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From Apophantics to Manifolds: the Structure of Husserl's Formal Logic

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Abstract.– A global picture of Husserl's architectonic view of the structure of formal science (including formal mathematics) is offered, as the view got its fullest (yet elliptic) articulation in the first three chapters of *Formale und transzendentale Logik* (1929). It is shown how Husserl's understanding of the structure of formal science (abstracting from the latter's subjective foundation) requires the independent consideration of at least three dimensions with respect to the formal, in terms, respectively, of 'approaches', epistemic 'interests', and 'successive layers'. First, there is the dimension of *apophantic* versus *ontological* approaches; second, the distinction of *combinatorial* (syntactic) versus *truth* (semantic) interest; and third, the consideration of the *three layers* of pure grammar, derivability relations, and systems or manifold theory. Moreover, it is shown how, in Husserl's view, the virtual identity of apophantic and ontological approaches on the top layer (deductive systems and/or manifolds) is supposed to give a kind of technical (if not philosophical) warrant for the unity of formal science.

In this paper, I will use expressions such as 'H-logic' as shorthand for 'Husserl's (conception of) logic' (in its definitive version of *Formale und transzendentale Logik*, unless otherwise mentioned).

1. The Context and the Problem

In his comments about *Formale und transzendentale Logik* (*FTL*) [Husserl 1929][1], Husserl was very explicit: this work was bringing to the fore the result of 'decades of reflection' (or, in Eugen Fink's words, *'den Erwerb langer Jahre literarischer Zurückhaltung'*). This was a finished book, complete in itself — which was definitely the exception, not the rule, for the later Husserl, as is well known. The given characterization, though not inadequate, is certainly more fitting for part one, the '*Objective Formal Logic*', than for the second half of the book, which lays out the 'transcendental foundations' for the former in a sometimes rather programmatic way. Both together would draw the contours of a theory of science: logic is and ought to be a theory of science. To be more precise a full-blown theory of science would consist of a whole system of superposed levels.[2]

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1 [Husserl 1929], refered as *FTL*. I will refer to Dorion Cairns' translation *Formal and Transcendental Logic* as [Husserl 1978].

2 To be even more precise, a full blown H-theory of science would roughly consist of the following system of superposed levels:

i) *objective formal logic*, including formal ontology (see sec. 3); *together with its transcendental analysis* it would give rise to a universal and formal theory of science, — or at least a first part of the latter, to be complemented then

ii) 1° by a *universal ontology* of a different kind, viz. uncovering universal but this time *non-analytic* structures of the world or of a world, and so differing

Of this broadely conceived system, only formal logic and its transcendental foundation are treated in *FTL*. As to my paper, it will be confined to some central problems concerning husserlian (henceforth H-) formal logic, disregarding not only non-formal ontology but also the question of transcendental foundations of formal logic. So my concerns will be limited here to some of the topics discussed in the first part of *FTL*. (This implies also that I will not systematically treat the question of the development of Husserl's thought on logic). It is a fact that the system of formal logic as Husserl conceived of it, had gained its shape ever more firmly over the years. And accordingly, he took its exposition as definitive, not only with respect to what he himself could have to say about the subject, but as a definitive clarification *tout court* of the structure, the scope and the sense of the science of formal logic.

In this connection 'formal logic' (or 'logical analytics') has to be taken in the broadest of the senses Husserl gives to that term: that is to say, as including traditional logic as well as its modern extensions, but especially also as including the whole of 'formal mathematics' as it had been developed since the mid nineteenth century. So what Husserl claims to achieve here, once and for all, is providing the outline (not the detail) of the true structure of *formal science*, including, *en passant*, a sketch of the answer concerning the philosophical question of the relation between logic and mathematics.

> In this work the diverse transformations are shown which the meaning of formal logic passes through [...] formal logic as 'formal ontology', as 'formal apophantics', [...] as purely formal *mathesis universalis*. The turbid polemics and misunderstandings about the relationship between logic and (formal) mathematics are finally being cleared away.[3]

The moderate interest Husserl's logic has aroused [4], ultimate as

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from formal ontology which conceives of possible worlds in 'empty generality', *and* 2° by a hierarchy of '*regional*', '*material*' logics or ontologies, each of which is the logic of a peculiar object region; examples of such material ontologies are geometry (when it is not conceived as a formal system but as a theory of 'real space': see sec. 5 (3')) and 'pure mechanics'. All of these logics/ontologies have to include *their* critical transcendental analysis as well.

3 'Selbstanzeige des Verfassers. Edmund Husserl, Formale und transzendentale Logik', prospectus of Niemeyer Verlag, 1929, reprinted in Husserl [1974], 340. (My translation).

4 But times might be changing, as a simple open list of names from recent times (i.e. long after the works of Weyl, Becker, Suzanne Bachelard or even Bar-

its 1929 exposition may have been, is a bit disproportionate to the enthusiasm and assurance testified in those claims. Interest, moreover, often took the shape of a rather negative attention. After all, shouldn't it only be fair to evaluate this aspect of Husserl's work against the background of results, both contemporary and more recent, acquired in formal science itself, at least as far as they have implications for the same or related problems as the ones considered in this work? And the *communis opinio* is that, taking this attitude, the verdict would seem to be rather in the negative.

Undoubtedly the point which has attracted most of the critical attention in this respect, is the one concerned with what Husserl called 'definite manifolds' (*definite Mannigfaltigkeiten*), or alternatively, 'complete axiom systems' (*vollständige Axiomensystemen*)[5]: as is well known, Gödel's incompleteness results, appearing two years after *FTL*, seemed to refute Husserl's idea that there can be (interesting) mathematical theories that manage to capture all truths about their domain by means of a finite axiom system.[6]

It is not the aim of this paper to go straight into these matters. It is just not plausible to suggest that interest in Husserl's work on logic should stand or fall by the issue of definite manifolds. Rather I would like to take up a task here, which is preliminary both to the issue mentioned and to related issues. What exactly is the scope of

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Hillel) may suggest: Schmit [1981], Tragesser [1984], Willard [1984], Tieszen [1989], Lohmar [1989], D. Bell, Martin-Löf, Føllesdal, Haaparanta...

5 But *FTL* is only the latest and most ample source on this husserlian topic. See also the known passage §72 of *Ideen I*. Besides, it is imperative today, in connection with this subject as well as with the problem context it belongs to, to mention still earlier lecture notes, research manuscripts and the like, that have in part been published in more recent years: a number of texts published as *Beilagen* and *Ergänzende Texte in Husserliana XII* (*Philosophie der Arithmetik*), a text in *Husserliana XXI* (*Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass 1886-1901* (ed. I. Strohmeyer)), and especially a number of sections in the lecture notes of 1911, to be published by U. Panzer as *Husserliana XXX* (1996), manuscript F I 12, 15a - 26b.

6 It was Cavaillès who, in 1942, pointed out that this should be what Husserl had in mind, and who accordingly launched the criticism of this conception [1987], 70-73. The subsequent controversy over the reading of husserlian definiteness mobilized a.o. S. Bachelard, Derrida, Tran duc Thao, Schmit, Lohmar,... See also my *Mathematical Dialectics: the Philosophy of Mathematics and Science of Jean Cavaillès*, Brussels (Kon. Academie voor Wetenschappen en Kunsten), forthcoming.

H-logical theory (which corresponds to logic or at least formal science in a straightforward, although slightly extended sense)? That is to say, what, in the husserlian perspective, is the essential structure one has to ascribe to logic, if logic is to fulfill its role and aim as a (formal) theory of science? A lot of things to be said here about Husserl's logic have been said by others, and more extensively so. (Some of the relevant literature is mentioned in the notes and in the bibliography). It is not my aim to discuss either the secondary literature or matters of exegesis and interpretation in detail; rather I would like to present an overall picture some aspects of which certainly cannot claim to be innovative. But I think that, by tying together a number of interpretative threads, the global outlook is different from what is found in the literature up to now.

