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N-body relativistic systems

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ABSTRACT. — Using the multi-time framework, we study the covariant dynamics of N point-like particles.

In particular we construct a N-body relativistic potential when a binary interaction is given. Bound states are obtained for equal-mass particles linked by harmonic forces.

Quantization allows for a class of N-body relativistic wave-equations. For $N > 2$ our point of view gives no evidence for a relativistic Schrödinger equation and the relative times are not eliminated.

1. INTRODUCTION. NOTATIONS

Under various names: Predictive Mechanics, quasi-potential approach, Instant—or front—form of dynamics, multi-local field theory, etc., a huge and disparate lot of litterature has been devoted to the relativistic theory of Action at a Distance, sometimes considered on its own ground, often considered as a simplification of field theory [*1*].

For obvious reasons of simplicity, most solvable models have been proposed for 2-particle systems only. Whatever is the formalism employed, the construction of an admissible interaction is not as arbitrary as it is, in principle, in gallilean mechanics.

As a result, the general form of a N-body relativistic interaction is far from being a trivial matter. We have elaborated a covariant formulation of dynamics of which the basic principles are valid for any number of

particles N [2] [3]. But it is only about the 2-body problem, so far, that we have developped a detailed treatment and constructed some examples of interaction [4].

We now intent to discuss the general case $N > 2$.

Our motivation is not only a conceptual need of completeness (with N-body dynamics as a foundation for statistical mechanics, covariant second quantization, celestial mechanics, etc.) but also the pressure of a more naive current of research oriented toward composite particles, in particular 3-quark systems.

Recently, 3-body relativistic dynamics has received some interest [5] and some 3-body relativistic wave equations have been proposed. Some years ago, L. Bel and J. Fustero have proposed an expansion procedure in order to define N-body classical interactions by formal series related with Liénard-Wiechert retarded formulas [6].

Their approach, like ours, is founded on the predictivity conditions but it is essentially a field theoretical perturbative treatment.

Our point of view is, as far as possible non perturbative, especially at the classical level, although the discussion will be generally *local* in the sense of differential geometry. Besides, our approach is not *a priori* connected with conventional field theory.

For the basic framework, namely predictivity conditions, position equations, Hamilton equations, Poisson brackets, etc., the N-body formulation goes without saying once the 2-body theory has been displayed.

We shall briefly recall these matters in the full N-body form.

Some other points, in particular those involving equal-time conditions, although treated in the same spirit as in the case $N = 2$, require more detailed pieces of phase-space geometry. And more specially, the practical problem of model construction turns out to be essentially more difficult for $N > 2$.

The rest of this article will be concerned with these questions.

According to the multi-time formalism [2-4] [7], the equations of motion for N point particles have the hamiltonian form [7]

$$\begin{aligned} \partial q_a/\partial \tau_b &= \{ q_a, \mathrm{H}_b \} \\ &\vdots \\ \partial p_a/\partial \tau_b &= \{ p_a, \mathrm{H}_b \} \end{aligned} \tag{1.1}$$

They are generated by the hamiltonians H_a. The indices $a, b = 1, 2, \ldots$ *are not* submitted to Einstein's summation convention. The unconstained variable q_a, p_a are submitted to the standard Poisson bracket relations, but q_a do not coincide with the positions.

The q_a have the transformation properties of a point in Minkowski space, while p_a have that of a four vector. Whenever possible, the greek indices are omitted. Signature $+---$, $c = \hbar = 1$, m_a are the masses.

We set

$$P = \sum_a p_a \qquad z_{ab} = q_a - q_b$$

$$P_{ab} = \frac{1}{2}(p_a + p_b) \qquad y_{ab} = \frac{1}{2}(p_a - p_b)$$

We call X_a the vector field generated by H_a in the 8N-dimensional phase space E^{8N}.

