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ALGEBRAIC WEYL SYSTEM AND APPLICATION

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ABSTRACT. -In order to define an l-sheaf due to [BZ 76] on the finite-dimensional p-adic symplectic space, we define an algebraic Weyl system, and its properties are investigated. In particular, we prove some necessary and sufficient conditions for Weyl system to be irreducible. As application, we give another proof of the Ston-Von Neumann Theorem of the p-adic Heisenberg group. From the Schrödinger representation associated to a selfdual lattice, we construct a Weyl system depending on a selfdual lattice and a p-adic valued function.

1. Introducton. First we begin with the notations: Let $\mathbb{N}_0$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{T}$ be the set of non-negative integers, the ring of integers, the rational number field, the real number field, the complex number field and the set of complex numbers of modulus 1, respectively. The field of p-adic numbers $\mathbb{Q}_p$ are constructed as follows: For a fixed prime number p, the p-adic valuation $|\cdot|_p$ on $\mathbb{Q}$ is defined in the following way. At the first, we define it for natural numbers. Every natural number n can be represented as the product of prime numbers $n = 2^{v_2}3^{v_3}\cdots p^{v_p}\cdots$. Then we define $|n|_p = p^{-v_p}$, we set $|0|_p = 0$ and $|-n|_p = |n|_p$. We extend the definition of p-adic valuation $|\cdot|_p$ to all rational numbers by setting for $m \neq 0$, $|n/m|_p = |n|_p/|m|_p$. The completion of $\mathbb{Q}$ with respect to the metric $d_p(x,y) = |x-y|_p$ is a locally compact field $\mathbb{Q}_p$. The p-adic valuation satisfies the strong triangle inequality

$$|x+y|_p \leq \max(|x|_p, |y|_p). \tag{1.1}$$

Any $x \in \mathbb{Q}_p$ can be expressed as $x = p^v \sum_{j=0}^{\infty} a_j p^j$ with $v \in \mathbb{Z}$ and $a_j \in \mathbb{Z}$ satisfying $0 \leq a_j \leq p-1$, $a_0 \neq 0$. To define the Fourier transform, an additive character $\chi_p(\lambda x) = \exp(2\pi i\{\lambda x\}_p)$ for every fixed $\lambda \in \mathbb{Q}_p$ on $\mathbb{Q}_p$ is used. Here $\{x\}_p = p^v \sum_{j=0}^{-v-1} a_j p^j$ is the fractional part of x.

In the Hilbert space $L_2(\mathbb{Q}_p)$ of $\mathbb{C}$-valued square integrable functions on $\mathbb{Q}_p$, we introduce the standard inner product and the norm

$$(\psi,\varphi) = \int_{\mathbb{Q}_p} \psi(x)\overline{\varphi(x)}dx, \quad ||\psi||^2 = (\psi,\psi), \tag{1.2}$$

where dx is the Haar measure on $\mathbb{Q}_p$ such that the volume of the ring of p-adic integers $\mathbb{Z}_p$ is 1.

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The n-dimentional p-adic space $\mathbb{Q}_p^n$ has the standard norm $|x|_p = \max_{1\leq j\leq n}|x_j|_p$, $x = (x_1, x_2, \cdots, x_n) \in \mathbb{Q}_p^n$. The Fourier transform is defined with respect to the character $\chi_p((k,x)) = \prod_{j=1}^n \chi_p(k_j x_j)$, where $(k,x) = \sum_{j=1}^n k_j x_j$.

It is well known that, traditionally, we use ordinary real numbers in theoretical and mathematical physics, since lengths of segments and angles etc. from Archimedean axiom should be measured precisely. However in quantum gravity and in string theory it was proved that a measurement of distances smaller than the Planck length (it is the smallest distance that can be measured, approximately 10^{-33}cm) is impossible. Vladimirov and Volovich [VV 84] proposed to consider the superanalysis and corresponding supersymmetric field theories not only over the field $\mathbb{R}$ but also over the field $\mathbb{Q}_p$ and other locally compact fields. The interest in physics of n.a. quantum models is based on that the structure of space-time for very small distances less than Planck length might conveniently be described by n.a. numbers. There are different mathematical ways to describe this violation of the Archimedean axiom. One of them is given by p-adic analysis.

Vladimirov and Volovich [VV 89] proposed a formalism of the p-adic quantum mechanics with $\mathbb{C}$-valued functions (cf. see also [FO 88], [RTVW 89] and [VVZ 94]). This formalism is based on a triple $(L_2(\mathbb{Q}_p), W(z), U(t))$. Here, $L_2(\mathbb{Q}_p)$ is the Hilbert space of $\mathbb{C}$-valued square integrable functions on $\mathbb{Q}_p$, $W(z)$ is the Weyl representation of the commutation relations, z is a point in the classical phase space and $U(t)$ is the time evolution operator where the time t is a p-adic number. The proposed formalism was extended, by Zelenov [Zel 91,92,93,94], to the case of many- and infinite-dimensional quantum mechanics, in which notion of Weyl system (H, W) on the p-adic symplectic space (V, B) was used, and the representation theory of the p-adic Heisenberg group was investigated (cf. see also [Meu 91]).

We recall that the definition of *l-sheaf* on the *l-space* by Bernshtein and Zelevinskii [BZ 76, pp. 6–9] which is an introduction to the representation theory of p-adic groups: A topological space X is said to be an l-space if it is Hausdorff, locally compact, and zero-dimensional. Denote by $C^\infty(X)$ and $S(X)$ the space of all locally constant $\mathbb{C}$-valued functions on X and the space of Schwartz-Bruhat functions on X, respectively. We say that an *l-sheaf is defined on* X if with each $x \in X$ there is associated a $\mathbb{C}$-vector space $\mathcal{F}_x$ and there is defined a family $\mathcal{F}$ of cross-sections (that is, mappings φ defined on X such that $\varphi(x) \in \mathcal{F}_x$ for each $x \in X$) such that the following conditions hold:

(1) $\mathcal{F}$ is invariant under addition and multiplication by functions in $C^\infty(X)$.

(2) If φ is a cross-section that coincides with some cross-section in $\mathcal{F}$ in a neighbourhood of each point, then $\varphi \in \mathcal{F}$.

(3) If $\varphi \in \mathcal{F}$, $x \in X$, and $\varphi(x) = 0$, then $\varphi = 0$ in some neighbourhood of x.