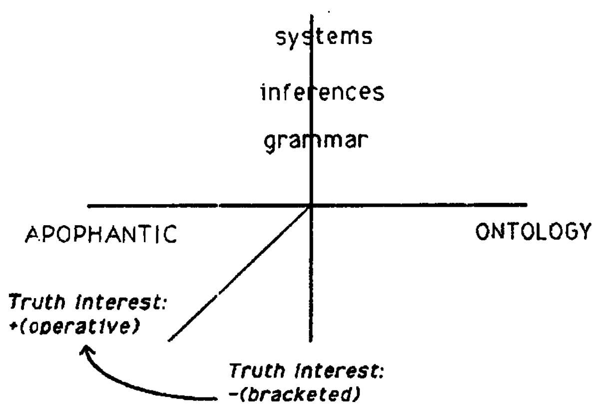
2. The Edifice and its Dimensions

Trying to reconstruct the husserlian architecture of (formal) logic, it appears that one can view it from different angles. Thus there are at least three different principles of construction or lines of division to be considered.[7] Indeed one could design an approximate picture of H-logic by means of a threedimensional structure, as I will try to show. (This happy coincidence, responding perhaps to an intuitive tendency for geometrical modeling not unpeculiar to Husserl, happens to make it apt for a ready visualization). So, if the geometrical analogy is not too misleading, it follows that it must be possible to start the description of the structure by taking either of three independent viewpoints, corresponding to its deployment relative to the three axes of a three-space. There can be no question of giving an elaborate description of this structure here. But then, no more can Husserl be taken to develop a detailed theory of logic in *FTL*. Let's interpret that as a further coincidence: the fact that *FTL*, part I, offers but a scheme of relatively empty boxes (meant to be filled out by the really detailed studies), helps to confine inquiries here to what is needed for our purpose. And that is just a rough draft of the 'H-logic skeleton', needed in order to determine the site of each branch and level from the basics of 'Baby Logic' up to the top

7 'At least': as should always be kept in mind, and as will become apparent in passing, there is also the all pervasive question of whether one is taking an objective or alternatively a subjective stance with respect to the several approaches or subdivisions distinguished below. Non-freaks of phenomenology shouldn't worry: here I will let the technical content of Husserl's logic prevail, looking away as much as possible from its subjective face. When at times it will prove inescapable to go into this deep running water, I shall attempt to keep it as untroubled as possible.

of Theory Forms. So I give the barest outline according to some kind of natural order, starting from what seems to me the most basic distinction.

In my view then, the three basic abstract dimensions along which H-formal science is organized are the following: *(i)* the distinction of two fundamental 'approaches' of the formal, an *ontological* versus an *apophantic* approach; *(ii)* the distinction of two ways of doing formal science: abstracting from its function for knowledge or, alternatively, putting this *truth function* at work; and *(iii)* the distinction of three 'layers' of the formal, respectively to be called a *grammatical*, an *inferential*, and a *systems* layer.



3. Apophantics and Formal Ontology

(Dimension i) One of the things logicians had always been lacking according to Husserl, was an understanding of the grand architecture of formal science, that is to say, an understanding of how its main 'parts' (or the main manifestations of formal thought) essentially relate to each other. And, he would add, such an understanding was hardly to be expected as long as the prevalence of a uniform logical tradition seemed to manifest an equally uniform logical order: how could anyone, in the absence of a possible comparison with another complexion of formal science fully understand or even grasp the idea that the logic handed down to us was only dealing with part and parcel of the logical order? But then the undreamt of came true. Already foreshadowed by Leibniz a.o., the second great manifestation of formal thought appeared with the great movement of formal abstraction in nineteenth century mathematics. The prime subject matter of the latter no longer being number and quantity, but general forms of operating with arbitrary 'entities', a new realm of formal concepts lay open for investigation.

As sharply as anyone, Husserl recognized the rise of *formal mathematics* as a major event in the history of cognition (as the rise of a new scientific style, one could say). This event forced one to reflect upon the nature of logico-mathematical knowledge itself. To him both formal logic in the sense of the tradition and formal mathematics are investigations of the realm of the formal, i.e. of the analytical *a priori*. How then could one have *two* disciplines revealing the structure of that realm of analytical forms? How did the newly discovered world of mathematical forms relate to the logical universe familiar from the formal part of philosophy from Aristotle to Kant?

Logic in the aristotelian tradition, one came to see now, hangs on one particular shape in which the formal shows itself. What has always been specific for the secular logical doctrine, is that its view of the formal takes as its basic unit the *judgement* or proposition (or more precisely, the possible forms of judgements). It takes the judgement form as a unit of meaningfully asserting something, and then studies the ways of combining these units into inferences and inference systems. Accordingly, Husserl calls this approach a *formal apophantics*. Apophantics is an investigation of judgements as judgements, or, broadly, of linguistic meanings as meanings: its topic is the nature of (a certain class of) our means of meaning (in so far as the latter possess a universal and apriorical core). This point of view remains in principle maintained when the classes either of the judgement forms or of the patterns of valid reasoning are extended beyond their initial boundaries (for example by taking into account relations), as long as one is still focusing on what on certain formal grounds deserves to be called valid or well-formed in our ways of *intending* something 'judgementally'. That is to say, this inquiry does not speak about the 'somethings' intended themselves — objects or states of affairs; it just talks about our ways of intending them. Logical analytics has taken the shape of an analytics of judgements.

But why couldn't (the form of) the 'somethings' themselves, which up to then were only involved as a background of object-poles intended, become equally well a thematic field of inquiry as lawlike as apophantics, to be dealt with in the same formal generality? Even when the object-pole was out of focus in the study of our apophantical intendings, stating and inferring was always implicitly understood to be *about* objects (c.q. states of affairs). And indeed, says Husserl playing one of his trump-cards, what in fact is formal mathematics talking about? What is the nature of abstract objects such as sets, sequences, complex and hypercomplex or even ideal numbers (concepts Husserl all allocates to formal mathematics), what is it that they have in common with each other and with logical concepts?

> When one considers the naturally broadest universality of the concepts set and number, and considers also the concepts element and unity wich respectively determine their sense, one recognizes that the theory of sets and the theory of numbers relate to the empty universe 'any object whatever' or 'anything whatever' (*Etwas-überhaupt*) [...] the *formal mathematical* disciplines are formal in the sense of having as fundamental concepts certain *derivative formations of 'anything-whatever'*. [Husserl 1978, 77 or *FTL*, §25]

That is to say, formal mathematics can be interpreted not only as a mere study of forms but as a study of forms pertaining to any object of our epistemic intendings: formal mathematics, when seen in this light, is nothing less than a *formal ontology*. Husserl can speak of an 'ontology' in that formal mathematics makes explicit and classifies the (formal) categories involved in any possible talk of objects, whatever their nature: the basic concepts of mathematics, when conceived according to the suitable level of abstraction, can be interpreted as doing exactly that (i.e. as *Gegenstandskategorien*), and can be put to that use in the hierarchical system of scientific knowledge. Seen in this light, formal mathematics is an apriorical theory of objects (*Gegenstandslehre*).

It might be useful to notice in passing the role (however restricted here) of the 'subjective' factor in making the distinction between apophantics and ontology. Indeed much as the philosopher finds before him the respective products of formal thought in theories objectively laid down, it remains true that one of the reasons for regarding them as *two* sides of *one and the same* formal science concerns the fact that they each represent a specific 'attitude' towards the formal (or a specific 'intentional focus' characterizing the kind of research involved, if you like). Ultimately, in Husserl's view, formal ontology and apophantics are distinct branches and can give rise to distinct logical or logico-mathematical traditions because they are commanded by an 'ontological', or alternatively, by an 'apophantical attitude'. Focusing on judgemental meaning, you get a *Bedeutungslehre* (with its *Bedeutungskategorien*); focusing on the object-pole of our meaning intentions, you are in the field of a *Gegenstandslehre*, that is to say of ontology. So the doctrines at issue, which appeared as objective products of a historical-conceptual development, can be considered at the same time as corresponding 'intentionalities' (or sense bestowing attitudes) objectified; attitudes dictated by the respective theoretical interests of the logician. (It is not 'subjective', of course, if the latter term is taken to mean 'depending on the interests of the individual logician').

4. Logical Combinatorics and Formal Logic of Truth

(Dimension ii) The latter morals concerning the *Doppelseitigkeit* subjective/objective, however, apply in a much more direct and permeating fashion to the second principle of distinction structuring H-logic. Indeed, its basis, as we will see, is at least in part 'subjective' in character.

Following Husserl the science of logic has the peculiarity that it can be practised in two ways: either exercising its 'function' of a preliminary doctrine stating conditions of possible truth, or, on the contrary, bracketing it. Logic done in the former way he calls a *formal logic of truth* (or logic practised in the interest of truth); in the latter way he talks about *consequence logic* (or *pure analytics [of non-contradiction]*). (In my opinion, the designation 'consequence logic' has to be regarded as confusing, for reasons to be explained in sec. 5; in general I will prefer to call the conception of logic at issue by the more appropriate name of *logical combinatorics*).

Saying that logic can be done in two ways, is not suggesting that logical combinatorics and truth logic are two disciplines or even branches of logic; and pointing at the relevance of the scheme of objective/subjective here, is even less to suggest that either of both 'practises' of logic has to do with a subjective rather than an objective way of doing logic (whatever that may mean). What is implied, is this: the very discovery (claimed by Husserl) of two distinct interpretations of formal science — the 'combinatorics' interpretation and the 'truth' interpretation — puts us on the track of an important fact, presumedly throwing a new light upon the objective prevalence of that science. The fact alluded to, is the relativity of this double reading of logic with respect to the specific *evidence type* leading the logician (again, not the individual logician, but any practitioner of logic) in either case. Logic in the sense of combinatorics is guided by the 'evidence of *distinctness*' (*Deutlichkeitsevidenz*), logic in the sense of truth by the 'evidence of *clarity*' (*Klarheitsevidenz*). And evidence, of whichever type, for Husserl, indicates activity (or at least functioning intentionality) of a subject, in this case of a subject of a special type of epistemic activity. Thus logic as guided by evidence of distinctness, for example, indicates the special role of one type of reasoning in the practice of the logician or formal mathematician.