The world-lines are defined by a differential system [2]

$$d^2x_a^\alpha/d\tau_a^2 = \xi_a^\alpha(x_1, \ldots x_N, v_1, \ldots v_N) \tag{1.2}$$

where x are the positions and

$$v_a = dx_a/d\tau_a$$

The existance of world-lines requires that the Lie brackets $[X_a, X_b]$ vanish.

In the case of interactions that are symmetric with respect to particle interchange, this condition is equivalent to

$$\{ H_a, H_b \} = 0 \tag{1.3}$$

Equations (1.1) and (1.2) can be considered as equivalent; however, the correspondence between the variables x, v and the canonical variables q, p rests on the solving of the *position equations* [3]

$$\{ H_a, x_b \} = 0 \qquad a \neq b \tag{1.4}$$

Equation (1.4) yields the x as functions of q, p. Then the generalized accelerations are given by

$$\xi_a = \{ H_a, v_a \} \tag{1.5}$$

where v is defined by $v_a = \{ H_a, x_a \}$.

Solving (1.4) gives rise to a kind of arbitrariness which does not appear in gallilean mechanics, where it is possible to take the positions as canonical variables. This is not possible here [8], but we can reasonably settle the question by requiring that x_a coincide with q_a on the surface Σ defined by

$$P.z_{ab} = 0 \tag{1.6}$$

This postulate is mathematically correct because actually (1.4) is, for each x_a, equivalent to a compatible system of $N - 1$ first order linear partial differential equations, while (1.6) which define Σ is just equivalent to $N - 1$ equations.

Moreover, it is easy to check that Σ is never invariant under the infinitesimal transformations generated from any system of $N - 1$ vector fields taken among the X_a [9].

From the physical point of view, Σ has a simple interpretation: it turns

out that for each point of Σ, all the time coordinates are equal in the center-of-mass frame. In this frame, the Poincaré invariant equation (1.6) reduces to

$$x_a^0 = x_b^0$$

Let us call Σ the equal-time surface. Note that in general the $\xi_a . v_a$ do not vanish, so the parameters τ_a which permit to write the hamiltonian equations are generally not the propertimes σ_a.

But assuming that $2H_a$ must be identified with the squared mass m_a^2 fixes the relationship [4] [10]

$$d\tau_a/d\sigma_a = (2H_a/v_a^2)^{1/2}$$

2. N-BODY POTENTIAL FROM BINARY ACTIONS

In equation (1.2) the generalized accelerations cannot be arbitrarily prescribed. They are submitted to a « predictivity condition » which is stronger [2] than a simple integrability condition because its role is also to insure the « individuality » of the particles, namely that each solution x_a will be a function of τ_a only. This condition

$$(v_a . \partial_a + \xi_a . \partial/\partial v_a)\xi_b = 0 \qquad a \neq b \tag{2.1}$$

being non linear, the N-body ξ_a *cannot be written as a simple sum of binary terms*

$$\xi_a = \sum_{b \neq a} \eta_a(x_a, v_a, x_b, v_b) \tag{2.2}$$

where η is the acceleration of a 2-body system.

Expressing the same situation in the hamiltonian langage, we see that, if the hamiltonians have the general form

$$H_a = \frac{1}{2} p_a^2 + V_a \tag{2.3}$$

the condition (1.3) cannot be satisfied when the « potentials » V_a are a simple sum of binary potentials, as it is usual in gallilean mechanics.

Hence, to construct a N-body system from a given binary interaction turns out to be a non trivial problem in relativistic dynamics.

(For instance, having worked out a relativistic version of the 2-body harmonic oscillator [4], we come to ask: how to describe N point particles mutually linked by harmonic forces?)

Solving the problem is a preliminary step in order to perform the program of second quantization [11], as well as in view of a possible action-at-a-distance theory of gravity.

Considering more practical matters and first quantization, the construc-

tion of N-body interactions should provide models for particles made of more than 2 quarks.