(4) For any $x \in X$ and $\xi \in \mathcal{F}_x$, there exists a $\varphi \in \mathcal{F}$ such that $\varphi(x) = \xi$.

The l-sheaf on X is denoted by $(X, \mathcal{F})$. The spaces $\mathcal{F}_x$ are called *stalks*, and the elements of $\mathcal{F}$ *cross-sections of the sheaf.* We call the set supp $\varphi = \{x \in X : \varphi(x) \neq 0\}$ the support of the cross-section $\varphi \in \mathcal{F}$. Condition (3) guarantees that supp φ is closed.

A cross-section $\varphi \in \mathcal{F}$ is called *finite* if supp φ is compact. We denote the space of finite cross-sections of $(X, \mathcal{F})$ by $\mathcal{F}_c$. It is clear that $\mathcal{F}_c$ is a $S(X)$-module, and that $S(X)\mathcal{F}_c = \mathcal{F}_c$. It turns out that this property can be taken as the basis for the definition of an l-sheaf.

Proposition 1.1 (cf. [BZ 76, Proposition 1.14]). *Let M be a $S(X)$-module such that $S(X)M = M$. Then there exists one and up to isomorphism only one l-sheaf $(X, \mathcal{F})$ such that M is isomorphic as an $S(X)$-module to the space of finite cross-sections $\mathcal{F}_c$.*

Proposition 1.1 means that defining an l-sheaf on X is equivalent to defining an $S(X)$-module M such that $S(X)M = M$.

In this paper, in order to define an l-sheaf $(V, \mathcal{F})$ on the finite-dimentional p-adic symplectic space (V, B), we define an algebraic Weyl system (H, W) on (V, B), and its properties are investigated, and we give an application.

This paper is organized as follows: In §2, we summerize the general properties of Weyl systems (H, W) on (V, B). In §3, we introduce the concept of an algebraic Weyl system (H, W) on (V, B), and prove some necessary and sufficient conditions for Weyl system (H, W) to be irreducible. In §4, as application, we prove the Stone-Von Neumann theorem. In §5, we construct a Weyl system depending on a selfdual lattice and a $\mathbb{Q}_p$-valued function. When it is irreducible, our construction coincides with the one of Zelenove [Zel 94].

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2. The general properties of Weyl systems. We review some of well-known results of Weyl systems (H, W) on the finite dimensional p-adic symplectic space (V, B). For proofs and more details, see [Zel 91] and [VVZ 94] (cf. for the ∞-dimensional case, see [Zel 92] and [Zel 94]).

For simplicity we assume that $p \neq 2$. By the definition of a p-adic symplectic space is the pair (V, B), where V is a finite demensional $\mathbb{Q}_p$-vector space and B is a nondegenerate antisymmetric $\mathbb{Q}_p$-bilinear form on V. Then $\dim_{\mathbb{Q}_p} V = 2n$ is even (cf. for the ∞-dimensional case, see [Zel 94, p. 423]).

Given $0 \neq e_1 \in V$, there must exist a $x \in V$ for which $B(e_1, x) \neq 0$, since B is nondegenerate. We choose a $a \in \mathbb{Q}_p$ so that $e_{n+1} = be_1 + ax$, and $B(e_1, e_{n+1}) = aB(e_1, x) = 1$. Then the hyperbolic plane $h_y = \operatorname{span}\{e_1, e_{n+1}\}$ has matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ with respect to the basis $\{e_1, e_{n+1}\}$. Since h_y is nondegenerate (i.e., the pair $(h_y, B|_{h_y \times h_y})$ is p-adic symplectic space), we have $V = h_y \oplus h_y^{\perp}$, where $h_y^{\perp}$ is also nondegenerate. Hence, we repeat the preceding construction in $h_y^{\perp}$, to obtain an orthogonal decomposition of V of the form

$$V = h_y^1 \oplus h_y^2 \oplus \cdots \oplus h_y^n, \tag{2.1}$$

where each h_y^j is a hyperbolic plane. Thus there is a basis $\{e_j : 1 \leq j \leq 2n\}$ for V for which the matrix of the form is $\begin{pmatrix} 0 & E_n \\ -E_n & 0 \end{pmatrix}$, where E_n is the $n \times n$ unit matrix, and 0 is the $n \times n$ null matrix. Also the map $P_j : V \to h_y^j$, $j = 1, \cdots, n$, which is defined by the formula

$$P_j x = B(e_j, x)e_{n+j} + B(x, e_{n+j})e_j, \ x \in V, \tag{2.2}$$

is the orthogonal projection map on h_y^j.

Let X_0 be a $\mathbb{Z}_p$-span of the symplectic basis $\{e_j : 1 \leq j \leq 2n\}$. Then X_0 is an open compact $\mathbb{Z}_p$-submodule of V and has the following properties:

$$B(x,y) \in \mathbb{Z}_p \ \forall x, y \in X_0, \quad \forall x \in V \backslash X_0, \ \exists y \in X_0 \text{ such that } B(x,y) \in \mathbb{Q}_p \backslash \mathbb{Z}_p. \tag{2.3}$$

Zelenov [Zel 91] defined a *Weyl system* on (V, B) as a pair (H, W) of a complex Hilbert space H and a continuous map $W : x \mapsto W(x)$ from V to the family of unitary operators on H satisfying the condition (the Weyl relation)

$$W(x)W(y) = \chi_p(B(x,y)/2)W(x+y). \tag{2.4}$$

Two Weyl systems (H, W) and (H', W') on (V, B) are *unitary equivalent* if there exists an intertwining unitary operator $U : H \to H'$. I.e. U is an unitary operator such that

$$UW(x)U^{-1} = W'(x), \ x \in V. \tag{2.5}$$

A Weyl system (H, W) is said to be *irreducible* if there exists no non-trivial subspace of H invariant under the $W(x)$, $x \in V$. We say that (H, W) can be *represented as a direct sum* $\oplus_{j \in I}(H_j, W)$ *of Weyl systems* (H_j, W) if H can be written as a direct sum $\oplus_{j \in I} H_j$ of subspaces H_j which are invariant under the action of operators $W(x)$.