Two kinds of question await an answer then: what, in fact, are logical combinatorics and truth logic, what are the relevant evidence types of distinctness and clarity?

Introducing the 1929 answer to the first, 'objective' problem area, one could begin by comparing Husserl's claimed discovery concerning the double reading of extant logic with a standard modern textbook view about the subject matter of logic (which corresponds rather well to Husserl's *initial* view). The usual conception then takes a start by defining logic — thus exorcizing in passing a flat psychologism — as stating laws or norms or rules about *valid* inference. Apart from its intrinsic consequences (or lack of consequences) for logic, such a definition allows to justify the latter's role (at least in a minimal sense) in epistemology, e.g. in a theory of science, since it could be read as stating preliminary conditions for the transference of true information incorporated in linguistic form into premises. This is just the position of the *Prolegomena*. Logic is characterized as *Geltungslehre*, the theory capturing the content of the notion of valid reasoning.[8] The commitment involved in Husserl's then defense of apriorical laws is much stronger, however, than the neutral attitude the usual textbook circumlocutions take in this respect. After all, (antiquated) psychologism was not finished with in those days.

Now, apart from the (alleged) shift concerning the (alleged) 'platonism' of the *Prolegomena*, a refinement has to be noticed in Husserl's view on the very idea of what logic is. It may well be that the refinement has to do with the influence of formal mathematics, and perhaps also with Hilbert's conception if the latter, but this question needs not bother us now. Anyway, Husserl holds that it is ambiguous to simply say that logic is the science dealing with principles of valid inference. Much as he sticked to the view of formal science as dealing with the analytical *a priori* (which is at odds with Hilbert's view), he stressed that notions such as validity and the like are indeterminate with respect to a 'double aspect' reading they leave open. Validity may aim at once at formal transference of truth *and* at the mere aspect of strict rule-following in formalized deductive contexts. Right now Husserl said it was vital to distinguish these aspects, to disambiguate the notions involved (say,

8 The view was embedded in a broader philosophy of logic, stressing that as a theory of validity, logic, for one thing, has a functional value (involves the possibility of being applied to any knowledge worth its name), *and* at the same time that this applicability rests upon purely theoretical grounds; i.e. logic is not just a useful or practical art of thinking: the norms it enacts are based on apriorical laws concerning *what is*. More precisely, 'to be', in this context, is to be understood, following Husserl, in an 'unreal', or rather 'irreal' sense, better: ideal sense (*ideale Gegenstände* being a species of *irreale Gegenstände*), perhaps as 'situated' in an ideal realm. (Realm or Irrealm? Hence the question of Husserl's 'platonism' in the *Logical Investigations*). Yet it would be more accurate to describe Husserl's view as of then as saying that logic is about what is possible (or compossible) than to say it is about 'what is'.

from now on, you use the notion of validity exclusively when talking in terms of truth preservation). It is not one and the same thing to describe modus ponens as a rule of separation allowing to write down separately the consequent of a material implication, and to describe it as a rule stating that the truth value *true*, established both for a material implication and for its antecedent, is transferred to the implication's consequent.

Does the difference matter? In the first mode one places oneself — using for the moment post-H terminology in a somewhat loose way – on a purely *syntactical* point of view — checking whether (and according to which rules) a certain transformation on distinctly defined strings of symbols are feasible. In the second mode — transference of truth — one chooses to interpret the corresponding rules, if operative, and transformations, if feasible, as indications of (the possibility of) acquiring true knowledge. They are the necessary conditions then, or at least those necessary conditions which are formal in nature, for 'reasoning truly'. (Besides there are sets of conditions of a different nature, pertaining to material H-logics of truth). Retrospectively stretching modern terminology a bit,[9] one could recognize a *semantic* intention in this way of looking at (H-logic).[10]

So the idea of characterizing logic (both apophantic and ontological) as a *Geltungslehre* has to be split up into two conceptions of logic, — logic as a theory calculus, and logic as an interpreted or interpretable theory,[11] pointing at possible truth (when filling in the 'empty places for variables in the formulas'). The former is a formal doctrine of the 'second power' one could say, a hyperformal science, abstracting from any interpretation of the moves to be made on the symbol strings. Husserl (spontaneously, and, for the reader, confusingly associating this feature with the one most obviously concerned level of combinatorics, the level of

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9 It is to be noticed for example that formal ontology, for all its object-directedness, is not *per se* semantic in the usual modern sense, as is witnessed by the fact that it can be practised without putting the truth function (or, for that matter, any meaning intention) into play, and indeed that is the way it is primarily exercised in formal mathematical practice according to Husserl.

10 Cf. The remarks of Tarski's (see the beginning of *Der Wahrheitsbegriff...*) and mainly of some of Tarski's commentators in this respect. (But Husserl's 'truth logic' is not a theory of truth.) See also Suzanne Bachelard [1974]; also T. Mormann [1991] (The latter study however involves more than a bit of hindsight in some respects).

11 Here it would be hardly anachronistic to say that formal truth logic introduces the notion of (formal) *model*.

derivability relations) calls it a logic of consequence or of non-contradiction (see also note 26). Indeed the most obvious candidates for singling out the permissible moves in such a formalist-formal science are the principles guiding derivation-transformations.

All this being said, the most remarkable fact about the distinction between *Logik der Widerspruchslosigkeit* (or rather, combinatorics) and *formale Wahrheitslogik* seems to be that, technically, there is almost no 'observable' distinction to be made! Once you have the laws pertaining to the former, you have — *modulo* a few rules of translation — the complete content of the latter. Formal logic of truth has no new laws or notions to offer at all, except for a few notions which seem straightforward paraphrases of the corresponding notions of logical combinatorics (just as the laws of truth logic are translations of the corresponding combinatorial laws using the new vocabulary). And it is precisely in this sense that it would be misleading to interpret the phrase 'truth logic' as suggesting that we have a somehow different branch of logic (let alone a different logic) before us. So what is the point of all this? The point is that it proves possible to conceive of logic and its apparatus irrespective of their truth function: in this sense the asymmetry among both conceptions is complete. The syntactical point of view is possible without any reference to the truth perspective. So the big thing about Husserl's second line of division within objective logic really comes to this: *it is possible to build up formal science (including the formal logic of natural language)*[12] without even taking notice of the fact that it is called upon to function as a prolegomenon to the expression of true propositions and valid reasoning (in the semantical sense). And this applies to syllogistics and the whole of apophantics as well as to formal mathematics (which probably gave birth to the consideration at stake). Reading this fundamental thesis of H-philosophy of formal science (a thesis emphasized also by [D. Lohmar 1989]: see his chapter 12) as a principle of self-sufficiency of the syntactical approach to the formal, it is obvious that the same tarskian critique can be launched against it as against the 1934 point of view of Carnap's *Logical Syntax*: viz. a criticism of the idea that it could be possible to reconstruct all important conceptual relations within formal mathématics in purely syntactical terms. But there is another side to the husserlian coin, as we will soon see.

The distinction cuts right through the division into apophantics and ontology. So already the question lays at hand what the different possible combinations of stances are (and also their technical

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12 See below on logical grammar sub (iii) in sec. 5.

repercussions) within logic, when letting the two principles considered thus far interact. *A priori* we have before us then: *(a) Apophantical* logic as a *calculus* or, *(b)* as (formally) *interpreted* doctrine (in the way traditional logicians always spontaneously conceived of syllogistics); *(c)* formal mathematical disciplines as *game mathematics* or calculi taking logical combinatorics as the ultimate constraint, versus *(d)* the conception of the same theories (e.g. set theory) as stating *ontological truths* which frame any possible scientific knowledge:

Truth interest	F. APOPHANTICS	F. ONTOLOGY
-	*(a)*	*(c)*
+	*(b)*	*(d)*

Within this scheme, it becomes straightforward to reidentify the nucleus of the analytically possible and to distinguish in it what belongs to the 'pure analytics of non-contradiction' from what is 'possible' in a sense distinct from the avoidance of mere syntactical impossibility. What *is* possible in the syntactical sense, however, is not evident at first sight or by mere inspection of analytical forms we are presented with: this raises the difficult question about Husserl's stance towards conventionalism. (Something will be said about it in sec. 5).