In this article, we consider only single-potential systems, i. e. all the V_a in (2.3) are taken equal, so we have

$$H_a = \frac{1}{2} p_a^2 + V \tag{2.4}$$

The condition (1.3) becomes simply

$$\{ V, P_{ab} \cdot y_{ab} \} = 0 \tag{2.5}$$

and the problem is to find V.

This question is completely and explicitly settled for the case N = 2 because in that case P_{ab} coincides with P, which gives rise to a lot of drastical [4] simplifications. We shall see throughout this paper that all the difficulty for N > 2 comes from the fact that $P_{ab} \neq P$.

3. THE EQUAL TIME PRESCRIPTION

Actually (2.5) is a set of N − 1 linear partial differential equations. We have from (2.4)

$$H_a - H_b = P_{ab} \cdot y_{ab}$$

which shows that among the $P_{ab} \cdot y_{ab}$ we have only N − 1 independant quantities, with vanishing mutual Poisson brackets.

Thus each solution of (2.5) can be determined by its values on a surface with co-dimension N − 1 (a submanifold of phase space determined by N − 1 equations). The best choice of initial surface seems to be Σ defined by (1.6).

This choice means that, at equal times (with respect to the center-of-mass frame) the potential V is assigned to coincide with some given function.

Example: for N = 2, requiring that V coincides with kz^2 at equal-times determines that $V = k(z^2 - (z.P)^2/P^2)$ on the whole phase space [4].

Naturally, it is essential that Σ *is not* characteristic for the partial differential system (2.5), in other word Σ is never invariant by the infinitesimal transformations generated by a set of N − 1 independant linear combinations of the $P_{ab} \cdot y_{ab}$. The proof is presented in ref. [9].

Finally, the receipe for solving (2.5) is just to prescribe the potential V at equal times.

Of course, Poincaré invariance is ensured by requiring that, on Σ, V coincide with a Poincaré invariant function. In general this potential has nothing to do with a binary interaction.

But it is natural to consider the most simple situation where V is constructed by a sort of composition of binary interactions.

As we have seen, the principle of such a composition cannot be linear on the whole phase space. But, since we are free to choose the form of V at equal times, we shall naturally assume that V is a sum of binary terms *modulo* a correction which vanishes on Σ.

Considering the single-potential models of 2-body systems, we know [4] that any binary potential takes the form

$$U(\tilde{z}_{12}^2, P_{12}^2, y_{12}^2, y_{12}.P_{12}, \tilde{y}_{12}.\tilde{z}_{12}) \tag{3.1}$$

with

$$\tilde{z}_{12}^2 = z_{12}^2 - (z_{12}.P_{12})^2/P_{12}^2$$
$$\tilde{y}_{12}^2 = y_{12}^2 - (y_{12}.P_{12})^2/P_{12}^2$$

The potential (3.1) can be characterized by the function

$$U(z_{12}^2, P_{12}^2, y_{12}^2, y_{12}.P_{12}, y_{12}.z_{12}) \tag{3.2}$$

which reduces to (3.1) when $P_{12}.z_{12}$ vanishes.

Let us define

$$U_{ab} = U(z_{ab}^2, P_{ab}^2, y_{ab}^2, y_{ab}.P_{ab}, y_{ab}.z_{ab}) \tag{3.3}$$

Applying the usual composition rule at equal times, let us require that, on Σ, the N-body potential V coincides with $\sum_{a<b} U_{ab}$ and write

$$V = \sum_{a<b} U_{ab} + W \tag{3.4}$$

where W vanishes on Σ.

Getting back to the general case of a single-potential model as defined by (2.4) (2.5), we can exhibit a useful combination of H_a and P^2 which is an interesting constant of the motion.

Let us first notice some identities. On one hand, we have almost obviously

$$\sum_{a<b} (p_a^2 + p_b^2) = (N-1) \sum p_a^2 \tag{3.5}$$

as can be easily checked by recurrence.