We denote by $V_0 = \{x \in V : |x|_p \leq 1\}$ the open compact subgroup of V. Let (H, W) be a Weyl system on (V, B). A vector $\varphi_0 \in H$ is called a *vacuum vector* of (H, W) if the condition $W(x)\varphi_0 = \varphi_0$ is satisfied for all $x \in V_0$. The set of vacuum vectors of (H, W) forms the vacuum subspace H_0 of H.

Every Weyl system (H, W) on (V, B) is, in a certain sense, determined by its restriction $(H, W|_{V_0})$ on $(V_0, B|_{V_0 \times V_0})$. Since V_0 is a compact abelian group, all irreducible unitary representations of V_0 are one-dimensional. Let $V_0^* = \mathrm{Hom}(V_0, \mathbb{T})$ be the group of characters of V_0. Then we have

$$V^*/V_0^{\perp} \cong V/V_0 \ni \hat{\alpha} \overset{\sim}{\longmapsto} \hat{\alpha}^* \in V_0^*, \ \ \hat{\alpha}^*(x) := \chi_p((\alpha, x)), \ x \in V_0, \tag{2.6}$$

where α is a representative of the coset $\hat{\alpha} \in V/V_0$. By the theory of unitary representations of compact groups, the representation space H can be expressed as an orthogonal sum

$$H = \oplus_{\hat{\alpha} \in V/V_0} H_{\hat{\alpha}}, \tag{2.7}$$

where $H_{\hat{\alpha}}$ is the maximal subspace on which V_0 acts as a multiple of $\hat{\alpha}^*$.

Let us choose an element α from each coset $\hat{\alpha} \in V/V_0$ and denote the family of such elements by J_0. Let

$$\Sigma = \{\varphi_\alpha = W(\alpha)\varphi_0 \in H : \alpha \in J_0\}. \tag{2.8}$$

If α_1 and α_2 belong to the same coset $\hat{\alpha} \in V/V_0$, then, using (2.4), the definition of the vacuum vector and (2.8), we obtain

$$\varphi_{\alpha_1} = W(\alpha_1)\varphi_0 = W(\alpha_2 + (\alpha_1 - \alpha_2))\varphi_0 = \chi_p(B(\alpha_1, \alpha_2)/2)\varphi_{\alpha_2}.$$

Therefore, a change of the element α in $\hat{\alpha}$ induces only a scalar multiplication of φ_α. We call Σ *the system of coherent states* of (H, W).

The investigation of Weyl systems on p-adic symplectic spaces is essentially based on the notions of vacuum vector and the system of coherent states as follows:

Theorem 2.1 (cf. [Zel 91]). *Weyl systems has the following properties:*

(i) *For any Weyl system, there exists the vacuum vector.*

(ii) *A Weyl system (H, W) is irreducible if and only if its vacuum subspace H_0 is one-dimensional. Otherwise, if we choose some orthonormal basis $\{\varphi_0^i\}_{i\in I}$ in H_0, (H, W) can be represented as a direct sum $\oplus_{i\in I}(H_i, W)$ of irreducible Weyl systems (H_i, W), where subspace H_i is the span of the vectors $\{\varphi_\alpha^i = W(\alpha)\varphi_0^i : \alpha \in J_0\}_{i\in I}$.*

(iii) *If (H, W) is irreducible Weyl system with the vacuum vector φ_0, then the system of coherent states Σ of (H, W) forms the orthonomal basis in H.*

(iv) *All irreducible Weyl systems are unitary equivalent.*

Example 2.2 (cf. [VV 89] and [Zel 89, in case of $p = 2$]). For 2-dimensional case, an irreducible Weyl system is constructed as a pair $(L_2(\mathbb{Q}_p), W)$ on $(\mathbb{Q}_p^2, B)$, where the unitary operator $W(z)$ is defined by

$$W(z)\varphi(x) = \chi_p(px + pq/2)\varphi(x + q), \quad z = (q, p) \in \mathbb{Q}_p^2, \ \varphi \in L_2(\mathbb{Q}_p),$$

the symplectic form $B : \mathbb{Q}_p^2 \times \mathbb{Q}_p^2 \to \mathbb{Q}_p$ is given by $B(z, z') = qp' - q'p$ ($z' = (q', p') \in \mathbb{Q}_p^2$). The vacuum vector has the form $\varphi_0(x) = \Omega(|x|_p)$, where $\Omega(x)$ is 1 if $0 \le x \le 1$ and 0 if $x > 1$.

Example 2.3 (cf. [VVZ 94, p. 243]). We denote the tensor product $\otimes^n(L_2(\mathbb{Q}_p), W)$ of n irreducible Weyl systems of Example 2.2 by $(L_2(\mathbb{Q}_p^n), W^{(n)})$ on $(\mathbb{Q}_p^{2n}, B)$. Hence

$$W^{(n)}(z) = \otimes_{j=1}^n W(z_j), \ z = (z_1, \cdots, z_n) \in \mathbb{Q}_p^{2n}$$

and the vacuum vector has the form

$$\varphi_0^{(n)}(x) = \Omega(|x|_p) = \prod_{j=1}^n \Omega(|x_j|_p), \ x = (x_1, \cdots, x_n) \in \mathbb{Q}_p^n.$$

On the group V, we normalize the Haar measure dx by the condition $\int_{V_0} dx = 1$. In the Hilbert space $L_2(V)$ of $\mathbb{C}$-valued square integrable functions on V, the standard inner product and the norm are given by (1.2).

We define a Hilbert subspace $L_2^{\chi_p}(V_0)$ of $L_2(V)$ by

$$L_2^{\chi_p}(V_0) = \{f \in L_2(V) : f(x + y) = \chi_p(B(x, y)/2)f(x) \ \forall y \in V_0\}, \tag{2.9}$$

and an operator $\widetilde{W}(x)$ by

$$\widetilde{W}(x)f(y) = \chi_p(B(x,y)/2)f(y-x) \quad (x \in V,\ f \in L_2^{\chi_p}(V_0)). \tag{2.10}$$

Example 2.4 (cf. [Zel 91] and [VVZ 94, p. 243]). The pair $(L_2^{\chi_p}(V_0), \widetilde{W})$ is an irreducible Weyl system on (V, B), and the vacuum vector has the form $\varphi_0(x) = \Omega(|x|_p)$.

