions of i, j for all l, then for an integer j such that $\sigma(j) + 1 \neq \sigma(j+1)$, $|S_{\sigma(j)} S_{\sigma(j+1)}| < 4$. Here $S_{\sigma(j)} = \{ y, y^{1-\sigma(j)} x y^{\sigma(j)} \}$ and $S_{\sigma(j+1)} = \{ y, y^{1-\sigma(j+1)} x y^{\sigma(j+1)} \}$. So we have a relation $x = x^{y^s}$ where $s(\neq 0)$ depends on σ and so on x, y. However note that there are only finitely many choices of s independent of x, y, say, $s_1, \ldots, s_m$. Let $k = \text{l.c.m.}\{ s_i : i = 1, \ldots, m \}$. Then $[x, y^k] = 1$ for all x, y.

Suppose that $|g(n-i, l)|$ or $|g(l, j)|$ is not strictly increasing.

Case (i). $|x| > n+1$.

Let $|g(l, j)| = |g(l, j+1)|$. Then $g(l, j+1) = g(l, j) \cdot y \cup g(l, j) x^{y^{\sigma(j+1)-1}} \cdot y$. So $g(l, j) = g(l, j) x^{y^r}$, where $r = \sigma(j+1) - 1$ and $y^{j-l+1} (x^{y^r})^h \in g(p, q)$, for any h. Since $|g(l, j)| \leq n+1$, $|x| \leq n+1$. This is a contradiction.

Case (ii). $|x| \leq n+1$.

For $S_{\sigma(1)} S_{\sigma(2)} \ldots S_{\sigma(n)}$, let j be an integer such that $\sigma(j) + 1 \neq \sigma(j+1)$. Now we can assume that $|S_{\sigma(j)} S_{\sigma(j+1)}| = 4$. Then since $|S_1 S_2 \ldots S_n| = |x|$, we can find p, q with $p \leq j < j+1 \leq q$ such that $|g(p, q)| = |g(p, q+1)|$ or $|g(p-1, q)| = |g(p, q)|$. Let $|g(p, q)| = |g(p, q+1)|$ (the other case is similar). Then we have a relation $g(p, q) = g(p, q) x^{y^r}$, where $r = \sigma(q+1) - 1$. So $g(p, q) = g(p, q)(x^{y^r})^h$ for any h, and $g(p, q) = \{ y^m, y^m (x^{y^r}), y^m (x^{y^r})^2, \ldots, y^m (x^{y^r})^{|x|-1} \}$, where $m = q - p + 1$. Thus for some integer t, we have relation $x^{y^{\sigma(j)-t}} = (x^{y^r})^a$ or $x^{y^{\sigma(j+1)-1-t}} = (x^{y^r})^b$ where $2 \leq a, b < |x|$. In any case we have $x^{y^s} = x^d$ for some $2 \leq d < |x|$. Since $|x| \leq n+1$, $[y^k, x] = 1$ for some k. In every case our s and k depend on x, y. However there are still only finitely many choices of s and k that are independent of x, y. This completes the proof. ■

A group G is restrained if there is an integer n such that $\langle x \rangle^{\langle y \rangle}$ is generated by n elements for all $x, y \in G$. In [4], the following is proved.

LEMMA 3.4. *Let G be a finitely generated restrained group. If H is a normal subgroup of G such that G/H is cyclic, then H is finitely generated.*

PROOF. For some $g \in G$, we can write G in the form $H\langle g\rangle$. Since G is finitely generated, there exist $h_1, h_2, \dots, h_r$ in H such that $G = \langle h_1, h_2, \dots, h_r, g\rangle$ and $H = \langle h_1, h_2, \dots, h_r\rangle^G$. For each $i = 1, \dots, r$, $\langle h_i^{\langle g\rangle}\rangle$ is finitely generated, say, $\langle h_i^{\langle g\rangle}\rangle = \langle h_{i1}, h_{i2}, \dots, h_{id(i)}\rangle$. Now let $H_1 = \langle h_{il(i)};\ 1 \leq i \leq r,\ 1 \leq l(i) \leq d(i)\rangle$. Then clearly g lies in $N_G(H_1)$, the normalizer of H_1 in G and $\langle h_1, \dots, h_r\rangle \leq H_1$. Hence $N_G(H_1) = G$. This means that $H_1 = H$ and H is finitely generated. ■