On the other hand, direct computation yields

$$P^2 = \sum p_a^2 + 2 \sum_{a<b} p_a p_b \tag{3.6}$$
$$4y_{ab}^2 = p_a^2 + p_b^2 - 2p_a p_b$$

Thus

$$P^2 + 4 \sum_{a<b} y_{ab}^2 = \sum p_a^2 + \sum_{a<b} (p_a^2 + p_b^2)$$

and by (3.5) we finally have

$$P^2 + 4\sum_{a<b} y_{ab}^2 = N\sum p_a^2 \tag{3.9}$$

which holds identically on the whole phase space, whatever the model can be. Now, we take (2.4) into account. Then by addition, we have

$$2N\Sigma H_a = N\Sigma p_a^2 + 2N^2V \tag{3.10}$$

Inserting (3.9) into (3.10) yields

$$2N\sum H_a = P^2 + 4\sum_{a<b} y_{ab}^2 + 2N^2V \tag{3.11}$$

Finally, by substraction of P^2 we obtain the combination

$$2N\sum H_a - P^2 = 4\sum_{a<b} y_{ab}^2 + 2N^2V \tag{3.12}$$

which remains constant in the motion. This is the N-body form of a quantity that we have already met in previous works about two-body dynamics [3] [12].

Let us emphasize that *only in the special* $N = 2$ *case*, where $y_{12}.P$ is conserved, it is also possible to substract from (3.12) the contribution of $(y_{12}.P)^2/P^2$ and to obtain a simpler constant of the motion, related to the relative energy.

In the case of $N > 2$, it would be useless splitting y_{ab}^2 by substraction of $(y_{ab}.P)^2/P^2$, since $y_{ab}.P$ is not constant.

In others words, the orthogonal decomposition of y_{ab} relative to P provides no simplification.

The decomposition relative to P_{ab} would not be better in general, but in the *case of equal masses*, all the $y_{ab}.P_{ab}$ vanish, which permits the statement that all the y_{ab} are purely spacelike, provided an appropriate assumption is added. This does not allow to define a conserved relative energy, as we did for $N = 2$, but permits a discussion involving the sign of V.

Finally (3.12) provides a criterium for bound states, as follows more precisely below:

Bound states

To make sure that we have a bound state, it is sufficient to prove that, with respect to the center-of-mass frame, $x_a^i - x_b^i$ remains bounded in the motion. At equal times in this frame (i. e. on the manifold Σ) $x_a - x_b$ coincide with $q_a - q_b = z_{ab}$, and all the z_{ab} are space-like when restricted to Σ (because orthogonal to P).

Thus all we have to prove is that

$$| z_{ab}^2 | < \text{positive const.} \tag{3.14}$$

holds on the surface Σ, for the motion that is considered.

Therefore, all we need, and shall use now, is a few inequalities valid at equal times.

In the rest of this section, let us assume that $V|_\Sigma \leqslant 0$.

For physically acceptable orbits $m_a^2 \geqslant 0$.

Let us consider the orbits corresponding to $m_a^2 > 0$. Then, as a result of (2.4), all the p_a^2 remain > 0 on Σ. Since the $p_a|_\Sigma$ remain time-like, they cannot change their orientation.

So we have the right to consider the case where all the $p_a|_\Sigma$ are future oriented (This property can be realized by suitable initial conditions).

Then all the $P_{ab}|_\Sigma$ are time-like vectors.

Now assume, for the rest of Section 3, that all the particles have *equal masses*

$$y_{ab} . P_{ab} = 0 \tag{3.15}$$

Then

$$y_{ab}^2 |_\Sigma \leqslant 0 \tag{3.16}$$

Therefore the constancy of (3.12) implies that $| V |$ is bounded on Σ. In order to satisfy (3.14) let us specify the form of V at equal times: assume that $V|_\Sigma$ is given by composition of binary terms, as in (3.3) (3.4), with

$$U_{ab} = - kf(- z_{ab}^2), \tag{3.17}$$

where k is some positive constant and f is a non-singular monotonously rising function.