Let $Sp(2n, \mathbb{Q}_p)$ be the group of automorphisms of the space (V, B) that preserve the symplectic form B. We fix a basis of V and express any $g \in Sp(2n, \mathbb{Q}_p)$ by a matrix $(g_{jk}) \in M_{2n}(\mathbb{Q}_p)$. Let

$$||g||' = \max\nolimits_{1 \leq j,k \leq 2n} |g_{jk}|_p. \tag{2.11}$$

Then $G = \{g \in Sp(2n, \mathbb{Q}_p) : ||g||' = 1\}$ forms a maximal compact subgroup of $Sp(2n, \mathbb{Q}_p)$.

Theorem 2.5 (cf. [Zel 91]). *Let (H, W) be an irreducible Weyl system on (V, B). Then a family of operators $\{U(g) : g \in G\}$ satisfying*

$$U(g)W(x) = W(gx)U(g), \quad x \in V \tag{2.12}$$

forms an unitary representation of G in H, and any vacuum vector of (H, W) is an eigenvector of the $U(g)$.

3. Algebraic Weyl system. In this section, in order to define an l-sheaf $(V, \mathcal{F})$ on (V, B), we define an algebraic Weyl system, and prove some necessary and sufficient conditions for Weyl systems (H, W) on (V, B) to be irreducible.

Let (H, W) be a Weyl system on (V, B) with a vacuum vector $\varphi_0 \in H_0$ and $S(V)$ the space of Schwartz-Bruhat functions on V. Then the convolution product

$$f \hat{*} g(x) = \int_V \chi_p(-B(x,y)/2)f(y)g(x-y)dy \tag{3.1}$$

makes $S(V)$ an associative $\mathbb{C}$-algebra without the unit element. For each $f \in S(V)$, we define a linear endomorphism $E_W(f)$ of H by

$$E_W(f)\varphi = \int_V f(x)W(x)\varphi\, dx, \quad \varphi \in H. \tag{3.2}$$

Since f is locally constant on V with compact support, this integral is well-defined. It is easy to see that $E_W(f \hat{*} g) = E_W(f)E_W(g)$. Indeed, for $f,\ g \in S(V)$ and $\varphi \in H$, using the Weyl relation (2.4) and (3.2), we get

$$\begin{aligned}
E_W(f \hat{*} g)\varphi &= \int_V \int_V \chi_p(-B(x,y)/2)f(y)g(x-y)W(x)\varphi\, dxdy \\
&= \int_V \int_V \chi_p(-B(t,y)/2)f(y)g(t)W(t+y)\varphi\, dtdy \\
&= \int_V g(t) \left(\int_V f(y)W(y)W(t)\varphi\, dy \right) dt \\
&= \int_V g(t)E_W(f)W(t)\varphi\, dt = E_W(f)E_W(g)\varphi.
\end{aligned}$$

Hence H is a $S(V)$-module.

Definition 3.1. a) Weyl system (H, W) is said to be *algebraic* if there exists an open compact subgroup K of V such that $W(x)\varphi = \varphi$ for all $\varphi \in H$ and $x \in K$. For some open compact subgroup K of V, let $H^K = \{\varphi \in H : W(x)\varphi = \varphi \ \forall x \in K\}$ be the subspace of K-invariant vectors in H. Then it is clear that (H^K, W) is an algebraic Weyl system, called the algebraic part of W. In particular, $(H_0 = H^{V_0}, W)$ is the algebraic part of W.

b) Weyl system (H, W) is said to be *admissible* if it is algebraic and if for each open compact subgroup K of V, H^K is finite dimensional.

Algebraic Weyl system gives an l-sheaf on (V, B) as follows:

Proposition 3.2. *Weyl system (H, W) is algebraic if and only if $S(V)H = H$.*

Proof. Let (H, W) be an algebraic. I.e. any $\varphi \in H$ is fixed by some open compact subgroup, say K, of V. Let $\xi_K = \text{volume}(K)^{-1}\times$ characteristic function of K. Then $E_W(\xi_K)\varphi = \varphi$. Conversely, let $S(V)H = H$. We can construct an open compact subgroup K of V such that $W(x)\varphi = \varphi$ for all $\varphi \in H$ and $x \in K$ as follows: Let $\varphi \in H$. Then φ can be written as $\varphi = \sum_i E_W(f_i)\varphi_i$. Then, using the linearity of W, (3.2) and the Weyl relation (2.4), we have

$$\begin{aligned} W(x)\varphi &= W(x)\sum_i E_W(f_i)\varphi_i = \sum_i W(x)E_W(f_i)\varphi_i \\ &= \sum_i W(x)\int_V f_i(y)W(y)\varphi_i \, dy = \sum_i \int_V f_i(y)W(x)W(y)\varphi_i \, dy \\ &= \sum_i \int_V f_i(y)\chi_p(B(x,y)/2)W(x+y)\varphi_i \, dy \\ &= \sum_i \int_V f_i(t-x)\chi_p(B(x,t)/2)W(t)\varphi_i \, dt. \end{aligned}$$

Since f_i is Schwarz-Bruhat function, there exists a positive integer l sufficiently large such that $\text{supp} f_i \subset p^{-l}X_0$ and for any $t \in \text{supp} f_i$, $f_i(t-x) = f_i(t)$, $|x|_p \le p^{-l}$, where X_0 is a $\mathbb{Z}_p$-span of the symplectic basis of V (see §2). Let $K = p^l X_0$. Then K is an open compact subgroup of V and for all t and x, $B(x,t) \in \mathbb{Z}_p$. Thus we have $W(x)\varphi = \varphi$ for all $\varphi \in H$ and $x \in K$. ∎

Let K be an open compact subgroup of V. Then $K \cap X_0$ is also an open compact subgroup of V. Let $\mathcal{O} = \{K \cap X_0 : K \text{ is an open compact subgroup of } V\}$ and $Y \in \mathcal{O}$, and let $\xi_Y = \text{volume}(Y)^{-1}\times$ characteristic function of Y. Then ξ_Y is an idempotent in $S(V)$ and $S(Y) = \xi_Y \hat{*} S(V) \hat{*} \xi_Y$ is an associative $\mathbb{C}$-algebra with the unit element ξ_Y.

Let $l(x)$ and $r(x)$ be a translations of $S(V)$ given by

$$l(x)f(y) = \chi_p(B(x,y)/2)f(y-x), \quad r(x)f(y) = \chi_p(B(x,y)/2)f(y+x). \tag{3.3}$$

The following is easily proved.