All this of course has implications for the status of logic itself, but also for its vocation as a preliminary theory of science. Before *FTL*, *Geltungslehre* had to act all roles almost out of nothing, because it had to assume all logical virtues (theoretical, normative, practical, philosophical...) *en masse*. Now things are being sorted out and one can move on in stages. In order to have a clean idea of models and of the multiple models virtue[13] so crucial for cognition you need a calculus; you need it as apophantic calculus (*(a)* above), if only to model the natural language part ineradicable in cognition, and you need it as formal mathematics *(c)*, in that science builds its further conceptual constructions upon the latter (or as constructions emergent relative to it). You are going to need *(b)* truth-interpreted

13 Nowadays some philosophers, impressed by some of the possible consequences (presumably things one would desperately but vainly try to get rid of) of the 'multiple modelizability' property, just treat it as a vice, a bad fate, nay a nightmare: as if the ghost of non-categoricity came clanking its chain at their bedside each night to catch them swimming in sweat while they scream '*please don't! don't skolemize me!*'

apophantic besides, because cognition is or contains at least *interpreted* calculi modeling natural language, and of course you will recur to *(d)* truth-formal ontology, because the progressive building of science requires to take sets, numbers, sequences and the like not just as empty hulls but as forms for 'real' concepts (from differential analysis e.g.), subsequently for 'real' quantities, and much more. And that picture is just going to be completed further by the introduction of the appropriately distinguished *levels* of logic (sec. 5).

So far, so good; but then yet another H-question had been posed: what about the ('subjective') foundation of it all? Suppose we have seen all the things Husserl taught us about combinatorics and truth in logic the way he would have it. Then he will insist and ask: but how is the whole thing possible as it is? How come we could get logic the way(s) we've got it, and how come we can have it *both* ways? Once you accept the picture, he will not let go: logic, if anything, is also an effectuation (*Leistung*) of reason (a rational effectuation), and so essential distinctions with respect to it will have to be elucidated in terms of the evidence or rather of the type(s) of evidence giving them originally to its practitioner. In the case at hand, the question then becomes: what are the evidence types giving originally a grasp of the character of logical structures respectively in the shape of combinatorial and of validity determining structures?

Here I will only give the flavour of the husserlian answer through an historical frame tale, as I promised not to bother the transcendentally exhausted reader as of today with problems too nocturnal to her mind.

Leibniz once praised Descartes for laying down the difference between the clarity of an idea and its distinct presentation to the mind; Leibniz once blamed Descartes for misusing his (Descartes') finding that we possess ideas which are both clear and distinct.

Descartes, so he said, was misled into his famous criterion for the apodictic certainty of an idea (in terms of distinctness *and* clarity) because he simply subordinated distinctness to clarity. Gambling altogether too high, Descartes proclaimed in science nothing less than clarity was to be achieved. But once you decide to accept only ideas within science which do satisfy also the requirement of clarity (i.e. of intuitive givenness to the mind),[14] over and above the

14 That is, of full intuitive evidence, all links in a chain being embraced '*uno intuitu*', says Descartes, so that the law or structure that governs their very linking is immediately perceived in an intellectual intuition (the way an intelligible though complex proof becomes transparent once its principle has really been grasped).

condition of distinct givenness, you throw away your ticket allowing you at least a glimpse of the paradise of the infinite. Finite intellects can have that glimpse, Leibniz held, but only when staying content with the infinite's symbolical representatives (as in the calculus of continuity). Concatenations of symbols are tractable because of their discreteness, reliable because of their formal recognizability. Upon these characteristics the evidence of distinctness (*Deutlichkeit*) is based, and all lawlikeness in the logical order is traceable from this kind of evidence.

Some centuries later, Hilbert (while safely remaining silent on the old clarity issue) in his turn tried to fix a ticket to Cantor's paradise which ought not to be too expensive: formalize mathematics and prove the consistency of mathematical theories by finite means. He thought this could be done, because the *distinct* intuitive givenness of sign-objects seemed intimately connected with elementary mathematical objects in a more ordinary sense.

And Husserl said: looking at what mathematicians really do when working in the purity of formal rigour, one sees that they concentrate on the manipulation of symbol strings up to the exclusion of any concern other than the combinatorial consistency of the moves. Everything which is needed in order to get across these moves is the evidence of *distinctness*. Here the mind is operating as a Turing machine (and able to do so), we would say, if we want to translate Husserl's view in terms more up to date once again. This, indeed, is the 'subjective' translation of the mentioned husserlian thesis of the sufficiency of the syntactical point of view.

On the other hand, and, irrespective of the question of mathematical infinity, we can also make sense of the evidence type the modern tradition used to call '*clarity*' (and of its surplus value), or so Husserl thought. This is the other side of the coin. Unaware of course of the tarskian and gödelian facts, he nevertheless claimed a special role for a semantical consideration of the formal (although he did not see any technical surplus value of the latter point of view, in the sense indicated by the relevant metatheorems). In his subjective mode of expression: There is more to formal theories than mere distinctness; as a matter of fact the true nature of symbolic thought in general is revealed by the fact that besides manoeuvres on the symbolic representatives of things, there is the interest in 'truth', which in its fullfledged form is the interest in dealing with things themselves as they are 'clearly' present — in a 'live' performance — to the intuiter. Applied to formal theories, this means that (even there) we want to be able to interpret the results as well as the principles of our operating, at work in our 'computer mode', as at the same time dealing with possible states of affairs, to be applied later on in the knowledge of real domains. When we have checked the formal consequences of positing a number system

$$Q = \{a+bi+cj+dk \mid a,b,c,d \in \mathbb{R} \}$$

with multiplication defined by axioms of the form

$$i^2=j^2=k^2=-1,\ ij=jk=ki=-1=-ji=-kj=-ik,$$

we want to be able to say what it means to be dealing with 'numbers' of this form (sacrificing the properties of order, and with an anticommutative composition law), in the sense we really confer to those concepts, -- and what it is the latter can be applied to. To sum up, *Deutlichkeit*, on the one hand, to Husserl, is the defining characteristic of the formally mathematizing mind; it is the necessary and sufficient subjective condition for the exercise of formal science as such. But at the same time, even in formal science, not *as* formal but *as* science, as a systematizing part and a theory of science, we are finalizing towards knowledge intended as true (whatever theory of truth we may adhere to). Moreover, already in the formulas handled we intend the concepts grasped to be lively present to our mind. In that sense, even in formal science, to Husserl's mind, *Deutlichkeit* is not enough: there is more to formal thought than combining symbols, and we do achieve more than that.[15]

5. Logical Hierarchy and Metalogical Unification

(Dimension iii) The third independent dimension which, according to my view, is determining for the structure of H-logic, as well as for its tuning to a theory of science, is the division with respect to *levels* of logical theory formation. Three different and superposed levels of theory have to be distinguished, corresponding to three consecutive 'tasks' the H-logician has to set himself in order to have at least a full idea of what his science ought to deal with. The respective levels and tasks can be circumscribed as elaborating (1 + 1') a logical grammar (or pure *theory of forms*); (2 + 2') a formal theory or theories of *derivability relations* among judgement forms on the one hand, among formal mathematical concepts on the other; and (3 + 3') a global *metatheory* about (the form of) theoretical units on the previous level. The succession is to be seen as hierarchical, in a

15 For a more thorough treatment of the evidences typical for logic, and of how the different 'ways of practising' the latter instantiate more general facts about these evidence types, see D. Lohmar's *Kommentar zur Formalen und transzendentalen Logik*, Wissenschaftliche Buchgesellschaft, Darmstadt, forthcoming.

sense to be specified, allowing to speak of subsequent 'levels'; the latter can be labeled a *grammatical*, an *inferential*, and a *systems* layer respectively.[16] And again the division cuts across the distinctions made according to dimensions *(i)* and *(ii)*, so ontology as well as apophantics, and logical combinatorics as well as truth logic are covered. Schematically:

F. APOPHANTICS	F. ONTOLOGY
(3) theory of deductive systems	(3') theory of manifolds
(2) derivability apophantics	(2') formal math. theories
(1) logical grammar	(1') ontological grammar

Rather than dimensions *(i)* and *(ii)*, it is the sequence of levels which finally will give us a grasp of the content of H-logic and of its relation to what we usually think of as the thing called logic.

(1 + 1') *A pure theory of forms*. Husserl's main idea here is that logic already starts long before questions of inference and derivability (let alone semantical validity) are being posed. Already *at the level of the grammar* of language — be it a formal or a natural language — there are universal and apriorical laws or rules (hence 'pure', hence 'logical' grammar), which, as logicians should realize, deserve to be united into one science together with the usual study of derivability (and the less usual study of theory forms). Indeed, in each case the things to be dealt with are matters of logical form. 'At the level of grammar': that is to say, the rules here at stake are

16 The question of 'levels' and 'layers' — both terms occur in the translations of and comments on relevant H-texts — is a typical commentator's problem, or rather pseudo-problem. It has been done away with satisfactorily by D. Lohmar [1989], 177ff. What is the case? From a certain moment on (i.e. after noticing the ambiguity of '*Geltung*', cf. sec. 3), Husserl realizes that he has to distinguish between consecutive 'tasks' (and accordingly 'levels' of theory construction) on the one hand, and alternative ways of doing logic or logical 'achievements' (basically what I called combinatorial and truth interest in sec. 3) on the other hand. For the former he usually speaks about '*Stufen*' (or '*Aufgaben*') of logic; for the latter he says '*Schichten*', '*Schichten der Leistungen*'. One could dispute the adequacy of these labels. On top of this, terminological confusion arises because Husserl sometimes mixes them up. But it is really not a worthwile problem to deal with, as long as one sees the substantive distinctions (the 'dimensions') behind it. And I see no reason to purify language usage beyond necessity (i.e. beyond the necessary disambiguations of H-terminology) by distinguishing between 'levels' and 'layers', especially when we can clarify things in a much less problematic way.

defining for the possible combinations of non-self-reliant syntactical forms into a larger unit, the judgement (and/or, ontologically, into the state of affairs[17] intended through it).

