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Equivalent-singular dichotomy for quasi-invariant ergodic measures

by

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ABSTRACT. — Let E be a locally convex Hausdorff space, H_1, H_2 be two linear subspaces of E and μ_1, μ_2 be two probability measures on E. We prove that if μ_i is H_i-quasi-invariant and H_i-ergodic ($i = 1, 2$), then μ_1 and μ_2 are equivalent or singular. If $H_1 = H_2$, the result is well-known. So our interest is in the case $H_1 \neq H_2$. This result unifies many known dichotomies such as the dichotomy for product measures on $\mathbb{R}^\infty$, Hajek-Feldman's dichotomy for Gaussian measures and Fernique's dichotomy for a product measure and a Gaussian measure.

RÉSUMÉ. — Soit E un espace vectoriel localement convexe séparé. Soient H_1, H_2 deux sous-espaces vectoriels de E, et μ_1, μ_2 deux mesures probabilités sur E. Nous prouverons que si μ_i est H_i-quasi-invariante et H_i-ergodique ($i = 1, 2$), alors μ_1 et μ_2 sont équivalentes ou singulières. Si $H_1 = H_2$, c'est bien connu. Notre intérêt est donc au cas $H_1 \neq H_2$. Ce résultat unifie les dichotomies des mesures produits dans $\mathbb{R}^\infty$, de Hajek-Feldman pour les mesures gaussiennes, et de Fernique pour la mesure produit et la mesure gaussienne.

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1. INTRODUCTION

Let μ, ν be probability measures on a measurable space $(\Omega, \mathscr{B})$. μ is said to be absolutely continuous with respect to ν (denoted by $\mu < \nu$) if $\nu(A) = 0$, $A \in \mathscr{B}$ implies that $\mu(A) = 0$. μ and ν are equivalent (denoted by $\mu \sim \nu$) if $\mu < \nu$ and $\nu < \mu$. μ and ν are singular (denoted by $\mu \perp \nu$) if there exists $A \in \mathscr{B}$ such that $\mu(A) = 1$ and $\nu(A^c) = 1$.

Let H be a subset of a locally convex Hausdorff space E and μ be a probability measure on C(E, E*) the cylindrical σ-algebra generated by the topological dual E*. We set $\tau_x(y) = y + x$. τ_x ($x \in E$) is C(E, E*)-measurable. μ is said to be H-quasi-invariant if it holds $\tau_x(\mu) \sim \mu$ for every $x \in H$, where $\tau_x(\mu)(A) = \mu(A - x)$, $A \in C(E, E^*)$. The H-quasi-invariant measure μ is said to be H-ergodic if $\mu(A\theta(A - x)) = 0$ for every $x \in H$, then $\mu(A) = 0$ or 1, where θ denotes the symmetric difference.

The aim of this paper is to prove the following theorem.

THEOREM. — *Let* E *be a locally convex Hausdorff space,* H_1, H_2 *be two linear subspaces of* E, μ_1, μ_2 *be two probability measures on* C(E, E*) *and assume that* μ_i *is* H_i*-quasi-invariant and* H_i*-ergodic* ($i = 1, 2$). *Then* μ_1 *and* μ_2 *are equivalent or singular.*

If $H_1 = H_2$, the result is well-known, for example see Skorohod [*13*], § 23. So our interest is in the case $H_1 \neq H_2$. This theorem unifies many known dichotomies such as *a*) dichotomy for product measures on $\mathbb{R}^\infty$ (this is a special case of the Kakutani dichotomy), *b*) Hajek-Feldman's dichotomy, *c*) Fernique's dichotomy, *d*) dichotomy for symmetric stable measures with discrete Lévy measures, *e*) dichotomy for the case $H_1 = H_2$ and *f*) Kanter's dichotomy, see section 3.

2. DICHOTOMY FOR QUASI-INVARIANT ERGODIC MEASURE

Let E be a locally convex Hausdorff space and μ be a probability measure on C(E, E*). We set $A_\mu = \{ x \in E ; \tau_x(\mu) \sim \mu \}$. A_μ is an additive subgroup of E but not necessarily a linear space. Denote by A_μ^0 the largest linear subspace contained in A_μ, that is, $A_\mu^0 = \{ x \in A_\mu ; tx \in A_\mu \text{ for every } t \in \mathbb{R} \}$.