Proposition 3.3. *We have the following properties:*

(i) *$W(x)E_W(f) = E_W(l(x)f)$ for all $x \in V$ and $f \in S(V)$.*

(ii) $f\hat{*}(l(x)g) = (r(-x)f)\hat{*}g$, $(r(x)f)\hat{*}g = f\hat{*}(l(-x)g)$ *for all* f, $g \in S(V)$.

(iii) $l(x)(f\hat{*}g) = (l(x)f)\hat{*}g$, $r(x)(f\hat{*}g) = f\hat{*}(r(x)g)$ *for all* f, $g \in S(V)$.

(iv) $l(x)\xi_Y = r(x)\xi_Y = \xi_Y$ *for all* $x \in Y$.

By (iv) of Proposition 3.3, $S(Y)$ is the space of elements f of $S(V)$ such that $l(x)f = r(x)f = f$ for all $x \in Y$. Clearly, the image $E_W(\xi_Y)H$ coincides with the subspace H^Y of Y-invariant vectors in H. It follows, inparticular, that for any exact sequence of $S(V)$-modules $0 \to H_1 \to H_2 \to H_3 \to 0$ the sequence $0 \to H_1^Y \to H_2^Y \to H_3^Y \to 0$ is also exact.

Let $H_W(Y) = \{\varphi \in H : W(-y)\varphi = \varphi \ \forall y \in Y\}$. Then the kernel of $E_W(\xi_Y)$ is the space $H(Y)$ spanned by all vectors of the form $W(y)\varphi - \varphi$, $y \in Y$, $\varphi \in H_W(Y)$. For it is clear that $E_W(\xi_Y)|_{H(Y)} = 0$ and that Y act trivially on $H/H(Y)$ since $H/H(Y) \cong \mathrm{Im}(E_W(\xi_Y)) = H^Y$, so that $E_W(\xi_Y)$ is the identity map on $H/H(Y)$.

Theorem 3.4. *Let (H, W) be a Weyl system on (V, B). Then (H, W) is irreducible if and only if for any open compact subgroup $Y \in \mathcal{O}$ of V either $H^Y = 0$ or (H^Y, W) is irreducible.*

Proof. Suppose that for any open compact subgroup $Y \in \mathcal{O}$ of V, $H^Y \neq 0$ and (H^Y, W) is reducible. Then there exists a non-trivial $S(Y)$-submodule $\mathfrak{a}$ of H^Y that is invariant with respect to the action of the operators $W(x)$, $x \in V$. Let $\mathfrak{b}$ be a $S(V)$-submodule of H generated by $\mathfrak{a}$. Since $\mathfrak{a} \subset \mathfrak{b}$, $\mathfrak{b}$ is non-trivial. Every $\varphi \in \mathfrak{b}$ is represented in the form

$$\varphi = \sum_{i=1}^{s} E_W(f_i)a_i + \sum_{j=1}^{t} n_j b_j \quad (s, t \in \mathbb{N},\ f_i \in S(V),\ a_i, b_j \in \mathfrak{a},\ n_j \in \mathbb{Z}).$$

Then, using (i) of Proposition 3.3, we have for $x \in V$,

$$W(x)\varphi = \sum_{i=1}^{s} W(x)E_W(f_i)a_i + \sum_{j=1}^{t} n_j W(x)b_j = \sum_{i=1}^{s} E_W(l(x)f_i)a_i + \sum_{j=1}^{t} n_j W(x)b_j.$$

Since $l(x)f_i \in S(V)$ and $W(x)b_j \in \mathfrak{a}$, $W(x)\varphi \in \mathfrak{b}$. Hence (H, W) is reducible. Conversely, suppose that $\mathfrak{b} \subset H$ is a non-trivial $S(V)$-submodule of (H, W). For any open compact subgroup $Y \in \mathcal{O}$ of V, the sequence $0 \to \mathfrak{b}^Y \to H^Y \to (H/\mathfrak{b})^Y \to 0$ is exact. Hence for $Y \in \mathcal{O}$ with $(H/\mathfrak{b})^Y \neq 0$, $\mathfrak{b}^Y$ is a non-trivial $S(Y)$-submodule of H^Y. ∎

Let (H, W) be an algebraic Weyl system on (V, B) and $H^* = \mathrm{Hom}_{\mathbb{C}}(H, \mathbb{C})$ the dual space of H. We define a Weyl system (H^*, W^*) on (V, B) by

$$\langle \varphi, W^*(x)\varphi^* \rangle = \langle W(-x)\varphi, \varphi^* \rangle, \tag{3.4}$$

for $x \in V$, $\varphi \in H$, $\varphi^* \in H^*$ and $\langle , \rangle$ is the natural pairing between H and H^*. This Weyl system (H^*, W^*) is not algebraic, so we take its algebraic part. More precisely, let $H^*_{al} = \bigcup_{Y \in \mathcal{O}} (H^*)^Y$ and $W^*_{al}(x) = W^*(x)|_{H^*_{al}}$, $x \in V$. Then (H^*_{al}, W^*_{al}) is an algebraic Weyl system on (V, B). Clearly, $\langle \varphi, E_{W^*_{al}}(f)\varphi^* \rangle = \langle E_W(\widetilde{f})\varphi, \varphi^* \rangle$ for $f \in S(V)$ where $\widetilde{f}(x) = f(-x)$.

Proposition 3.5. *Let (H, W) be an admissible Weyl system on (V, B). Then we have:*

(i) (H^*_{al}, W^*_{al}) *is also admissible.*

(ii) (H, W) *is irreducible if and only if* (H^*_{al}, W^*_{al}) *is.*

Proof. (i) We show that for all $Y \in \mathcal{O}$, $(H^*_{al})^Y$ is finite dimensional. Let $\varphi^* \in (H^*_{al})^Y$. Then we have for $\varphi \in H$,

$$\langle \varphi, \varphi^* \rangle = \langle \varphi, E_{W^*_{al}}(\xi_Y)\varphi^* \rangle = \langle E_W(\widetilde{\xi_Y})\varphi, \varphi^* \rangle = \langle E_W(\xi_Y)\varphi, \varphi^* \rangle.$$

This show that $(H^*_{al})^Y = (H^Y)^*$. I.e. $(H^*_{al})^Y$ is finite dimensional, since (H, W) is an admissible.