Ad (1). *'Apophantical grammar'*. Here the project of a logical or universal grammar had been touched upon, if only programmatically, long before Husserl (and has been taken up after him). Very briefly, what is *logical grammar* about? Before the compatibility of signitive constituents into the unity of a judgement can be meaningfully questioned in the sense of non-contradiction, another problem of compatibility must have been solved: do the given constituents (say, in a given word order) combine to a judgement, i.e. (speaking loosely) to a closed and separable unity of sense? If not, they do not even constitute a candidate for an evaluation as either contradictory or non-contradictory (let alone as possibly true). So for example *'All some presidents or'* is not a cognitively meaningful unit; it is an example of what Husserl calls *Unsinn*. In contrast, *'All presidents are liers, and this president is not a lier'* is a perfectly acceptable judgement from the point of view of logical grammar, although it is an example of what Husserl calls *Widersinn*.[18] Only constituent-strings endorsed as well-formed judgements (i.e. checked with respect to the opposition *Sinn/Unsinn*) are open to a further investigation as to their deductive relations within the total set of sentences (i.e. with respect to the opposition *Sinn/Widersinn*).[19] It is not hard to see that Husserl's distinction between the level of pure grammar and the level of derivability logic exactly corresponds to Carnap's distinction, made a few years later,

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17 It is most noticeable that for Husserl (more or less in the same way as for a number of philosophers, often those sensitive to the so-called 'slingshot argument' as Davidson termed it), *states of affairs* are not just 'out there'. To him they are constituted in the strong sense of being produced (viz. by the synthetic categorial activity of judging, by the synthetic categorial object called a jugdement). *Objects*, on the other hand, appearing in states of affairs, are not in this sense produced by synthetical acts.

18 *'All or some presidents'*, in turn, would have to be considered as a possibly meaningful constituent (of a meaningful judgement), but not as a closed and separable unit of meaning.

19 More precisely the given examples are (non-formalized instances of) formal *Unsinn* and *Widersinn* respectively. Furthermore Husserl makes the distinction between *formal* and *material Sinn/Unsinn/Widersinn* questions. Indeed there is material *Unsinn*, and material *Widersinn* as well; they are forms of 'synthetic' nonsense or countersense, each originating in questions of material logics (which implies that the reasons for falling short of criteria of sense are apriorical in their case as well). *'This colour plus one equals four'* is an example of material

between *formation rules* and *transformation rules* of a language.[20] The difference between Husserl and Carnap lies in the conception of the nature of such rules: for Carnap they are purely conventional, for Husserl at least the apophantical formation rules express the nature of given meaning relations, leaving little room for arbitrary variation. One can see Husserl's point by considering the range of application of his grammatical rules. To be sure, at the level of logical grammar of a language *two* entries are needed to define the language: the vocabulary of the language as well as the rules of combination (the formation rules). When taking the judgement as the entity to be studied, it seems that Husserl sees both the types of constituents making up a vocabulary — that is, the *Bedeutungskategorien* — *and* their admissible combinations as fixed (at least up to a high degree). This is, I suggest, because Husserl sees apophantic as having the primary task of reconstructing the logic of natural language (in so far as there are universals to be found in it); and the availabity either of basic types of categorematics in natural langage (such as subject term

Unsinn. This square is round would be an example of material not of formal *Widersinn*, since non-formalized geometry is synthetic for Husserl. Here's a summary, illustrating the view by means of some additional examples (without worrying about their appropriate formalizations):

	Unsinn	*Widersinn*
formal	*C not D if then*	*If some K are not-L, all K are L*
material	*Snow is prime*	*Some ellipses are not conics*

All of these husserlian doctrines (expounded in the IVth *Logical Investigation*) concerning criteria of meaningfulness are more or less inspired by meinongian and other queries originating in the 'austrian semantics' tradition, and have their bearing on classical questions in the philosophy of logic and language. (Of course these doctrines do raise a number of problems related to syntactical c.q. semantical categories; just think of the following disjunction problem: if '*Snow is prime*' is materially 'malformed', is '*Snow is prime or snow is white*' also malformed?)

20 *Logische Syntax der Sprache*, e.g. Part I, A. The similarity was seen by Y. Bar-Hillel [1970], 93-94. Note however, that although it is indeed the distinction of levels (1) and (2) of the 1929 H-logic, *syntactically interpreted* that is, which gives the exact analogue of Carnap's distinction, already the *Prolegomena* had isolated the level of logical grammar (level (1)), — without giving at that moment an exact analogue of Carnap's transformation rules (which required the distinction between syntax and semantics, as in dimension (ii), to be clearly defined).

and predicate term, nominalized predicate and the like), or of syncategorematics (*some*, *or*, *possibly*...) in natural language does not seem to result from arbitrary conventions. Neither the elements making up the vocabulary nor their authenticated combinations seem subject to arbitrary variation. H-Logical grammar seems to be intended as a discovery of the true formal structure of natural language sentences and their constituents (the logical form hidden behind the superficial analysis given by traditional ideas of grammar), an undertaking rather in the spirit of fregean/russellian theories of meaning.[21]

Ad (1'). *'Ontological grammar'*. Analogously it is the task of the lowest level of *Gegenstandslehre* to fix, first, the categories (the vocabulary) of the formal mathematical languages involved and, second, the formation rules permitting to combine these categories (the object categories) into well-formed formulas. Here language and conceptualizing not only are open to formalization (as in apophantic) but, as I think Husserl suggests, they belong to linguistic fragments extending the capacities of expression of natural language, or making this expression more precise. When intuitive content gets minimized in the end, these fragments organize within formal systems. Obviously the first stage in defining a formal system is to lay down its grammar. The choice of the elements of syntax of a formal system (and their combinations) for a formal mathematical theory seems to be less bound to 'pregivens' than in the case of natural language syntactical rules. Thus Husserl does not speak about 'judgements' here, or 'predication' etc. Does this mean that conventionalism is at home in this division of H-logic? That is a rather difficult question; it is safe to say however that in this field grammar is more open to diversity. But looking more closely at the *Gegenstandskategorien* and their mutual relations, it is at the same time plausible (H-plausible) to suppose that ontological grammar rests upon meaning relations which are also, to a large extent, given. Indeed, notions such as 'set' and 'element', to stick to the paradigm case, must appear to Husserl as relying on concepts (or operations such as collecting),

21 No doubt such a similarity obtains unwittingly. On the other hand a detailed study would show that in practice Husserl might propose rather strawsonian solutions to specific questions about logical form and contextuality, the applicability of excluded middle to concrete *Sinn/Unsinn*-cases, etc. See *FTL*, part II, chapter 4: 'Evidential criticism of logical principles carried back to evidential criticism of experience' (Cairns, 202-222, esp. §§ 87-89). (Though in cases like *'the present king of France'*, presumably he would have to be on Russell's side, if pressed to examine it according to his own criteria): there is no reason to suppose that the requirements of "*material affinity between syntactical nuclei*" are violated in such cases.

possibilities already naturally or intuitively open to the mathematizing mind; which is not very indicative of a conventionalist view. We have notions which are given in so far as they are expressions of ideas intelligible to and intelleged by the mind: in that sense knowledge interest not combinatorics is the guide, even in formal theorizing. *Formal theorizing* in the sense of theorizing about analytical a priori relations is not exactly synonymous with *formalizing*: just as in the case of apophantics, there is a level of meaningful concept formation preceding problems of formalization of these formal contents. When, for instance, Husserl says, his formal science is built up in a way so as to exclude the appearance of any paradoxes [*FTL*, Erg. Text III, 343], he might be thinking of excluding the formation of certain sign strings (say, such as '$x \in x$') as pseudo-propositions on such very grounds. Realizing however the gap between formal notions and the intuitive contents they are called upon to capture, he sees that the role of conventions is not escapable here. And since the purest variety of combinatorial thought is conventionalism, the distinction *(ii)* between truth logic and logical combinatorics is important when filling in the content of the boxes to be found on Husserl's 'levels', beginning from this basic level of the theory of forms.

