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Erratum

Erratum to "Fading absorption in non-linear elliptic equations" [Ann. I. H. Poincaré – AN 30 (2) (2013) 315–336] ☆

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The purpose of this note is to correct an error that occurred in the proof of Theorem 1.2 of the paper 'Fading absorption in non-linear elliptic equations' which appeared in Ann. I. H. Poincaré – AN, 2013.

The theorem itself is correct as stated. However Proposition 3.1 (used in its proof) and relation (3.18) are wrong. We restate Proposition 3.1 and provide a modified argument to replace the part of the proof from (3.18) to the end.

Let U_j, $j = 1, 2, \ldots$ be the unique solution of the boundary value problem

$$\begin{aligned} &-\Delta U_j + \bar{h} U_j^q = 0 \quad \text{in } \mathbb{R}^N_+, \\ &U_j(x', 0) = \gamma_j(x') \quad \text{for } x' \in \mathbb{R}^{N-1} \end{aligned} \tag{0.1}$$

dominated by the harmonic function $\int_{\mathbb{R}^{N-1}} P(x, y')\gamma_j(y')\,dy'$. Here $\bar{h}$ and γ_j are given by (1.3) and (3.2) respectively. Proposition 3.1 is replaced by:

Proposition 3.1′. *Under the assumptions of Theorem* 1.2,

$$\lim_{j\to\infty} U_j(0, x_N) = \infty \quad \forall x_N > 0. \tag{0.2}$$

Proof. The proof is based on (3.17) and the inequality $u_j \leqslant U_j$ in Ω_j. This inequality follows from the comparison principle and the fact that $\bar{h} \leqslant a_j$ in Ω_j while $u_j \leqslant U_j$ on $\partial\Omega_j$. This inequality and (3.17) yield

$$u_{j-1}(x', 0) \leqslant U_j(x', \tau_j) \quad \forall j \geqslant j_0,\ |x'| < r_{j-1}.$$

Therefore by the comparison principle applied in Ω_{j-1},

$$u_{j-1}(x', x_N) \leqslant U_j(x', x_N + \tau_j) \quad \forall j \geqslant j_0,\ x \in \Omega_{j-1}. \tag{0.3}$$

Let $j > j_0$ and $0 \leqslant k \leqslant j - j_0$. Using (0.3), (3.17) and induction on k we obtain,

$$u_{j-k-1}(x', x_N) \leqslant U_j\left(x', x_N + \sum_{i=0}^{k} \tau_{j-i}\right) \quad \forall x \in \Omega_{j-k-1}. \tag{0.4}$$

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By (1.10) and (3.16) $\sum_{j=0}^{\infty}\tau_j=\infty$ and $\sup_{m\geqslant 0}\tau_m=\bar\tau<\infty$. Therefore, if $b>\bar\tau$ then, for every $j\geqslant j_b$, there exists an integer $\lambda_j=\lambda_j(b)$ such that $0\leqslant b-\sum_{\lambda_j+1}^{j}\tau_m=:\delta_j\leqslant\tau_{\lambda_j}$ and $\lambda_j\to\infty$ as $j\to\infty$. Hence by (0.4)

$$u_{\lambda_j}\left(x',\delta_j\right)\leqslant U_j\left(x',\delta_j+\sum_{\lambda_j+1}^{j}\tau_m\right)=U_j\left(x',b\right)\quad\forall x'\colon\ \left|x'\right|<r_{\lambda_j}. \tag{0.5}$$

Applying the comparison principle to u_{λ_j} in Ω_{λ_j} and using (3.12) we find that the inequality $0\leqslant\delta_j\leqslant\tau_{\lambda_j}$ implies $u_{\lambda_j}(0,\delta_j)\geqslant\frac{1}{4\alpha}u_{\lambda_j}(0,\tau_{\lambda_j})$. Therefore (0.5) and (3.17) imply,

$$\frac{1}{4\alpha}A^{-1}_{\lambda_j-1}\leqslant\frac{1}{4\alpha}\gamma_{\lambda_j-1}(0)\leqslant\frac{1}{4\alpha}u_{\lambda_j}(0,\tau_{\lambda_j})\leqslant U_j(0,b).$$

Finally, as $\lim_{k\to\infty}A_k=0$, this inequality implies (0.2) for $x_N>\bar\tau$. It is easy to see that if (0.2) fails for some $x_N>0$ then it fails for all larger values of x_N. Therefore (0.2) holds for every $x_N>0$. $\square$

Completion of proof of Theorem 1.2. Let v_j be the solution of (3.3) where γ_j is replaced by $\Gamma_j=A_j^{-1}r_j^{N-1}\delta_0$ on $\partial\Omega_j\cap[x_N=0]$. As in Section 3.1, the function $\tilde v_j$ defined by $\tilde v_j(y)=A_jv_j(r_jy)$, $y\in D_0$ satisfies the boundary value problem (3.9) with $\tilde\gamma$ replaced by $\tilde\Gamma=\delta_0$. Denote the solution of this problem by $\tilde v$. Next we apply Lemma 3.1 to $\tilde v$ in $D_0\cap[x_N>b]$ (b a fixed positive number). We conclude that choosing $\beta>0$ sufficiently large $0<c(\beta)\leqslant\frac{\tilde v(y',\beta)}{\phi_1(y')}\leqslant 1$ for $|y'|<1$. Hence, if $\gamma_j'(x'):=v_j(x',r_j\beta)$ then $c(\beta)\leqslant\frac{\gamma_j'(x')}{A_j^{-1}\phi_1(x'/r_j)}\leqslant 1$ in the ball $|x'|<r_j$. Obviously $u_j'(x):=v_j(x',x_N+r_j\beta)$ satisfies (3.3) with γ_j replaced by γ_j'. Proceeding as in Section 3.2 we obtain a sequence $\{\tau_j\}$ satisfying (3.16) and

$$\gamma_{j-1}'\left(x'\right)\leqslant u_j'\left(x',\tau_j\right),\quad\left|x'\right|\leqslant r_j,\ j\geqslant j_0.$$

Let U_j' (resp. V_j) be defined in the same way as U_j except that γ_j is replaced by γ_j' (respectively Γ_j) extended by zero for $|x'|\geqslant r_j$. Then Proposition 3.1′ applies to $\{U_j'\}$ so that

$$\lim_{j\to\infty}U_j'(0,x_N)=\infty. \tag{0.6}$$

Furthermore $V_j\geqslant v_j$ in Ω_j so that $V_j(x',r_j\beta)>\gamma_j'(x')$, $|x'|<r_j$. By the comparison principle, $V_j(x',x_N+r_j\beta)\geqslant U_j'(x)$ in $\mathbb{R}^N_+$. Hence

$$\lim_{j\to\infty}V_j(0,x_N)=\infty\quad\forall x_N>0.\qquad\square$$