(ii) If $\mathfrak{a}$ is a non-trivial $S(V)$-submodule of H, then $\mathfrak{a}^\perp = \{\varphi^* \in H^*_{al} : \langle \mathfrak{a}, \varphi^* \rangle = 0\}$ is a non-trivial $S(V)$-submodule of H^*_{al}. Hence (H^*_{al}, W^*_{al}) is reducible. The converse follows from that $H \to (H^*_{al})^*_{al}$ is an isomorphism, i.e., $(W^*_{al})^*_{al} = W$. ∎

Proposition 3.6 (Schur's Lemma). *If (H, W) is an irreducible Weyl system on (V, B), then $D = Hom_{S(V)}(H, H)$ is a division ring. In other words, $D = \mathbb{C}$, i.e., if $\sigma : H \to H$ is a $S(V)$-module homomorphism, then σ is a scalar multiple of the identity morphism.*

Proof. Clearly, any non-zero $\sigma \in D$ is bijective, hence is invertible. Consequently, every non-zero element of D is a unit and thus D is a division ring. Also, if σ is not a scalar multiple of the identity map Id, then $\sigma - \lambda \cdot \mathrm{Id}$ is invertible for any $\lambda \in \mathbb{C}$. Let $0 \neq \varphi \in H$. If a sequence $(\sigma - \lambda_1 \cdot \mathrm{Id})^{-1}\varphi, (\sigma - \lambda_2 \cdot \mathrm{Id})^{-1}\varphi, \cdots$, for $\lambda_i \in \mathbb{C}$ distinct, of elements in H is linearly dependent, then there exists a sequence $z_1, z_2, \cdots$ of elements in $\mathbb{C}$ such that not all the z_i, say z_1 and z_2, are equal to 0 and $z_1(\sigma - \lambda_1 \cdot \mathrm{Id})^{-1}\varphi + z_2(\sigma - \lambda_2 \cdot \mathrm{Id})^{-1}\varphi = 0$. It implies $(z_1 + z_2)\left(\sigma - \frac{\lambda_1+\lambda_2}{z_1+z_2} \cdot \mathrm{Id}\right)\varphi = 0$, which contradicts the fact that $\sigma - \lambda \cdot \mathrm{Id}$ is invertible for any $\lambda \in \mathbb{C}$. Thus $(\sigma - \lambda \cdot \mathrm{Id})^{-1}\varphi$ $(\lambda \in \mathbb{C})$ are linearly independent. But, indeed, by (iii) of Theorem 2.2.1 H is spanned by $W(\alpha)\varphi_0$, $\alpha \in J_0$, which is countable. So H can not contain uncountablly many linearly independent vectors. Therefore σ is a scalar multiple of the identity morphism. ∎

4. Application. In this section, using Theorem 3.4 we give another proof of the Stone-Von Neumann Theorem of p-adic Heisenberg group.

The Stone-Von Neumann Theorem. Heisenberg group $N = N(V, B)$ of a p-adic smplectic space (V, B) is the set of pairs $(t, x) \in \mathbb{Q}_p \times V$ with the multiplication law

$$(t, x) \cdot (t', x') = (t + t' + B(x, x')/2, x + x'),$$

and it satisfies an exact sequence $0 \longrightarrow \mathbb{Q}_p \xrightarrow{\iota} N(V, B) \xrightarrow{\kappa} V \longrightarrow 0$, where ι and κ are given by $\iota(t) = (t, 0)$ and $\kappa(t, x) = x$, respectively.

The group $\mathrm{Im}(\iota) = \{(t, 0) : t \in \mathbb{Q}_p\}$ is the center and the commutator subgroup of N. Thus a character η of $\mathrm{Im}(\iota)$ is given by the formula $\eta(\iota(t)) = \chi_p(\lambda t)$ for some $\lambda \in \mathbb{Q}_p$. From now we assume $\lambda = 1$. Let ℓ be a Lagrangian subspace of V. Then $L = \mathbb{Q}_p \times \ell$ is an abelian subgroup of N, and there exists a unique character ω of L such that ω induces η on $\mathrm{Im}(\iota)$ and ω induces the identity map on ℓ. Explicitly, such ω is given by

$$\omega(t, x) = \chi_p(t) \quad (t \in \mathbb{Q}_p,\ x \in \ell). \tag{4.1}$$

We denote by $(H_\omega, T_\omega) = \mathrm{Ind}(N, L; \omega)$ the unitary representation of N induced by the character ω of L. Hence, the Hilbert space H_ω is the completion of the space of all functions $\varphi : N \to \mathbb{C}$ such that

$$\varphi(nh) = \omega^{-1}(h)\varphi(n) \quad (n \in N,\ h \in L), \tag{4.2}$$

$$\varphi \in L_2(N/L) \text{ for the Haar measure } d\dot{n} \text{ on } N/L, \tag{4.3}$$

and the unitary operator $T_\omega(n_0)$ on H_ω $(n_0 \in N)$ is the left multiplication of n_0^{-1}:

$$(T_\omega(n_0)\varphi)(n) = \varphi(n_0^{-1}n). \tag{4.4}$$

We have the Stone-Von Neumann Theorem (in case of real, see [Car 66, p. 368]):

Theorem 4.1 (The Stone-Von Neumann Theorem).

(a) *(H_ω, T_ω) is an irreducible unitary representation of N.*

(b) *For any Hilbert space H, every unitary representation (H, T) of N satisfying*

$$T(t, 0) = \chi_p(t) \cdot \mathrm{Id}_H \quad \text{for } (t, 0) \in Im(\iota) \tag{4.5}$$

is a multiple of T_ω.

Remark 4.2. It can be seen from [Per 81, pp. 371–372] that the Stone-Von Neumann Theorem is connected with Weil [Wei 64] as follows: If $V = \ell \oplus \ell'$ is a decomposition of (V, B) into a sum of two Lagrangian subspaces, then we can define a map

$$H_\omega \ni \varphi \longmapsto \varphi|_{\ell'} \in L_2(\ell').$$

This is an intertwining unitary operator for the unitary representation (H_ω, T_ω) and the irreducible unitary representation $(L_2(\ell'), \Phi)$ of N, where Φ is given by

$$(\Phi(t, x)f)(v^*) = \eta(t + \langle u, v^* \rangle - \langle u, u^* \rangle/2)f(v^* - u^*)$$

for $x = (u, u^*) \in V$ and $f \in L_2(\ell')$ if $\ell' \cong \ell^*$ (Pontrjagin dual of ℓ) and $\langle\ ,\ \rangle$ is the canonical bilinear form on V.