Now we mention some properties of $PC(2)$ as immediate consequences of Theorem 3.3. For closure properties, we follow notations in [7]. Consider the restricted direct product $G = \mathrm{Dr}\, A_n$, where A_n is the alternating group of degree $n > 4$. Then G is locally finite but has no center. Clearly the standard wreath product of two infinite cyclic groups is not center-by-finite exponent. Neither is a free product of two infinite cyclic groups.

COROLLARY 3.5. (i) *A $PC(2)$-group is collapsing.*

(ii) *A $PC(2)$-group is restrained.*

(iii) *The class of $PC(2)$-groups is not closed under any of the closure operations P, D, C, W, F, R, L.* ■

QUESTIONS. (i) For $G, H \in PC(2)$, is $G \times H$ in $PC(2)$?

(ii) Is $PC(2)$ quotient-closed?

COROLLARY 3.6. *A finitely generated soluble $PC(2)$-group G is center-by-finite.*

PROOF. By Theorem 3.3, G is center-by-(finite exponent). And a finitely generated soluble group with finite exponent is finite. ■

Locally graded groups are those groups in which every finitely generated non-trivial subgroup has a finite non-trivial quotient.

THEOREM 3.7. *If G is a finitely generated locally graded $PC(2)$-group, then G is center-by-finite.*

PROOF. Let N be the finite residual of G. By Theorem 3.3 G is center-by-(finite exponent). Thus G/N is a finitely generated residually finite center-by-(finite exponent). It was shown in [11] that a finitely generated residually finite group of finite exponent is finite. Hence G/N is center-by-finite. G is restrained and so N is finitely generated by repeated applications of Lemma 3.4. Let $N \neq 1$. Since G is locally graded, N has a non-trivial finite factor group N/K. But then

$N/\mathrm{core}_G(K)$ is finite and $G/\mathrm{core}_G(K)$ is finite-by-(center-by-finite). This group is polycyclic-by-finite and so it is residually finite, contrary to the choice of N. ■

An element g of a group G is called an FC-element if it has only a finite number of conjugates in G. In particular if there is a positive integer m such that no element of G has more than m conjugates, then G is called a BFC-group. The subgroup of all FC-elements is called the FC-center.

THEOREM 3.8. *A finitely generated non-periodic* $PC(2)$*-group* G *is center-by-finite.*