A lot more would have to be said about (1') as well as about (1); for example about the way *object* categories often mirror corresponding *apophantical* categories *in detail*; and exactly which of the former give rise to a rigorous mathematical treatment. The former point, besides, would give a first element of the answer to the question how, technically, apophantics and ontology relate to each other; there must be more than points of contact between the two approaches of the formal. Without giving up each other's specificity, a certain kind of unification remains a *desideratum*, already on this technical level.[22] This point would become especially clear by showing two things: 1° how specific object categories arise out of specific meaning categories;[23] 2° how the *operational* character of

22 And the chapters 4 and 5 of *FTL* part I, which are purely philosophical in character, treat among other things the same question of how to conceive the unity of apophantics and ontology without reducing either to the other. But then the question is not treated as a technical problem relating to connections among strata within logical theory formation, but rather as a general question of what we would call philosophy of language: how do we come to have such things as grammatical categories; which of them are 'prior' in senses to be specified; why are two kinds of them needed?, etc.

23 To 'arise out of' can take at least two meanings here: 1° to originate by *thematizing* an apophantical syntactical notion or operation, as in the case of the

apophantical as well as ontological formation unifies the picture: every item (even simple predication) results from operating on (syntactical) forms in order to get more and more complex forms, the point of view of operations representing a direct loan from the Hankel-Grassmann kind of abstract mathematics. All this however would lead beyond the 'macroscopic' aims of this study; just as filling in exactly how the basic object category *Etwas-überhaupt* specifies into its successive 'derivatives' (set, element, ..., permutation, combination, etc.) would.

(2 + 2') *The level of derivability relations*. In the same way as the theory of forms described the way categories are available and to be combined by means of formation rules, the theory of derivability describes the way initial propositions (premises, axioms) lead to other well-formed propositions (conclusions, theorems) by means of transformation rules.

Ad (2). *Derivability apophantics*, or consequence logic, as Husserl says,[24] is really nothing else than a common label for all theories that are usually called (and have usually been called) logic in the ordinary sense of 'a theory about the validity of deductive patterns of inference'. It is a box where you can put, in principle, traditional syllogistics, as well as propositional and predicate logic in

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object category 'plurality', arising from the syntactic form of a plural (applied to categorematic terms), and 2° to *mirror* a corresponding but prior apophantical category (as in the case of the notion of 'state of affairs' with respect to the notion of 'statement').

24 Again: confusingly, because at the same time this term should also delineate H-logical combinatorics (at this level) from truth logic (at the same level), as we have seen (sec. 3). The same remark applies to the alternative designation Husserl uses for this level: analytics of non-contradiction. The reason Husserl sticks to these names is obvious. Indeed, what is decisive here is the fact of being a (presumedly syntactical) consequence, or of 'being included' of one sentence with respect to another. Alternatively, when of two sentences A and A' neither is 'included' in the other, either A and A' are 'neutral' regarding inclusion with respect to each other, or they 'exclude' each other. When two sentences exclude each other in this (syntactical) sense, that is another way of saying that they are mutually (syntactically) inconsistent: the determination 'analytics of non-contradiction' indicates that Husserl takes over the leibnizian (and traditional) view concerning the ultimate character of the principle of non-contradiction. Of course and perhaps unwittingly as far as Husserl is concerned, the semantical reading of the notions of consequence and consistency is equally well possible in these contexts. But that is exactly why I say it is necessary to distinguish dimension (ii) from (iii) in a more explicit and adequate way than Husserl himself does.

the modern way, and modal logics ancient or novel... The fact that different ways of formalizing can be involved (e.g. aristotelian or fregean) poses a problem for Husserl which already occurs at the grammatical level, comparing standard categorical proposition forms like '*All S are p*' with modern formalizations. It is not detrimental however, neither on the grammatical nor on the derivability level, as soon as one introduces at least a mimimal conventionalism about formal representations of languages, say a 'translation manual'. This minimum, I think, was accepted by Husserl, although presumably he did find that there should be a natural preferability ordering among different formalizations, the 'ultimate' logical form of sentences and inferences being unique.[25]

Ad (2'). *'Mathesis formalis'*. This is the level of formal mathematics *stricto sensu*, the level of theories made up when developing deductive chains on the basis of well-formed propositions involving only formal objects (or formal concepts pertaining to any object) like sets, numbers etc. Since we are completely in the realm of formal entities and their properties, we can forget about the restrictions typical of the predicative structure of natural languages now. Small wonder there is no immediately recognizable correspondence in the detail anymore between (2)-theories and (2')-theories, as there *was* among (1) and (1'): the proper object categories being acquired once and for all, we go on building on *them*. Indeed, it is even the case that, *within* formal ontology, immediate connections among developments based on separate object categories get looser as we go along: the several groups of deductive chains show the tendency towards clustering around theoretical kernels relatively isolated from each other, kernels we are able to axiomatize and we call *formal mathematical theories*. These relatively autonomous theories remain multiply interconnected of course, and from the logical point of view their connections are mainly revealed when curling up the Penelope's thread back to the basic categories they are talking about: these basic categories stood in logical relations of relative primitiveness or

25 It seems that Husserl saw modern treatments of deductive inference as essential improvements upon its traditional formal representations, while it is questionable whether he thought so with respect to judgement forms in natural language. Anyway, the uniqueness of logical form of a judgement has at least this meaning: to Husserl it is an apriorical question which constituents of a sentence should be kept fixed as 'categorematics' (nonlogical terms) and which are to be conceived as 'syncategorematics' (logical constants). But consider from a contemporary point of view: is 'previous' (in a sentence about the previous and the present prime minister of Belgium) a nonlogical constituent (as older conceptions would have it), or is it a logical constant (as in temporal logics)?

dependence, mutually and with respect to (their degree of lineage from) the original category of *Etwas-überhaupt.*

Again without going into details, it is easy to get an idea of the contents of box (2') by checking a list of theories Husserl includes:

- set theory
- formal analysis
- formal number theory
- vector spaces
- combinatorial analysis
- (presumably also) mereology[26]

(3 + 3') *Theory of deductive systems and theory of manifolds.* It is to Husserl's credit that, in 1900, he saw that the most interesting piece of theorizing in formal science still had to start when the two previous '*Aufgaben*' (Cf. note 17) would have been fulfilled. Indeed one of the ways Husserl introduces his idea of a third level is by noticing that questions about formalized derivation systems *as such* will have to be addressed, and that they are key questions pertaining to logic. What Husserl calls a theory of deductive systems (in apophantical terms), or alternatively a theory of manifolds (in ontological terms), clearly belongs to the level of *metatheoretical* investigations.

Ad (3). *A theory of formal-deductive systems.* Apophantics at this level is a 'theory of theories', a formal theory of formal theories. One way to introduce the transition to this metalevel is obvious: go one step beyond what you've been doing (setting up deductive theories) by taking it as a further object of discourse (theorizing about theories).

> The big step forward taken by modern mathematics [...] consists not only in having clarified the possibility of isolating the form of a deductive system [...] — it consists moreover in this, that mathematics went on to consider such system forms themselves as mathematical objects. [*FTL*, §30, 97 or Husserl 1978, 93]

We have to conceive another way of expressing the same is in terms of the opposition local/global:

> a logical discipline relating to the deductive sciences as deductive and considered as theoretical *wholes.* The earlier level[s] of logic had taken for [their] theme the pure forms of all significational formations that, as a matter of apriori possibility, can occur *within* a science: judgement-forms... argument-forms, proof-forms... Now

26 The subject of the IIId *Logical Investigation.*

> *judgement-systems in their entirety* become the theme-systems, each of which makes up the unity of a possible deductive theory. [Husserl 1978, 90]

What can a theory of systems have to say about these systems of judgements? We have to pose this question, because it is not immediately clear that Husserl could identify with hilbertian conceptions.[27] But they are rather close. H-metamathematics is certainly not only philosophical in nature, although its technical contents seem neither to rely on the same justificatory function as Hilbert's (consistency proofs as the overpowering objective) nor to be bound by the same finitistic doctrine.

> Besides the task of formally defining th[e] concept [*form of a deductive theory*], there is yet the endless task, of differentiating that concept, of projecting, in their explicitly systematic developed state, possible forms of deductive theories, but also of recognizing various deductive theory-forms of this sort as singularities of higher form-universalities. [*ibid.*, 92]

And that is not all. Besides the task of classifying theories belonging to all kinds of types (take as an example all formal theories belonging to a theory type with a fixed axiom $a \oplus b = b \oplus a$ for a given one of the operations involved), in the end the ambition of the theory of deductive systems is even to itself adopt the form of a systematic *theory*

> [in which] the particular determinate forms [are] subsumed under each of those higher form-universalities — and ultimately, under the highest idea, that of any theory-form, any deductive theory whatever.[28]

Whether Husserl is here talking about a theory in the most rigorous sense, i.e. whether metatheory itself could ever be a deductive axiomatic theory, remains in the dark.[29]

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27 The task of building up a theory of theory forms was clearly circumscribed already in 1900: see Husserl [1975], §§69ff.