Let $\{ x_i^* \} \subset E^*$ be arbitrary but fixed countable subset. Consider the σ-subalgebra $\mathscr{A} = C(E, \{ x_i^* \}) \subset C(E, E^*)$, the σ-algebra generated by

$\{ x_i^* \}$. For every $x \in E$, the translation $\tau_x(\tau_x(y) = y + x)$ is $\mathscr{A} - \mathscr{A}$-measurable. Let μ_0 be the restriction of μ to $\mathscr{A}$. For every $x \in A_\mu^0$ (that is, $tx \in A_\mu$ for every $t \in \mathbb{R}$), we can define the one parameter group $\{ U_t^x ; t \in \mathbb{R} \}$ of unitary operators on $L^2(\mathscr{A}, \mu_0)$ by

$$(U_t^x f)(y) = \left[\frac{d\tau_{tx}(\mu_0)}{d\mu_0}(y) \right]^{1/2} f(y - tx).$$

This one parameter group depends on the sequence $\{ x_i^* \}$.

LEMMA 1. — Let $\{ x_i^* \}$ be fixed and $\mathscr{A} = C(E, \{ x_i^* \})$ as above. Let $x \in A_\mu^0$ be also fixed. Then the one parameter group $\{ U_t^x ; t \in \mathbb{R} \}$ on $L^2(\mathscr{A}, \mu_0)$ is strongly continuous, that is, for every $f \in L^2(\mathscr{A}, \mu_0)$, $t \to U_t^x f \in L^2(\mathscr{A}, \mu_0)$ is continuous on $\mathbb{R}$.

Proof. — Remark that $\mathscr{A}$ has a countable generator $\{ C_i \}$, that is, $\{ C_i \}$ forms an algebra of subsets and generates $\mathscr{A}$. To prove that $\{ U_t^x \}$ is strongly continuous on the separable Hilbert space $L^2(\mathscr{A}, \mu_0)$, it is sufficient to show that for every $f, g \in L^2(\mathscr{A}, \mu_0)$, $t \to (U_t^x f, g) = \int (U_t^x f)(y) g(y) d\mu_0(y)$ is Lebesgue measurable on $\mathbb{R}$, since $\{ U_t^x \}$ is a one parameter unitary group, see von Neumann [*10*], Hewitt and Ross [*5*], (22.20).

Consider the transformation $T_x : \mathbb{R} \times E \to \mathbb{R} \times E$ given by $T_x(t, y) = (t, y + tx) = (t, \tau_{tx}(y))$. T_x is $\mathscr{B}(\mathbb{R}) \otimes \mathscr{A}$-measurable, where $\mathscr{B}(\mathbb{R})$ is the Borel field of $\mathbb{R}$. Moreover, if we put λ be the Lebesgue measure on $\mathbb{R}$, then we have $T_x(\lambda \otimes \mu_0) \sim \lambda \otimes \mu_0$. In fact for every $A \in \mathscr{B}(\mathbb{R}) \otimes \mathscr{A}$, we have $T_x(\lambda \otimes \mu_0)(A) = \int \tau_{tx}(\mu_0)(A_t) d\lambda(t)$, where $A_t = \{ y ; (t, y) \in A \}$. So it follows that $T_x(\lambda \otimes \mu_0)(A) = 0$ if and only if $\tau_{tx}(\mu_0)(A_t) = 0$ for λ-a. e. t, and if and only if $\mu_0(A_t) = 0$ for λ-a. e. t since $tx \in A_\mu$, that is $\lambda \otimes \mu_0(A) = 0$.

By the Radon-Nikodym theorem, there is a $\lambda \otimes \mu_0$-measurable function $\alpha(t, y)$ such that $\lambda \otimes \mu_0(T_x^{-1} A) = \int_A \alpha(t, y) d(\lambda \otimes \mu_0)(t, y)$ for every $A \in \mathscr{B}(\mathbb{R}) \otimes \mathscr{A}$. For every $B \in \mathscr{B}(\mathbb{R})$ and $C \in \mathscr{A}$, we have

$$\int_B \mu_0(\{ y ; y + tx \in C \}) d\lambda(t) = \int_B \int_C \alpha(t, y) d\mu_0(y) d\lambda(t).$$

Hence we have $\tau_{tx}(\mu_0)(C) = \mu_0(\{ y ; y + tx \in C \}) = \int_C \alpha(t, y) d\mu_0(y)$ λ-a. e. (the null set depends on C). Let $\{ C_i \}$ be a countable generator of $\mathscr{A}$.