Then $| U_{ab} |$ and therefore also $| z_{ab}^2 |$ remains bounded on Σ, for a given motion of the class that we have defined above. This is characteristic of a bound state.

In particular, this argument is applicable to the *harmonic potential* [13]

$$U_{ab} = k z_{ab}^2 \tag{3.18}$$

4. N-BODY WAVE EQUATIONS

By the most straightforward correspondance principle

$$p_a^\alpha \to \partial / \partial x_a^\alpha \tag{4.1}$$

$$q_a^\alpha \to \text{multiplication by } x_a^\alpha \tag{4.2}$$

the hamiltonians become operators acting on a wave function $\Psi(x_1, \ldots, x_N)$.

More precisely, $2H_a$ becomes a squared-mass operator and we write the wave equations [3]

$$2H_a\Psi = m_a^2\Psi \tag{4.3}$$

This procedure comes out very naturally since it provides N Klein-Gordon equations when applied to the free particle hamiltonians $\frac{1}{2}p_a^2$.

The Poisson bracket condition (1.3) has now the quantum mechanical counterpart

$$[H_a, H_b] = 0 \tag{4.4}$$

which insures the compatibility of (4.3). Actually (4.4) is stronger than the compatibility condition, this feature owing to the fact that our picture is supposed to admit a classical analogue with world-lines.

As in previous sections, we shall consider only single-potential models. Then (4.3) is equivalent to the system

$$2\Sigma H_a\Psi = \Sigma m_a^2\Psi \tag{4.5}$$

$$y_{ab}.P_{ab}\Psi = (m_a^2 - m_b^2)\Psi \tag{4.6}$$

with

$$y_{ab} = -i(\partial/\partial x_a - \partial/\partial x_b), \text{ etc.}$$

By means of (3.11) which holds as well in terms of operators, eq. (4.5) can be equivalently replaced by

$$\left(P^2 + 4\sum_{a<b} y_{ab}^2 + 2N^2V\right)\Psi = N\sum m_a^2\Psi \tag{4.7}$$

And for eigenstates of P^2, (4.7) reduces simply to the equation

$$\left(4\sum_{a<b} y_{ab}^2 + 2N^2V\right)\Psi = \left(N\sum m_a^2 - K^2\right)\Psi \tag{4.8}$$

where K^2 is the eigenvalue of P^2.

Similarly to a situation which occurs in various different approaches of [14] the N-body relativistic problem, we have a principal wave equation, (4.7) or (4.8), involving the interaction, supplemented with $N - 1$ subsidiary conditions namely the system (4.6).

Of course, (4.4) imposes on V the commutation condition

$$[V, P_{ab}.y_{ab}] = 0 \tag{4.9}$$

which is a source of difficulty from the practical point of view, since, for $N > 2$, we do not know in explicit form the solutions of (4.9).

Moreover, our equations (4.6) (which are not really « subsidiary », but the necessary consequences of a certain approach related to classical

equations of motion) do not permit immediatly (except for N = 2) to separate the dependence of Ψ upon the relative times $x_a^0 - x_b^0$. This is related to the difficulty of separating the relative energy, that we have met at the classical level in Section 3, and in contrast with most subsidiary conditions usually proposed in the litterature, which would for instance involve $y_{ab}.P$ instead of $y_{ab}.P_{ab}$.

Owing to the fact that $P_{ab} \neq P$ it must be also stressed that, except for N = 2 [*12*] there is not evidence that (4.8) can be reduced to a so-called relativistic Schrödinger equation involving the relative laplacians instead of the relative Dalembert operators $- y_{ab}^2$.

The three difficulties we just mentioned deserve a little discussion. For the determination of V, which is of crucial importance if one wishes to write down explicit models, a possible line of investigation is the following: eq. (3.4) and the condition that W vanishes on Σ, suggests that the classical W admits formally an expansion in powers of the different $z_{ab}.P$, each term being at least linear.

