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MINIMALITY AND UNIQUE ERGODICITY FOR SUBGROUP ACTIONS

by S. MOZES and B. WEISS

Let G be a Lie group, H a closed subgroup, and Γ a lattice in G. H acts on the homogeneous space G/Γ by left translations: $h \cdot (g\Gamma) = (hg)\Gamma$. We call such an action a *subgroup action*. The action is *minimal* if every H-orbit is dense in G/Γ, and *uniquely ergodic* if the G-invariant probablity measure is the only H-invariant Borel probability measure on G/Γ. In [F], Hillel Furstenberg asked whether for subgroup actions with Γ cocompact, minimality implies unique ergodicity. Furstenberg proved that for $G = SL(2, \mathbb{R}), \Gamma$ a cocompact lattice in G, and

$$H = \left\{ \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} : t \in \mathbb{R} \right\},$$

the flow is both minimal and uniquely ergodic. In [V], W.A. Veech proved the same for G a semisimple Lie group without compact factors, Γ a cocompact lattice, and $H = N$, where $G = KAN$ is an Iwasawa decomposition of G. In [St], A.N. Starkov proved that when G is any connected Lie group, Γ any lattice, and H a one-parameter subgroup, the H action is minimal if and only if it is uniquely ergodic. Using Marina Ratner's results obtained in proving Raghunathan's conjectures (see [R]), it is easy to prove that minimality and unique ergodicity are equivalent when G is a connected Lie group, Γ is a lattice in G and H is a subgroup of G generated by the one-parameter Ad-unipotent subgroups of G contained in H. In this paper we give an affirmative answer to Furstenberg's question in another case. Namely, we prove:

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THEOREM 1. — *Let* $\mathbf{G}$ *be a reductive algebraic* $\mathbb{R}$*-group with compact center, and let* $\mathbf{H}$ *be an* $\mathbb{R}$*-subgroup of* $\mathbf{G}$. *Let* H *and* G *denote* $\mathbf{H}^0_{\mathbb{R}}$ *and* $\mathbf{G}^0_{\mathbb{R}}$, *respectively, and let* Γ *be a lattice in* G. *If the action of* H *on* G/Γ *is minimal then it is uniquely ergodic.*

Regarding the converse (whether unique ergodicity implies minimality), it is easy to see that when Γ is cocompact and H is amenable, unique ergodicity implies minimality. However, if Γ is not cocompact, Raghunathan (see [W]) has given an example of a (nonalgebraic) subgroup action which is uniquely ergodic but not minimal.

Our method of proof relies on Ratner's results obtained in proving Raghunathan's conjectures, and the method of 'locally linearizing' G/Γ, developed by S.G. Dani, G.A. Margulis, and Nimish Shah in connection with the same conjecture. See the survey articles [R] and [D2] and the references therein. It should be noted that the results of G.A. Margulis and G.M. Tomanov in [MaT] may be used to deduce the results of the present note. In particular, our Corollary 6 follows directly from their results.

It's likely that in Theorem 1, the assumptions on G and Γ can be relaxed. In particular, does Theorem 1 hold assuming only that G is a connected Lie group and Γ is a closed Lie subgroup such that there exists a G-invariant probability measure on G/Γ?

Notation. — Throughout this paper, boldface letters such as $\mathbf{G}$ will denote $\mathbb{R}$-algebraic groups and uppercase letters such as G will denote $\mathbf{G}^0_{\mathbb{R}}$ (the connected component of the identity in the real points of $\mathbf{G}$). We often write properties of G as properties of $\mathbf{G}$, e.g., we may say that $\mathbf{G}$ has no compact factors, and so on.

The following result is due to Mostow ([M]) for cocompact lattices and to Prasad and Raghunathan ([PR]) in general:

PROPOSITION 2. — *Let* G *be a semisimple Lie group,* Γ *a lattice in* G *and* H *a Cartan subgroup in* G. *Then there is* $g \in G$ *such that* $g^{-1}Hg \cap \Gamma$ *is a lattice in* $g^{-1}Hg$. *In particular, the orbit* $Hg\Gamma$ *is closed in* G/Γ.