Then we can find a λ-null set $N \subset \mathbb{R}$ such that for every $t \in \mathbb{R}\backslash N$, it holds that $\tau_{tx}(\mu_0)(C_i) = \int_{C_i} \alpha(t, y)d\mu_0(y)$ for every i. So we obtain for every $t \in \mathbb{R}\backslash N$, $\tau_{tx}(\mu_0)(C) = \int_C \alpha(t, y)d\mu_0(y)$ for every $C \in \mathscr{A}$, since $\{ C_i \}$ generates $\mathscr{A}$. By the uniqueness of the Radon-Nikodym derivative, it follows that for every $t \in \mathbb{R}\backslash N$, $(d\tau_{tx}(\mu_0)/d\mu_0)(y) = \alpha(t, y)$ μ-a. e. (the μ-null set depends on t). Consequently for every $t \in \mathbb{R}\backslash N$ and for every $f, g \in L^2(\mathscr{A}, \mu_0)$,

$$(U_t^x f, g) = \int_E \alpha(t, y)^{1/2} f(y - tx)g(y)d\mu_0(y),$$

which implies that $t \to (U_t^x f, g)$ is λ-measurable since $\alpha(t, y)$ is $\lambda \otimes \mu_0$-measurable.

This proves the Lemma.

LEMMA 2. — Suppose that $x \in A_\mu^0$. Then for every $A \in C(E, E^*)$, it holds that $\mu(A \cap (A + tx)) \to \mu(A)$ as $t \to 0$.

Proof. — We show that $\mu(A\theta(A + tx)) \to 0$ as $t \to 0$. By the definition of the cylindrical σ-algebra, there exists a sequence $\{ x_i^* \} \subset E^*$ such that $A \in \mathscr{A} = C(E, \{ x_i^* \})$. We set $\mu_0 = \mu \mid \mathscr{A}$ (the restriction) as before. Since $A + tx \in \mathscr{A}$, it is sufficient to show that $\mu_0(A\theta(A + tx)) \to 0$ as $t \to 0$. We have

$$\begin{aligned}\mu_0(A\theta(A+tx)) &= \int |\chi_A(y) - \chi_A(y-tx)|^2 d\mu_0(y)\\ &\leqq 2\|U_t^x(\chi_A) - \chi_A\|^2 + 2\int |(d\tau_{tx}(\mu_0)/d\mu_0)(y)^{1/2}\chi_A(y-tx) - \chi_A(y-tx)|^2 d\mu_0(y)\\ &\leqq 2\|U_t^x(\chi_A) - \chi_A\|^2 + 2\|U_t^x 1 - 1\|^2 \to 0 \quad \text{as} \quad t \to 0 \text{ by Lemma 1},\end{aligned}$$

where χ_A is the characteristic function of A and $\| \quad \|$ denotes the L^2-norm.

This completes the proof.

Now we prove the main dichotomy theorem.

THEOREM. — Let E be a locally convex Hausdorff space and H_1, H_2 be two linear subspaces of E. Let μ_1, μ_2 be two probability measures on C(E, E*) such that μ_i is H_i-quasi-invariant and H_i-ergodic ($i = 1, 2$). Then μ_1 and μ_2 are equivalent or singular.

Proof. — Since H_i is linear, we have $H_i \subset A_{\mu_i}^0$, hence μ_i is $A_{\mu_i}^0$-ergodic. If $A_{\mu_1}^0 = A_{\mu_2}^0$, then it is well-known that $\mu_1 \sim \mu_2$ or $\mu_1 \perp \mu_2$ holds, since μ_1 and μ_2 have the same quasi-invariant subspace $A_{\mu_1}^0 = A_{\mu_2}^0$, see for example Skorohod [*13*], 23. We prove that if $A_{\mu_1}^0 \neq A_{\mu_2}^0$, then μ_1 and μ_2 are singular.

Suppose that $A^0_{\mu_1} \neq A^0_{\mu_2}$ and μ_1 and μ_2 are not singular. Without loss of generality we may assume that there exists $x_0 \in A^0_{\mu_1} \backslash A^0_{\mu_2}$ (if not, consider $A^0_{\mu_2} \backslash A^0_{\mu_1} \neq \phi$). Since $x_0 \notin A^0_{\mu_2}$, we can find a $t_0 \in \mathbb{R}$ such that $t_0 x_0 \notin A_{\mu_2}$. Put $x = t_0 x_0$, then we have $\tau_x(\mu_2) \perp \mu_2$ since both $\tau_x(\mu_2)$ and μ_2 are $A^0_{\mu_2}$-quasi-invariant and $A^0_{\mu_2}$-ergodic by the same subspace $A^0_{\mu_2}$.