PROOF. Let $G = \langle x_1, x_2, \ldots, x_r \rangle$ be a $PC(2, n)$-group and let z be an element of infinite order in $Z(G)$, the center of G. For $w \in G$, let Ny be a right coset of N, the normalizer of $\langle x \rangle$ where $x = wz$ if w has finite order, and $x = w$ if not. Suppose that y is reduced and $l(y) = m \geqslant n$, where $l(y)$ denotes the length of the shortest word for y. Write $S = \{x_i^{\pm 1}: i = 1, \ldots, r\}$ and $y = y_1 y_2 \ldots y_m$ where $y_i \in S$. Now we consider an n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G where $S_i = \{y_i, x^{\pi_{i-1}} y_i\}$, $\pi_0 = 1$, $\pi_j = y_1 y_2 \ldots y_j$. Since G is a $PC(2, n)$-group, there is $\sigma (\neq 1) \in \Sigma_n$ such that $|S_1 S_2 \ldots S_n| = |S_{\sigma(1)} S_{\sigma(2)} \ldots S_{\sigma(n)}|$. Write $g(i, j) = S_{\sigma(i)} S_{\sigma(i+1)} \ldots S_{\sigma(j)}$ for $i \leqslant j$. Since x is of infinite order, $|g(n-i, l)|$ and $|g(l, j)|$ are strictly increasing functions of i, j for all l. Let j be an integer for which $\sigma(j) + 1 \neq \sigma(j+1)$. Note that $S_1 S_2 \ldots S_n = \{y_1 y_2 \ldots y_n, x y_1 y_2 \ldots y_n, x^2 y_1 y_2 \ldots y_n, \ldots, x^n y_1 y_2 \ldots y_n\}$, and $|S_1 S_2 \ldots S_n| = n + 1$. Hence $|S_{\sigma(j)} S_{\sigma(j+1)}| < 4$. Since $S_{\sigma(j)} = \{y_{\sigma(j)}, x^{\pi_{\sigma(j)-1}} y_{\sigma(j)}\}$ and $S_{\sigma(j+1)} = \{y_{\sigma(j+1)}, x^{\pi_{\sigma(j+1)-1}} y_{\sigma(j+1)}\}$, we get $x^{\pi_{\sigma(j)}} = x^{\pi_{\sigma(j+1)-1}}$, or $(x^{-1})^{\pi_{\sigma(j)}} = x^{\pi_{\sigma(j+1)-1}}$. Hence $\pi_{\sigma(j)} \pi_{\sigma(j+1)-1}^{-1}$ lies in N. So $N\pi_{\sigma(j)} = N\pi_{\sigma(j+1)-1}$. By the repeated applications of the above argument, we can assume that $Ny = Ny'$, where $l(y') < n$. Hence N has finite index in G and so does $C(wz) = C(w)$. In fact there is an integer m such that $|G : C(w)| < m$ for all $w \in G$. Hence G is a BFC-group. Since G is finitely generated, it is center-by-finite. ■

COROLLARY 3.9. A torsion-free $PC(2)$-group is abelian. ■

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REFERENCES

[1] R. D. BLYTH, *Rewriting products of group elements I*, J. Algebra, **116** (1988), pp. 506-521.
[2] R. D. BLYTH - A. H. RHEMTULLA, *Rewritable products in FC-by-finite groups*, Canad. J. Math., **41**, No. 2 (1989), pp. 369-384.
[3] M. CURZIO - P. LONGOBARDI - M. MAJ - D. J. S. ROBINSON, *A permutational property of groups*, Arch. Math. (Basel), **44** (1985), pp. 385-389.
[4] Y. K. KIM - A. H. RHEMTULLA, *Weak maximality condition and polycyclic groups*, in *Proceedings of the American Mathematical Society*, Vol. 123, No. 3, March 1995, pp. 711-714.
[5] A. H. RHEMTULLA - A. R. WEISS, *Groups with permutable subgroup products*, in *Group Theory: Proceedings of the 1987 Singapore Conference*, edited by K. N. Cheng and Y.K. Leong, de Gruyter (1989), pp. 485-495.
[6] D. J. S. ROBINSON, *A Course in the Theory of Groups*, Springer-Verlag, New York (1980).
[7] D. J. S. ROBINSON, *Finiteness Conditions and Generalized Soluble Groups*, Part I, Springer-Verlag, New York (1972).
[8] J. F. SEMPLE - A. SHALEV, *Combinatorial conditions in residually finite groups I*, J. Algebra, **157** (1993), pp. 43-50.
[9] M. J. TOMKINSON, *FC-groups*, Research Notes in Mathematics, **96**, Pitman Advanced Publishing Program, Boston (1984).
[10] J. S. WILSON, *Two-generator conditions for residually finite groups*, Bull. London Math. Soc., **23** (1991), pp. 239-248.
[11] E. I. ZELMANOV, *On some problems of group theory and Lie algebras*, Math. USSR Sbornik, **66** (1990), No. 1, pp. 159-168.

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