28 This is an example given in *Prolegomena* §70 and in *FTL*, §28.

29 Compare: 'At this point there arises the idea of a universal task: to strive towards a highest theory, which would comprise all possible forms of theories (correlatively, all possible forms of manifolds) as mathematical particularizations — accordingly, as *deducible*' (*FTL*, §32, transl. adapted from Cairns, 98). While in *Prolegomena* Husserl still held the opinion that theory of systems could not be thus conceived: 'for fundamental principles in the strict sense cannot be given here' (§69, 249), in contrast to the construction of the theory of forms and derivability.

Anyway, when climbing up to level (3), one thing is certain: now we are really far out in the realm of convention. If rules and axioms can ever be subject to the game of variation, then this is a case in point. This has to do with the fact that, besides being defined, for one thing, by means of a metatheoretical viewpoint in the obvious reading of that term, and moreover by the idea of an ascent from the local to the global, the theory of theory forms is also characterized by the notion of completing the process of formalization. Formal science is driven by the aim of finding out what will be left when the last remnants of contentual thought will have been replaced by formal counterparts. Lift all intuitive constraints on the scope of mathematical operations, and you will discover what the really formal bounds on the latter are. The rationale is straightforward: only so can we get an idea of the global class of possible combinations, and of where combinations of any given type lead to. Could one coherently pose for a certain system of 'numbers' that $a \oplus b = - (b \oplus a)$? Yes, under specified conditions for the 'constituents' of hypercomplex '*a*, *b*' it is straightforward to formulate such an axiom (Hamilton). Does this mean, as for Carnap, that 'anyone can build his own language form the way he likes'? We must not forget that basic syntactical categories have to remain the building blocks. And truth logic is present at this level too, in order to sanction those combinations which prove 'useful'. So truth logic operates as a selector connecting some of those formal systems which have been ratified from the formal point of view, with real mathematical theories, (e.g. those involving abstract number concepts, as in the higher regions of analysis). Anyway, in the process, apophantics itself definitely has transgressed the border of a logic of natural language.

But the main thing is this: apart from the legitimate tendency to carry on the movement of formalization as long as this continuation proves possible, is there a special reason why we should play the variation game? There is a practical bonus to be gained here, beyond the sanction given by truth logic at this level. Indeed, however much logic differs from an *art de penser*, it is clear that the theory of systems could play the role both of a logic of discovery in the leibnizian sense, and of an instrument allowing to economize theoretical work. That's indeed what it does, by revealing connections between the seemingly unconnected (theories from highly remote origins and application showing unexpected structural similarities under the scrutiny of formal reformulation). Who would have prefigured the isomorphy between the intended models of the axiomatic theory of probability and of the theory of additive functions of sets? The theory of systems achieves such feats, at least if sufficiently developed. And it does so, while

> the stringency of deductive procedure depends only on the lawfulness of logical form, [and one may] forget about the conceptual content of the terms involved [...] one is dispensed from wondering whether it is numbers we are talking about, or forces, or energies, or light rays [Husserl 1984, 83]

In other words, once you know enough about the degree of *equiformity* — as Husserl calls it — of two theories, irrespective of their subject matters, a whole lot of the theoretician's job has already been done.

Ad (3'). *A theory of manifolds.* A manifold (*Mannigfaltigkeit*) in the husserlian sense is a notion directly in line with the nineteenth century rise of formalizing abstraction: it designates a purely formally defined domain of 'objects' ruled by abstract operations obeying certain general laws. As such it is the basic concept of the theory of manifolds (*Mannigfaltigkeitslehre*).[30] The latter, then, constitutes the third and highest layer of formal mathematics conceived as a formal *ontology*. The theory of manifolds is the investigation of the possible forms of object domains as such; that is, when objects of thought have been cleared (*entleert*) of the last remnants of intuitive content, which survived even in such notions as 'set' or 'number' (even in the formal sense),[31] there still remains something to be said about the form of a domain of objects as it appears in all formal mathematical theories alike.

It is clear that here again, on the object side, we have a move from the local to the global consideration or metalevel: instead of talk of objects we just look at the form of any collectivity to which any mathematical object could belong. And instead of defining the collectivity on the basis of the objects collected for the purposes of one theory, we go the other way around: whatever fields of objects may belong to specific mathematical theories, they are considered as mere instantiations of a general notion of a domain of objects now.

Now what is the relevance of making such a move on the ontological side? To answer this question convincingly a detailed study would be required; nevertheless here something like the key to

30 Both 'manifold' and 'theory of manifolds' are used in present day mathematics in a more narrow sense (see e.g. Torretti [1978], sec. 2.2.), which derives however in part from the same source (Riemann) as one of Husserl's usages.

31 Indeed for Husserl the notion of a set is still based upon an operation of collecting, which is not devoid of intuitive content; and more evidently, as is obvious to scholars of *Philosophie der Arithmetik* and of the studies in the sequel of that book, intuitive aspects are involved in his conception of (even the formal notion of) number.

the unity of the whole building of H-logic is to be found. It is possible to give a flavour of the answer by confining to the example of geometry here; while keeping in mind that it is but an example, at the same time it should become clear it is a *privileged* example.

Perhaps the reader will have noticed that nowhere in the list of mathematical notions and theories appearing in previous levels has there been any reference to geometry. Geometry didn't appear on level (2'); geometrical concepts were absent from level (1'). Why is that? Geometry is so much an example of intuitive, contentful thought for Husserl, that it can hardly be inserted in the list of piecemeal formalization levels. On the other hand it had been the subject of 'counterintuitive', formal treatments since Gauss and Riemann. Either geometry is meant in its original sense and then it is a study of real space, axiomatic or otherwise, but not formal: rather it is the material ontology *par excellence* for the sphere of rigorous knowledge of nature. Or else, geometry is taken in the sense of modern mathematics, and then it is only well understood when taken as a science of the *formal category* of a space, and not mistaken for a science of space itself. So the geometry apt to ever become part and parcel of formal science cannot be geometry in its initial sense, but only a formalized version or rather formal versions each developing different analytical[31] *a priori* components corresponding to the several possibilities left open when divesting the *a priori* of geometrical thought from its synthetic core. That something apriorical yet pertaining to concepts typical of spatial thought should remain when separated from this synthetic core is the discovery to be accounted for. This 'geometry' has to be integrated in formal mathematics all at once, as a formal encoding (formal geometry) of a whole parallel body of knowledge, readily developed as a contentual theory, coming from without. So this formal version of geometry should be integrated immediately at the top level, since it cannot and does not contain any intuitive element of the notion of space. It should be integrated in the theory of manifolds, which thus, as a coping-stone of formal abstraction, makes the link with material theory of science (theory of real space), paradoxical as that may seem at first sight. The notion of a category or form of a space is what then comes in place of the notion of space, — of *the* space we 'see and live in'. And the former notion, devoid of all intuitive evidence, can be nothing but

31 'Analytical' (as well as 'synthetic'), incidentally, has to be understood here both in the sense prevailing in the philosophy of logic, and in the sense which is standard in characterizing mathematical theories: 'analytical' geometry, 'analysis' (especially when conceived as the corpus of analytical methods launched to study geometrical problems, as in differential geometry).

the notion of a domain of objects in the generic sense, as I will now try to indicate briefly.[32]

To be brief, Husserl here invokes the whole 19th century development of formal mathematics, from *Hankel and Grassmann* on the one hand, as forerunners of abstract algebra (where properties of formal operations are taken to define a domain of objects conceived as any elements apt to be combined by the operator in such a way that the structure considered is closed under the operation, and the defining properties hold), to *Riemann* on the other, as founder of the specifically so-called theory of manifolds generalizing on problems from (differential) geometry and analysis. The idea of a theory of manifolds itself seems to draw mainly on the oldest of deductive-axiomatic disciplines. The *Elements* incorporate the ideal of a deductive science as such. But it is the 'purification' of geometrical thought in the course of the first metatheoretical researches (on independence) which finally led to the questioning of the initial euclidean content of the more general ideal. Husserl struggled for a long time to come to grips with these daring generalizations in the foundations of geometry, and in particular to fix his position on the relation between space concepts of n-dimensional 'manifolds' and non-euclidean geometries on the one hand, and physical space and intuitive space on the other. He finally settled for the view of radical formalization of the space-concept as a transformation of the euclidean ideal itself. To be more precise, when abstracting from the essentially material directedness of geometry, some core element remains intact: the euclidean prototype 'deductive theory as such'. The possibility of reducing essentials of euclidean thought like the parallel postulate to mere particular cases of analytical formulas about the degree of curvature evidences the presence of a purely formal element within geometry itself, however much the latter is a material mathematical discipline when practised according to its initial sense and finality as a study of space. (So Husserl wants to show, the geometrical theory of manifolds is the practise of geometry as an instance of the Hankel-Grassmann type of abstract algebra alluded to above, so to speak).

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32 The hypothesis I would be prepared to develop in a different context (in a study on 'Husserl's Use of Riemann's Notion of a Manifold', in preparation) concerning the relation between husserlian manifold theory and geometry is threefold: (1) one of the main functions to be served by the theory of manifolds was to clarify the relation between two kinds of mathematics, in Husserl's terms, between material and formal mathematics; (2) this was especially the case concerning the relation between material and formal geometry; (3) the latter distinction was so important for Husserl because it permitted him to fix his position on the status of non-euclidean and differential geometries.