Let us consider also the subgroup $\Delta = \{(t, 0) : t \in \mathbb{Z}_p\}$ of the center $\mathrm{Im}(\iota)$ of N. Thus all unitary representations of N satisfying (4.5) are trivial on Δ. Hence we may consider them as representations of $\widetilde{N} = N/\Delta$. We can identify $\widetilde{N}$ with $\mathbb{T} \times V$ via $(t, x) \mapsto (\alpha, x)$. Then the multiplication law of $\widetilde{N}$ is given by

$$(\alpha, x) \cdot (\beta, y) = (\alpha\beta\chi_p(B(x, y)/2), x + y). \tag{4.6}$$

We call $\widetilde{N}$ the *p-adic Heisenberg group* of (V, B). The center $C(\widetilde{N})$ of $\widetilde{N}$ consists of the elements

$$C(\widetilde{N}) = \{(\alpha, 0) : \alpha \in \mathbb{T}\} \simeq \mathbb{T}$$

is a subgroup of the commutative group $\widetilde{L}$. Let ω' be a character of $\widetilde{L}$ extending the character $(\alpha, 0) \mapsto \alpha$ of the subgroup $C(\widetilde{N})$. We can consider ω' as a character of L satisfying (4.1).

Remark 4.3. If (H, W) is a Weyl system on (V, B), then the family of operators $T(t,x) = \chi_p(t)W(x)$, $(t,x) \in N$ (resp. $\widetilde{T}(\alpha,x) = \alpha W(x)$, $(\alpha,x) \in \widetilde{N}$), forms an unitary representation of N (resp. $\widetilde{N}$) on H. Conversely, if $T(t,x)$, $(t,x) \in N$ (resp. $\widetilde{T}(\alpha,x)$, $(\alpha,x) \in \widetilde{N}$), is some unitary representation of N (resp. $\widetilde{N}$) on the Hilbert space H satisfying the condition $T(t,0) = \chi_p(t) \cdot \mathrm{Id}_H$ (resp. $\widetilde{T}(\alpha,0) = \alpha \cdot \mathrm{Id}_H$), then the pair (H, W), $W(x) = \chi_p(-t)T(t,x)$ (resp. $W(x) = \widetilde{T}(1,x)$), $x \in V$, is a Weyl system on (V, B).

Another proof of the Stone-Von Neumann Theorem. Let (H_ω, T_ω) be the unitary representation of the Heisenberg group N. Each element of the Heisenberg group N is written uniquely as $(t,x) = (0,x)\cdot(t,0)$. Hence, if $\varphi \in H_\omega$, (4.2) implies $\varphi(t,x) = \varphi((0,x)\cdot(t,0)) = \chi_p(-t)\varphi(0,x)$. Thus φ is determined by its restriction to V. Hence the mapping

$$H_\omega \ni \varphi \xmapsto{R} \varphi|_V \in L_2(V)$$

is an intertwining unitary operator. The unitary representation $\widetilde{T}(n) = RT_\omega(n)R^{-1}$ acts on $L_2(V)$ by the following formula: For $n = (t,x) \in N$,

$$\begin{aligned}(\widetilde{T}(t,x)\varphi|_V)(y) &= RT_\omega(t,x)\varphi(0,y) = R\varphi((-t,-x)\cdot(0,y)) \\ &= R\varphi(-t - B(x,y)/2, y-x) \\ &= R\varphi((0,y-x)\cdot(-t-B(x,y)/2, 0)) \\ &= \chi_p(t)\chi_p(B(x,y)/2)\varphi|_V(y-x).\end{aligned}$$

Let $W(x) = \chi_p(-t)\widetilde{T}(t,x)$. It does not depend on t and satisfies the Weyl relation (2.4). Thus $(L_2(V), W)$ is a Weyl system on (V, B). In particular, if $\varphi|_V \in L_2^{\chi_p}(V_0)$, then $(L_2(V), W) = (L_2^{\chi_p}(V_0), \widetilde{W})$ is an irreducible Weyl system on (V, B) (see Example 2.4). To complete proof, we must show that $(L_2(V), W)$ is an irreducible. Let $H = L_2(V)$ and $Y \in \mathcal{O}$. Then $H^Y = S(Y) \neq 0$, since $l(x)\varphi = \varphi = W(x)\varphi$ for all $x \in Y$. Let f_0 be a vaccum vector of $(S(Y), W)$. Then we have for $x \in V_0$ and $y \in V$,

$$f_0(y) = W(x)f_0(y) = \chi_p(B(x,y)/2)f_0(y-x).$$

Since $f_0 \in S(Y)$, for any point $y \in V$ there exists an integer l such that $f_0(y-x) = f_0(y)$, $|x|_p \le p^l$. Thus $\mathrm{supp} f_0 \subset B_r = \{x \in V : |x|_p \le p^r\}$, $r = \min\{0, l\}$ and $f_0(y) \equiv$ constant, $y \in B_r$. Therefore $f_0(y) = c\cdot\Omega\left(p^{-r}|x|_p\right)$, $c \in \mathbb{C}$. It follows from (ii) of Theorem 2.1 and Theorem 3.4 that $(L_2(V), W)$ is an irreducible Weyl system on (V, B). ∎

5. A Weyl system depending on a selfdual $\mathbb{Z}_p$-lattice and a $\mathbb{Q}_p$-valued function. Let $\mathfrak{L}$ be a lattice in (V, B). The dual lattice $\mathfrak{L}^*$ is defined by

$$\mathfrak{L}^* = \{x \in V \ : \ B(x,y) \in \mathbb{Z}_p \ \text{ for all } \ y \in \mathfrak{L}\}. \tag{5.1}$$

If $\mathfrak{L} = \mathfrak{L}^*$, then $\mathfrak{L}$ is called selfdual. From now we consider only case where $\mathfrak{L}$ is a selfdual lattice. We consider a commutative subgroup $\Gamma = \{(t,x) \ : \ t \in \mathbb{Q}_p, \ x \in \mathfrak{L}\}$ of N. Let $\widetilde{\Gamma}$ be the image of the group Γ in $\widetilde{N}$. The fact that the lattice $\mathfrak{L}$ is selfdual is equivalent to the fact that $\widetilde{\Gamma}$ is a maximal commutative subgroup of $\widetilde{N}$. Let τ be a character of $\widetilde{\Gamma}$ extending the character $(\alpha, 0) \mapsto \alpha$ of the subgroup $\mathbb{T}$. By the Stone-Von Neumann Theorem, $(H_\tau, T_\tau) = \mathrm{Ind}(N, \Gamma; \tau)$ is an irreducible unitary representation of N.