Up to the well-known factor-ordering problems which are not specific of relativistic dynamics, formal operator expressions will be assigned to V, U_{ab} and W by quantization. At least, the operator expression for W is made simpler because all the $z_{ab}.P$ commute with each other.

Consider the vector space of functions $\varphi_{(x_1,\ldots x_n)}$ satisfying

$$z_{ab}.P\varphi = 0 \tag{4.10}$$

(where z_{ab} means multiplication by $x_a - x_b$, etc.)

and beware that these functions *are not solutions* of (4.7) (4.6) in general. The matrix element $\langle \varphi_1 W \varphi_2 \rangle$ vanish when φ_1 and φ_2 are solutions of (4.10).

This formal argument suggests that the quantum mechanical analog of the equal-time prescription (3.4) could be the same formula with the condition that

$$\langle \varphi_1 W \varphi_2 \rangle = 0$$

when φ_1 and φ_2 satisfy (4.10).

It goes without saying that any attempt to be more rigorous will require correct definitions about norms, scalar product of wave-functions, etc. The question rised by the lack of Schrödinger equation and the apparent discrepancy between our equations (4.6) and the subsidiary conditions commonly proposed in the litterature can be answered as follows [*15*]: our wave equations are just the diagonalization of N squared-mass operators. But, even if completed by the linear momentum, they do not form a complete set of commuting observables (Classicaly they do not form a maximal set of integrals in involution, owing to the number of dimension of phase-space). Therefore, it is always possible, at the price of a lost of generality, to write extra-equations compatible with the system (4.7) (4.6) so it may

happen that after taking these extra-equations into account, our system reduce to something more familiar.

That is why it is sometimes difficult to claim that different ways of writing wave-equations are in conflict.

Let us end with a little remark. It seems tempting to write

$$y_{ab} \cdot \mathrm{P}\Psi = (m_a^2 - m_b^2)\Psi$$

instead of (4.6), because (4.9) would be replaced by a simpler condition which is explicitly solvable by taking for V a function of the spatial square

$$z_{ab}^2 - (z_{ab} \cdot \mathrm{P})^2/\mathrm{P}^2$$

This would provide an explicit exemple of the system (4.3), satisfying (4.4). Such a system is not a single-potential model, which is nothing horrible in itself (because (2.4) is assumed only for simplicity). But it does not reduce to a set of Klein-Gordon equations when V is zero.

That is why, in the multi-time approach, and we may like it or not, the most reasonable wave-equation system remains (4.6) (4.7)—or alternatively (4.6) (4.8)—where condition (4.4) is to be fulfilled.

5. CONCLUSION

It may look disappointing that, even for the simple harmonic model, we miss the explicit knowledge of the potential outside the equal-time surface. Fortunately, the result that we have obtained concerning boundness is a hint for the hope that we can go without this information in many cases.

After all, it seems very well that the contents of a model is determined by its equal-time behavior, the extension of the potential outside Σ having just to exist, as a price paid for the redundance of formalism which insures the manifest covariance.

At least, this point holds classically, if it remains delicate at the quantum level (our argument about $\langle\, \mathrm{W} \,\rangle$ cannot pretend to be more than heuristic). In so far as the harmonic interaction is concerned, we expect that a more subtle analyzis would produce a general result valid for inequal masses as well. We have left open the question of solving the equations of harmonic motion (in the center-of-mass frame, of course).

It would be interesting to look for qualitative results about another kind of interactions, namely when $\mathrm{U}_{ab} \to 0$ for large spatial separations. In particular, the delicate problem of cluster decomposition is worthy of investigations.

From the point of view of quantum mechanics, the comparison with other approaches should be a little sharpened.

The introduction of spin by standard methods seems naturally possible.

The question remains whether it will only involve longer calculations, or will present special difficulties.

Far from exhausting the subject, we have only opened a way of research, with a lot of basic principles and a short piece of application.

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