Since μ_1 and μ_2 are not singular, there exists $A \in C(E, E^*)$ such that $\mu_1(A) > 0$, $\mu_2(A) > 0$ and

$$(1) \qquad \mu_1 \sim \mu_2 \qquad \text{on } A\,.$$

Since $x \in A^0_{\mu_1}$ and $A^0_{\mu_1}$ is linear space, we have

$$(2) \qquad \mu_1 \sim \tau_{x/n}(\mu_1) \qquad \text{for every} \quad n = 1, 2, \ldots$$

Consider the translates $\tau_{x/n}(\mu_2)$, $n = 1, 2, \ldots$ By the $A^0_{\mu_2}$-quasi-invariance and $A^0_{\mu_2}$-ergodicity, $\tau_{x/n}(\mu_2) \sim \mu_2$ or $\tau_{x/n}(\mu_2) \perp \mu_2$ $(n=1, 2, \ldots)$. If $\tau_{x/N}(\mu_2) \sim \mu_2$ for some N, then it follows that

$$\mu_2 \sim \tau_{x/N}(\mu_2) \sim \tau_{x/N}(\tau_{x/N}(\mu_2)) = \tau_{2x/N}(\mu_2) \sim \ldots \sim \tau_x(\mu_2),$$

which contradicts to $x \notin A_{\mu_2}$. Thus we have

$$(3) \qquad \mu_2 \perp \tau_{x/n}(\mu_2) \qquad \text{for every} \quad n = 1, 2, \ldots$$

By (1), we have

$$(4) \qquad \tau_{x/n}(\mu_1) \sim \tau_{x/n}(\mu_2) \qquad \text{on } A + \frac{x}{n} \quad \text{for every } n\,.$$

If we can show that $\mu_1(A \cap (A + x/m)) > 0$ (by (1) it follows also $\mu_2(A \cap (A + x/m)) > 0$)) for some m, then (2), (3) and (4) imply $\mu_1 \perp \mu_2$ on $A \cap (A + x/m)$ which contradicts to (1).

By Lemma 2, we have $\mu_1(A \cap (A + x/n)) \to \mu_1(A) > 0$ as $n \to \infty$. So there exists m such that $\mu_1(A \cap (A + x/m)) > 0$ as desired.

This completes the proof.

Remark. — In the above theorem, if μ_1 and μ_2 are Radon probability measures on the Borel field $\mathscr{B}(E)$, then the same dichotomy $\mu_1 \sim \mu_2$ or $\mu_1 \perp \mu_2$ holds. In fact, for two Radon probability measures μ_1, μ_2, $\mu_1 \sim \mu_2$ on $C(E, E^*)$ implies that $\mu_1 \sim \mu_2$ on $\mathscr{B}(E)$, see Sato and Okazaki [*12*].

3. APPLICATION

We shall point out that many well-known dichotomies are unified by our result. The following dichotomies are known.

a) Let μ, ν be probability measures on $\mathbb{R}$ equivalent to the Lebesgue measure ($l \in \mathrm{I}$) and $\mu = \otimes \mu_l$, $\nu = \otimes \nu_l$ be the product measures. By Kakutani's dichotomy [*8*], it follows that $\mu \sim \nu$ or $\mu \perp \nu$. We remark that Kakutani's dichotomy is more general and it requires no linear structure.

b) Let E be a locally convex Hausdorff space and μ, ν be two Gaussian measures on C(E, E*). Then Hajek and Feldman [*2*] [*4*] proved that either $\mu \sim \nu$ or $\mu \perp \nu$ holds.

c) Let $\mathbb{R}^\infty$ be the countable product of real numbers. Let μ_i be a probability measure on $\mathbb{R}$ equivalent to the Lebesgue measure and $\mu = \otimes \mu_i$ be the product measure. Let ν be arbitrary Gaussian measure on $\mathbb{R}^\infty$. Then Fernique [*3*] proved that either $\mu \sim \nu$ or $\mu \perp \nu$ holds, answering the problem of Chatterji and Ramaswamy [*1*].