Theorem 5.1 (cf. [LV 80, p. 143]). *There exists a canonical isomorphism $\Theta^\tau_{\mathfrak{L},\ell}$ between H_ω and H_τ intertwining the representations T_ω and T_τ:*

$$(\Theta^\tau_{\mathfrak{L},\ell}\varphi)(n) = \sum_{x \in \mathfrak{L}/(\mathfrak{L}\cap\ell)} \tau(x)\varphi(n\cdot(0,x)). \tag{5.2}$$

For any $\varphi \in H_\tau$, $x \in V$ and $y \in \mathfrak{L}$, we have

$$\begin{aligned}\varphi(0, x+y) &= \varphi((0,x)\cdot(-B(x,y)/2, y)) = \tau^{-1}(-B(x,y)/2, y)\varphi(0,x) \\ &= \chi_p(B(x,y)/2)\tau^{-1}(0,y)\varphi(0,x).\end{aligned}$$

Let $H(\mathfrak{L}, \sigma)$ be the Hilbert space obtained by completing the space of continuous functions $f : V \to \mathbb{C}$ satisfying the following two conditions

$$f(x+y) = \chi_p(B(x,y)/2 - \sigma(y))f(x) \quad (x \in V, \ y \in \mathfrak{L}), \tag{5.3}$$

where σ is a $\mathbb{Q}_p$-valued function on V satisfying $(\sigma(x+y) - \sigma(x) - \sigma(y))/p^n \in \mathbb{Z}_p$;

$$f \in L_2(V/\mathfrak{L}) \text{ for the Haar measure } d\dot{x} \text{ on } V/\mathfrak{L}. \tag{5.4}$$

We define a unitary operators $W_{\mathfrak{L},\sigma}(x)$ by

$$W_{\mathfrak{L},\sigma}(x)f(y) = \chi_p(B(x,y)/2 - \sigma(x))f(y-x), \quad y \in V, \ f \in H(\mathfrak{L}, \sigma). \tag{5.5}$$

Example 5.2. The pair $(H(\mathfrak{L}, \sigma), W_{\mathfrak{L},\sigma})$ is a Weyl system over (V, B). Indeed, the unitarity of the $W_{\mathfrak{L},\sigma}(x)$ is obvious. It is sufficient to check the Weyl relation. We have

$$\begin{aligned}W_{\mathfrak{L},\sigma}(x)W_{\mathfrak{L},\sigma}(y)f(z) &= W_{\mathfrak{L},\sigma}(x)\chi_p(B(y,z)/2 - \sigma(y))f(z-y) \\ &= \chi_p(B(x,z)/2 - \sigma(x))\chi_p(B(y,z-x)/2 - \sigma(y))f(z-x-y) \\ &= \chi_p(B(x,y)/2)\chi_p(B(x+y,z)/2 - \sigma(x) - \sigma(y))f(z-(x+y)) \\ &= \chi_p(B(x,y)/2)\chi_p(B(x+y,z)/2 - \sigma(x+y))f(z-(x+y)) \\ &= \chi_p(B(x,y)/2)W_{\mathfrak{L},\sigma}(x+y)f(z).\end{aligned}$$

Theorem 5.3. *The Weyl system $(H(\mathfrak{L},\sigma), W_{\mathfrak{L},\sigma})$ is irreducible if and only if $\sigma(\mathfrak{L}) \subset \mathbb{Z}_p$.*

Proof. Let H_0 be the vacuum subspace of $(H(\mathfrak{L},\sigma), W_{\mathfrak{L},\sigma})$, and let $f_0 \in H_0$ (by Theorem 2.1, such a vector exists). Then, using (5.3) and (5.5) we obtain

$$f_0(x) = W_{\mathfrak{L},\sigma}(z) f_0(x) = \chi_p(B(z,x)) f_0(x) \quad \text{for } z \in \mathfrak{L}.$$

Thus it satisfies supp $f_0 \subset \{x \in V \mid \chi_p(B(z,x)) = 1 \ \forall z \in \mathfrak{L}\} = \mathfrak{L}$. By (5.3), we have $f(x+y) = \chi_p(-\sigma(y)) f(x)$. Thus

$$\begin{aligned}
&(H(\mathfrak{L},\sigma), W_{\mathfrak{L},\sigma}) \text{ is irreducible} \\
&\qquad \Longleftrightarrow \text{ the vacuum subspace } H_0 \text{ is one-dimensional} \\
&\qquad \Longleftrightarrow f(x) \equiv \text{constant}, \ x \in \mathfrak{L} \\
&\qquad \Longleftrightarrow \sigma(\mathfrak{L}) \subset \mathbb{Z}_p. \quad \blacksquare
\end{aligned}$$

Remark 5.4. (i) We denote by $(H(\mathfrak{L}), W_{\mathfrak{L}})$ the irreducible Weyl system of Theorem 5.3. Then an irreducible unitary representation of the Heisenberg group $\widetilde{N} \cong \mathbb{T} \times \mathbb{Q}_p^{2n}$ is defined as a pair $(H(\mathfrak{L}), T_{\mathfrak{L}})$, where $T_{\mathfrak{L}}(\alpha, x) = \alpha W_{\mathfrak{L}}(x)$. This representation is a p-adic analogue of the Cartier representation [Car 64] of the real Heisenberg group.

(ii) For the Heisenberg group $\widetilde{N} \cong \mathbb{T} \times V$ of a p-adic symplectic space (V, B) of arbitrary dimension, $\mathfrak{L}$-representation corresponding Weyl $\mathfrak{L}$-system, which is analogues of Fock representations of commutation relations, was constructed by Zelenov [Zel 94].

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