COROLLARY 3. — *Let* $\mathbf{G}$ *be a reductive* $\mathbb{R}$*-algebraic group with compact center, let* Γ *be a lattice in* G *and let* $\mathbf{H}$ *be an* $\mathbb{R}$*-torus in* $\mathbf{G}$. *Then* H *has an orbit whose closure* $\overline{Hg\Gamma}$ *is contained in a closed orbit* $Lg\Gamma$, *where* L *is an abelian subgroup in* G *containing* H. □

We say that a group H acts *pointwise minimally* on a topological space X if for any $x \in X$, if $y \in \overline{Hx}$ then $x \in \overline{Hy}$. It is easy to see that if the action of H on X is pointwise minimal then X is the disjoint union of closed subsets on which H acts minimally.

PROPOSITION 4. — *Let G be a Lie group, let Γ be a closed subgroup of G and let H be a subgroup of G which acts minimally on G/Γ. If H_0 is a cocompact normal subgroup of H then H_0 acts pointwise minimally on G/Γ. If in addition G is semisimple, connected and without compact factors, and Γ is a lattice in G then H_0 acts minimally on G/Γ.*

Proof. — We first show the first assertion. Suppose $y \in \overline{H_0 x}$. Since H acts minimally, there is a sequence $j_n \in H$ such that $j_n y \to x$. Writing $j_n = k_n h_n$ where the k_n belong to a compact subset of H and $h_n \in H_0$, and passing to a subsequence, we get $k^{-1} h_n y \to x$, where k is some fixed element of H. We have $kx \in \overline{H_0 y}$ and since H_0 is normal in H,

$$k\overline{H_0 x} \subset \overline{H_0 y} \subset \overline{H_0 x}. \tag{1}$$

Let us show the reverse inclusion. Let $\pi : H \to H/H_0$ be the natural map and $e \in H$ the identity. The sequence $\{\pi(k^n) : n = 1, 2, \ldots\}$ has an accumulation point in H/H_0 and this implies that $\pi(e) \in \overline{\{\pi(k)^m : m = 2, 3, \ldots\}}$. Therefore there are $m_i \in \mathbb{N}, h_i \in H_0$ such that $k^{m_i} h_i \to e$. Applying k to both sides of (1) we see that for any $n \geq 1$,

$$k^n \overline{H_0 x} \subset k\overline{H_0 x}. \tag{2}$$

Now let $z \in \overline{H_0 x}$ and let $l_i \in H_0$ such that $l_i x \to z$. We obtain $k^{m_i} h_i l_i x \to z$, and by (2), $z \in k\overline{H_0 x}$.

Now let us prove the second assertion. We now know that if any point in G/Γ has a dense orbit under H_0, so do all other points. So to show that the action of H_0 is minimal, it is enough to show that it is ergodic with respect to the G-invariant measure on G/Γ. It is known that the action of H is ergodic if and only if $\psi(H)$ is noncompact whenever $\psi : G \to G'$ is a nontrivial homomorphism with $\ker(\psi) \cap \Gamma$ a lattice in $\ker(\psi)$. Indeed, for Γ irreducible (i.e., when for any such ψ, $\ker(\psi)$ is either finite or has finite index in G), this is Moore's theorem (see [D2], Theorem 2.11), and for Γ a general lattice this follows from a more general form of Moore's theorem (see [D2], Theorem 2.12). Since H acts minimally on G/Γ it acts minimally on any such factor $\psi(G)/\psi(\Gamma)$, and thus $\psi(H)$ is noncompact; therefore so is $\psi(H_0)$; and hence the action of H_0 on G/Γ is also ergodic. □