d) Let μ, ν be symmetric stable measures with discrete Lévy measure on a separable Banach space E. Then it holds that $\mu \sim \nu$ or $\mu \perp \nu$ (this result was informed in Janssen [*7*]). In general, a symmetric stable measure μ has the characteristic functional of the form $\exp\left(- \int_S | \langle y, x^* \rangle |^p d\sigma(y) \right)$, where $0 < p \leqq 2$ and σ (the Lévy measure) is a bounded measure on the unit sphere $S = \{ x ; \| x \| = 1 \}$, see Tortrat [*14*]. If σ is concentrated on a countable subset, then μ is said to have discrete Lévy measure.

e) Let E be a locally convex Hausdorff space and H be a linear subspace of E. Let μ, ν be H-quasi-invariant and H-ergodic Probability measures on C(E, E*). Then it holds either $\mu \sim \nu$ or $\mu \perp \nu$, see for example Skorohod [*13*], 23.

f) Let G_1, G_2 be countable additive subgroups of $\mathbb{R}^\infty$ and μ_1, μ_2 be two probability measures such that μ_i is G_i-quasi-invariant and G_i-ergodic, $i = 1, 2$. Suppose also that μ_i is $L(G_i)$-quasi-invariant where $L(G_i)$ denotes the linear hull of G_i, $i = 1, 2$. Then Kanter [*9*] proved that either $\mu_1 \sim \mu_2$ or $\mu_1 \perp \mu_2$ holds.

Our dichotomy is a direct generalization of *e*) and *f*). But we have used the dichotomy *e*) in the proof of our theorem. We observe the dichotomies *a*), *b*), *c*) and *d*) from our viewpoints, and see that our dichotomy unifies them.

In *a*), μ and ν are $\bigoplus_l \mathbb{R}$-quasi-invariant and $\bigoplus_l \mathbb{R}$-ergodic, where $\bigoplus_l \mathbb{R}$ is the direct sum of R, see Zinn [*15*]. In the dichotomy *b*), the Gaussian measures μ and ν are quasi-invariant and ergodic under the translations of their reproducing kernel Hilbert spaces, for details we refer to Rozanov [*11*]. In the Fernique's dichotomy *c*), μ (resp. ν) is quasi-invariant and ergodic under the translation of the direct sum $\bigoplus_i \mathbb{R}$ (resp. the reproducing kernel Hilbert space of ν). Hence the dichotomies *a*), *b*) and *c*) are derived by our Theorem.

We examine the dichotomy *d*). We shall see that μ and ν are quasi-invariant and ergodic by suitable linear subspaces. Let $\{ x_i \}$ be the discrete support of the Lévy measure σ of μ, $a_i = \sigma(\{ x_i \}) > 0$ and $y_i = a_i x_i$. Then the characteristic functional of μ is given by $\exp(- \Sigma_{i=1}^{\infty} | \langle y_i, x^* \rangle |^p)$, $x^* \in E^*$. Let $\{ f_i(\omega) \}$ be an independent identically distributed symmetric p-stable random variables with the characteristic functional $\exp(- | t |^p)$. Then it follows that $\int \exp(i\langle \Sigma_{i=1}^n f_i(\omega) y_i, x^* \rangle) dP(\omega) = \exp(- \Sigma_{i=1}^n | \langle y_i, x^* \rangle |^p)$ converges to the characteristic functional of μ. So by Ito-Nisio's theorem [*6*], $\Sigma_{i=1}^n f_i(\omega) y_i$ converges almost everywhere in E and the distribution of $\Sigma_{i=1}^{\infty} f_i(\omega) y_i$ equals μ. Let λ_i be the p-stable measure on $\mathbb{R}$ with the characteristic functional $\exp(- | t |^p)$ and $\lambda = \otimes \lambda_i$ be the product measure on $\mathbb{R}^{\infty}$. Set $T : \mathbb{R}^{\infty} \to E$ by $T(t_m) = \Sigma_{n=1}^{\infty} t_n y_n$. Then we have $\lambda(\{ (t_m) \in \mathbb{R}^{\infty} ; \Sigma_{n=1}^{\infty} t_n y_n$ converges in $E \}) = 1$ and the image $T(\lambda)$ coincides with μ. Since λ is $\bigoplus_i \mathbb{R}$-quasi-invariant and $\bigoplus_i \mathbb{R}$-ergodic, see Zinn [*14*], it follows easily rhat μ is also $T(\bigoplus_i \mathbb{R})$-quasi-invariant and $T(\bigoplus_i \mathbb{R})$-ergodic. Thus the dichotomy *d*) is a special case of our Theorem.

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