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Corrections to "How to explicitly solve a Thue–Mahler equation"

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In Appendix A3 of our paper we make use of Corollary 2 of [BGMMS]. This paper [BGMMS] contains a serious misprint, as was pointed out to us by Professor A. Baker, to whom we are grateful. Namely, from the lower bound for $|\Lambda|$ in [BGMMS, Corollary 2] a rather substantial factor n^{2n+1} is missing, as was confirmed to us by the authors of [BGMMS]. As a consequence, our constant c_7, defined on p. 284, should be multiplied by m^{2m+1}.

It will be clear that a larger upper bound for B can be derived from the corrected result of [BGMMS], and that the general method of our paper is insensitive to the actual value of the constants. However, in any particular example the computations do of course depend on the correct value of the constants. Therefore we have to reconsider the details of our example, treated in the $^{\mathrm{Ex}}$-sections.

It follows that a correct value for c_7 is 7^{15} times the value given in the paper, thus $c_7 = 1.08672 \times 10^{46}$. We computed that with $c_6 = 0.6$ this yields an optimal value for the upper bound for B, namely $c_{\mathrm{real}} < 1.511 \times 10^{50}$. This is only slightly worse than the value given in Section 11^{Ex}.

Fortunately the correct value for c_{real} is small enough to do the first p-adic and real reduction steps with the same lattices as used in the paper, i.e. without having to do any new computations. Namely, in Section 15^{Ex} we now have $K_0 = N_0 = 1.511 \times 10^{50}$, and with $W_1, \ldots, W_6$, m unchanged the condition of Proposition 15 is again fulfilled in all cases, and hence $N_1 = 1153$ still holds.

In Section 16^{Ex} we now take $K_0 = 1.511 \times 10^{50}$, and $N_1, W_1, \ldots, W_5, C$ unchanged. Now $R = 3.023 \times 10^{50}$, $S = 6.591 \times 10^{100}$, and with $c_{16} = 0.0388479$ the condition of Proposition 16 is fulfilled in all cases. This leads to $H \leqslant 4919$. Thus certainly the old bound $H < 9.844 \times 10^{49}$ holds, and the reduction procedure as described in the paper shows that the gap in the proof has been fixed.

However, we can do even better. Namely, recently A. Baker and G. Wüstholz proved a new lower bound for linear forms in logarithms of algebraic numbers, which is consideraby sharper than the one given in our Appendix A3. This result will be published shortly in [BW], and we are grateful to Professor Baker for communicating it to us.

Using this new result we found that we can take $c_7 = 2.2044 \times 10^{38}$, $c_8 = 0$. This leads, by taking $c_{16} = 10^{-9}$, again to $c_{\text{real}} < 9.844 \times 10^{49}$, which is exactly the upper bound found in the paper. This shows that the reduction procedure as worked out in the Section 15^{Ex} and 16^{Ex} is in fact adequate to prove the main result on our particular Thue–Mahler equation.

Finally we note the following minor misprints:

- page 228, lines -3 and -6: $|N\ldots|$ should be $|N\ldots|_p$,
- page 283, lines -3 and -4: all $\geqslant$ and $\leqslant$ should be $=$ symbols,
- page 286, line 3 below the table: 2.289×10^3 should be 2.289×10^{33}.

References

[BGMMS] J. Blass, A.M.W. Glass, D.K. Manski, D.B. Meronk and R.P. Steiner: Constants for lower bounds for linear forms in logarithms of algebraic numbers II. The homogeneous rational case, *Acta Arithmetica* **55** (1990) 15–22.

[BW] A. Baker and G. Wüstholz: Logarithmic forms and group varieties, *Journal für die reine und angewandte Mathematik*, (1993) to appear.