Recall that two lattices are called *commensurable* if their intersection is of finite index in both. In the sequel we will need the fact that if G is a connected Lie group, Γ is a lattice and H is a subgroup, minimality of the action of H is a property which does not change when exchanging Γ with a commensurable lattice. The minimality of the action of H on G/Γ is equivalent to the minimality of the action of Γ on $H\backslash G$. If Γ' contains Γ then it is obvious that the minimality of the action of Γ implies that of Γ'. If Γ' is of finite index in Γ then we can replace it by a normal subgroup contained in it which is also of finite index, and by Proposition 4, we see that its action on $H\backslash G$ is pointwise minimal. Therefore $H\backslash G$ is a finite union of closed Γ' invariant sets, on each of which Γ' acts minimally. By connectedness the action of Γ' on $H\backslash G$ must be minimal. We remark that if G is also assumed to be semisimple, then unique ergodicity of the action of H also depends only on the commensurability class of Γ. As before it is obvious that if Γ' contains Γ as a subgroup of finite index and the action of H on G/Γ is uniquely ergodic, then so is the action of H on G/Γ'. One can use Moore's ergodicity theorem to prove that the same holds if Γ' is a finite index subgroup of Γ. We will not be using this fact.

The following lemma is where we utilize Ratner's results and the concept of 'tubes'. The lemma was not stated explicitly in [Mo] but nevertheless its proof is contained in (and actually comprises the main part of) the proof of Theorem 1 in that paper. For the last part of the lemma, see [DMa], Theorem 3.4.

LEMMA 5. — *Let $\mathbf{G}$ be a connected $\mathbb{R}$-algebraic group, let Γ be a lattice in G, and let H be a Lie subgroup. Let V be the normal subgroup of H generated by the one-parameter Ad-unipotent subgroups contained in H, and suppose μ is an H-invariant H-ergodic probability measure on G/Γ. Let $\mathfrak{G}$ denote the Lie algebra of G.*

Then there is a subgroup F of G and $g_0 \in G$ such that the following hold:

1. *$g_0 F g_0^{-1}$ contains V.*
2. *$F \cap \Gamma$ is a lattice in F.*
3. *$g_0 F\Gamma = \overline{V g_0 \Gamma}$.*
4. *There is an ergodic decomposition of μ with respect to V, $\mu = \int_{\mathcal{P}} \nu d\pi(\nu)$, where $\mathcal{P}$ is the set of V-invariant V-ergodic Borel probability measures on G/Γ, π is a measure on $\mathcal{P}$, and for π-almost every ν, ν is invariant under a conjugate of F.*

5. *Let X denote the vector space $\bigwedge_1^d \mathfrak{G}$, where $d = \dim F$. Let $P(X)$ denote the projective space of lines in X, and if x is a nonzero vector in X, let $\bar{x}$ denote its image in $P(X)$. Let $0 \neq f \in \bigwedge_1^d \mathfrak{F}$, where $\mathfrak{F}$ is the Lie algebra of F, let $\rho : G \to GL(X)$ be the representation $\rho = \bigwedge_1^d \mathrm{Ad}$, and denote by $\bar{\rho}$ the induced representation on $P(X)$. Then for any one-parameter subgroup $\{h(t) : t \in \mathbb{R}\}$ of H, there is a sequence $t_j \to \infty$ such that $\bar{\rho}(h(t_j))\bar{\rho}(g_0)\bar{f} \underset{j\to\infty}{\longrightarrow} \bar{\rho}(g_0)\bar{f}$, and $|\det(\mathrm{Ad}(h(t_j))|\,\mathrm{Ad}(g_0)\mathfrak{F})| \underset{j\to\infty}{\longrightarrow} 1$.*

6. *Let $N = \{g \in G : \rho(g)f = f\}$. Then the orbit $N\Gamma$ is closed in G/Γ. Equivalently, $\rho(\Gamma)f$ is a discrete subset of X.*

COROLLARY 6. — *Retaining the notation of Lemma 5, suppose H is the real points of a real algebraic group $\mathbf{H} < \mathbf{G}$. Then there is a cocompact normal subgroup H' of H which stabilizes $f_0 = \rho(g_0)f$.*

Proof. — Let W denote the subspace spanned by $\rho(H)f_0$ in X, so W is a $\rho(H)$-invariant subspace. Since V is contained in $g_0Fg_0^{-1}$, it stabilizes the line through f_0, and since V is generated by Ad-unipotent elements, there is no nontrivial rational character on V. Therefore $\rho(V)$ fixes f_0. Since V is normal in H, this implies that $\rho(V)$ fixes every vector in W.