So the idea of a form of a domain of objects thought to be in all kinds of abstract relations to each other, without invoking any of the ordinary metric or even more generally, projective or topological relations usually thought of as defining for geometrical relations *tout court*, is the correlate of a radically formalized axiomatic system. A particular pregiven domain possessing the familiar properties of space stands to the domain of formal geometry exactly as an axiom in the traditional sense of a basic truth about a particular domain stands to an axiom form about a domain form. They are pairwise equally far removed from each other. In this sense a space, when divested of its real patterns, is a mere manifold, a domain of arbitrary objects; objects apt to be filled in as numbers, as geometrical objects, as collections of either of them, etc. *The notions of manifold or object domain are nothing but the correlate of the apophantical notion of a theory form*: what is a theory form (a deductive system) supposed to talk about but an arbitrary domain of objects, and conversely, what is an arbitrary object domain but the residual object-constituent of a theory form?

So in the end apophantics and ontology meet again and join. Even if no more than a technical unification is involved, leaving open the philosophical question of relating apophantical and ontological intentionalities, isn't it a remarkable fact that the coping-stone of H-formal science is a level where *mathesis universalis* is defined as a theory 'spanning both formal logic and formal mathematics'? [*FTL*, Ergänzender Text II, 342] In fact nothing can distinguish the approach in terms of theory forms from the approach in terms of object domains beyond the fact that each reflects a different origin (apophantical focus on propositional systems, versus ontological interest in formal objects) and a different direction of application (relating formal systems to linguistic structures involved in knowing, versus relating formal object systems to material theories incorporating knowledge of the world). In all other respects theory of deductive systems and theory of manifolds are intrinsically equivalent; they translate each other's notions and concerns.[33] Given the fact that a *reine Mannigfaltigkeit* is nothing beyond a sphere of objects by definition to be conceived as all and any things satisfying the apophantically given form of a theory, there is nothing here like a separate theory on these objects extending the sphere of particular mathematical ontologies (set theory, combinatory analysis...) already fixed on the previous level. In contrast to the previous level, where those formal mathematical theories were really novel and separate

33 This means that all those interpretations privileging either an apophantical reading (Cavaillès, Miller) or, more often, an ontological reading of the top level are unsatisfactory in this respect.

with respect to apophantical counterparts not exactly correlated to them, it is precisely characteristic for the top level that the theory of manifolds cannot add any new information to whatever could be learnt in apophantical metatheory. Since the same reasoning would have to hold in the opposite direction, one cannot but conclude that we need the top level to see how apophantical and ontological theories technically converge to make apparent the unity of formal science 'from above'. The special additional use of a theory of manifolds consists merely in the epistemologically crucial link to material mathematics. Without going into the matter, I just want to add the suggestion that the conception of the ideal mathematical object domain as a *definite manifold* is there to stress this point. It is being invoked as a warrant, a warrant that nothing crucial from the point of view of rigorous knowledge should get lost in the drastic process of formalizing its contents. To quote from Dieter Lohmar, who most clearly expressed this insight:

> What it is that [Husserl wants to] capture here, is a kind of exceptional case within the set of eidetic sciences possessing a material principle of unity [i.e. disciplines that can be practised according to the non-formal principles derived from the nature of their proper domain]. In the range of eidetic disciplines — which include a.o. the apriorical part of physics, of chronometry and of the theory of motion, as well as of acoustics and colour optics, etc. — the fields of geometry and also of mathematical physics represent such exceptions. It is hardly the typical case that disciplines with a material principle of unity are apt to be axiomatically reformulated and formalized without leaving any structural features out of the picture. [Lohmar 1989, 188] (my translation).

Indeed, from the point of view of theory construction it would become irrelevant whether we know and understand the nature of the specific object domain we are dealing with at a certain moment, once we have the assurance that it is a definite manifold: even more than a knowledge of the degree of equiformity of two theories (supra, (3)), a proof of definiteness would represent an enormous advancement permitting us to discover and ascribe a lot of properties to entities the grasp of which (for the time being) remains merely or primarily formal. So the need for an husserlian way of understanding the status of the new geometries and other formal devices of scientific thought was one of the factors that led to the articulation — possibly for the first time in the history of formal thought in such an explicit way — of an ideal of completeness which was to be deceived, at least in one way of giving it a precise content, just a few years later.

6. Some Questions and a Vista

Some surprisingly modern features of Husserl's views on logic and formal science might have come to light. Leaving aside the more familiar phenomenological requirement of 'subjective' foundations by means of intentional analyses, it is undeniable that Husserl had (and kept) a keen interest in 'objective' logic. As a first feature, it is striking how a certain kind of double orientation of formal studies – towards linguistic ('apophantic') as well as towards object-oriented ('ontological') aspects is anticipated in Husserl's dimension *(i)*. Second, although unaware of course of the subsequent tarskian and gödelian facts, Husserl claimed a special role for a *semantical* consideration of the formal (although he did not see any technical surplus value of the latter point of view, in the sense indicated by the relevant metatheorems). On the other hand, he sticked to the view of the *syntactical*, combinatorial, algorithmic mode as *the* essential characteristic of formal thought (dimension *(ii)*). Third, there is a broad view of *levels* of logical investigation, permitting to see its connections with the idea of universal grammar at the base level, with a metatheoretical field of investigations at the top (dimension *(iii)*).

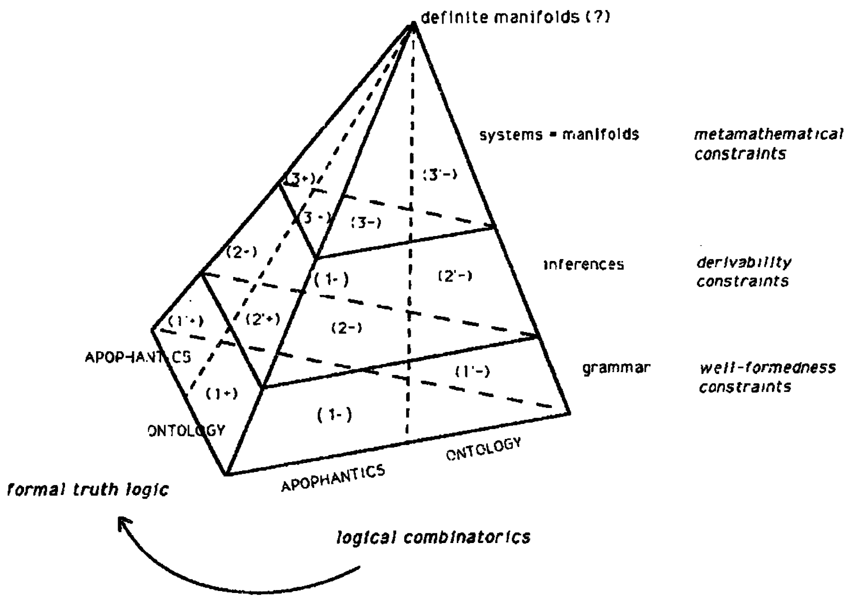
On the other hand, a lot of problems remain open (or are just opened up) at the end of this exploration of the edifice of Husserl's logic. To cite just a few examples: in how far is it possible to reformulate without remainder some of Husserl's distinctions in terms of modern logic as we conceive of it today? In particular, how far did Husserl follow Hilbert's efforts of formalization, and at which point did he stick to his own, older views concerning what has to count as 'formal' in formal science? Even more to the point: doesn't Husserl's notion of deductive system stand half the way between merely '*abstract* axiomatics' and the subsequent notion of a *'pure' formal system*? Further questions concern of course the notion of definiteness,[34] and the relation between formal and material mathematics,[35] as well as details of his edifice. Whereas all of these questions require further (and critical) inquiries, I think the most elegant way of summarizing the results of this preliminary journey is

34 A notion, by the way, for which Husserl could claim a priority, as recognized a.o. by Zemerlo (who made use of it in some variant connotation in the context of what he called 'definite properties' in axiomatic set theory).

35 Here we have questions such as: Will Husserl be bound to adhere to a Poincaré-type philosophy of (formal) geometry? Will he be forced to an instrumentalist interpretation of general relativity?, etc.

to give, literally, a picture of the structure of H-logic in the large, as it appears at the macro-stage of inquiry here presented.

Combining the levels of sec. 5 with the distinction of truth logic (x +) versus truth bracketing logical combinatorics (x -), as well as with apophantical (x) versus ontological (x') branches, we get the promised three-dimensional visualization:



Legend

	Logic	
	as combinatorics	*in formal truth interest*
Logical grammar	(1 -)	(1 +)
Ontological grammar	(1'-)	(1'+)
Derivability logic	(2 -)	(2 +)
Formal math. theories	(2'-)	(2'+)
Theory of systems	(3 -)	(3'+)
Theory of manifolds	(3'-)	(3'+)

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