Let $\mathbf{G}_0$ denote the algebraic subgroup of $\mathbf{G}$ leaving W invariant, let ρ_0 denote the restriction of ρ to $\mathbf{G}_0$ and W, and let $H_0 = \rho_0(H)$. It will suffice to show that H_0 is a compact subgroup of $GL(W)$, for then $H' = \ker \rho_0 \cap H$ satisfies the required conclusions.

Now H_0 is closed in $GL(W)$, since the map $\rho_0 : \mathbf{G}_0 \to GL(W)$ is $\mathbb{R}$-algebraic, and since $\mathbf{H}$ is an $\mathbb{R}$-algebraic subgroup of $\mathbf{G}$. Let us show that H_0 is bounded in $GL(W)$. It is an image of H/V and therefore contains no $\mathbb{R}$-unipotents. An $\mathbb{R}$-algebraic group without unipotents is reductive (its unipotent radical is defined over $\mathbb{R}$ and therefore trivial), with compact semisimple part (since non-compact semisimple $\mathbb{R}$-groups are generated by $\mathbb{R}$-unipotents). Therefore H_0 is an almost direct product of a compact subgroup and a connected abelian group, and we only have to show that this abelian subgroup, which we denote by A, is bounded. For this it suffices to show that for any $w \in W$, the orbit Aw is bounded. For the last statement it suffices to show that Af_0 is bounded, for if $Af_0 \subset K$, where K is a compact subset of W, and $w = \sum a_i\rho_0(h_i)f_0$, then $Aw \subset \sum a_iA\rho_0(h_i)f_0 = \sum a_i\rho_0(h_i)Af_0 \subset \sum a_i\rho_0(h_i)K$, which is also compact.

Choose a basis for W in which all the matrices in A are put simultaneously in real Jordan form (see e.g. [DP], p. 43). This Jordan form has eigenvalues (characters $A \to \mathbb{R}^*$) and two by two blocks (homomorphisms $A \to \mathbb{R}^* \times SO(2,\mathbb{R})$) on the diagonal. Since A is reductive, there are no nonzero entries outside these blocks. From part 5 of Lemma 5, we see that the diagonal blocks must all have determinant one. By connectedness, this implies that all the eigenvalues are identically one, and the homomorphisms are in fact into $SO(2,\mathbb{R})$. From this it follows that Af_0 is bounded in W.□

Proof of Theorem 1. — *Step 1:* First let us show that we may assume that for any surjective homomorphism $\pi : \mathbf{G} \to \mathbf{K}$, where K is compact, we have $\overline{\pi(\Gamma)} = K$.

To see this, suppose $K_1 = \overline{\pi(\Gamma)}$ is a proper subgroup of K, let K_0 be the component of the identity in K_1, $\mathbf{G}_0 = \pi^{-1}(K_0)$, $\Gamma_0 = \Gamma \cap \mathbf{G}_0$, $\mathbf{H}_0 = \mathbf{H} \cap \mathbf{G}_0$. The subgroup $\mathbf{G}_0$ is also reductive with compact center. Since K_0 is of finite index in K_1, Γ_0 is of finite index in Γ and therefore without loss of generality we may assume that $\Gamma \subset G_0$. Let $\bar{\pi} : G/\Gamma \to K/K_0$ be the map induced by π. The action of $\pi(H)$ on K/K_0 is a factor of the action of H on G/Γ and therefore is minimal. Also since H is algebraic, $\pi(H)$ is compact and therefore $\pi(H)K_0 = K$. This implies that $HG_0 = G$ and that H_0 is cocompact in H. Let x and y be two points in $G_0/\Gamma \subset G/\Gamma$. Since the action of H is minimal there is a sequence $h_n \in H$ such that $h_n x \to y$. Since H_0 is cocompact in H we can write $h_n = l_n h'_n$, where the l_n are in a compact subset of H and $h'_n \in H_0$. Passing to a subsequence we see that there is $l_0 \in H$ such that $l_0 h'_n x \to y$. This implies that $\pi(l_0)K_0 = \pi(l_0)\bar{\pi}(h'_n x) \to \bar{\pi}(y) = K_0$. Therefore $l_0 \in G_0$. This shows that the action of H_0 on G_0/Γ_0 is minimal.

Now let μ be an H-invariant measure on G/Γ. The projection $\bar{\pi}_*\mu$ of μ to K/K_0 is $\pi(H)$-invariant and therefore is K-invariant. Decompose μ with respect to $\bar{\pi}$ and write $\mu = \int_{K/K_0} \nu_x d\bar{\pi}_*\mu(x)$, where for almost every $x \in K/K_0$, ν_x is a measure on $\bar{\pi}^{-1}(x)$ invariant under the conjugate of H_0 in H stabilizing $\bar{\pi}^{-1}(x)$. Replacing H with a conjugate we can assume in particular that $\nu_{[K_0]}$ is an H_0-invariant measure on G_0/Γ and that almost every ν_x is a translation of $\nu_{[K_0]}$. We have proved that the action of H_0 on G_0/Γ_0 is minimal and therefore by an induction on the dimension of G, it is uniquely ergodic. Therefore the measure $\nu_{[K_0]}$ is G_0-invariant and since $HG_0 = G$, the measure μ is G-invariant.

Step 2: We now prove that if $\mathbf{N}$ is a normal $\mathbb{R}$-subgroup of $\mathbf{G}$, such that

$\pi(\Gamma)$ is a lattice in G/N (here $\pi : \mathbf{G} \to \mathbf{G}/\mathbf{N}$ is the projection) and G/N contains a noncompact semisimple subgroup, then $\pi(H)$ contains nontrivial one parameter Ad-unipotent subgroups. Indeed, π induces an equivariant map $G/\Gamma \to (G/N)/\pi(\Gamma)$ and $H' = \pi(H)$ acts minimally on $(G/N)/\pi(\Gamma)$. (See diagram)

$$\begin{array}{ccc} G & \longrightarrow & G/\Gamma \\ \pi \downarrow & & \downarrow \\ G/N & \longrightarrow & (G/N)/\pi(\Gamma) \\ \tau \downarrow & & \downarrow \\ G' & \longrightarrow & G'/\tau(\pi(\Gamma)) \end{array}$$

If H' does not contain one-parameter Ad-unipotent subgroups, then H' contains a cocompact torus H'_0. If G' is the product of all non-compact simple factors of G/N, $\tau : G/N \to G'$ the projection, and $K' = \ker\tau$, then by Proposition 4, the action of $\tau(H'_0)$ on $G'/\tau(\pi(\Gamma))$ is minimal, and therefore so is the action of $K'H'_0$ on $(G/N)/\pi(\Gamma)$. By Corollary 3, there is $x \in G/N$ such that the closure of $H'_0 x\pi(\Gamma)$ is contained in the closed orbit $Lx\pi(\Gamma)$, where L is an abelian subgroup of $\mathbf{G}/\mathbf{N}$. This means that the orbit $K'Lx\pi(\Gamma)$ is also closed in $(G/N)/\pi(\Gamma)$. By minimality of the action of $K'H'_0$, $(G/N)/\pi(\Gamma) = K'Lx\pi(\Gamma)/\pi(\Gamma)$. Since G/N is connected and $\pi(\Gamma)$ is discrete, this implies that $G/N = K'L$ so $\mathbf{G}/\mathbf{N}$ is a compact extension of an abelian group, which contradicts the assumption on $\mathbf{G}/\mathbf{N}$.

Step 3: Now let μ be an H-invariant H-ergodic probablity measure on G/Γ, let us show that μ is G-invariant. Let V be the subgroup of H generated by Ad-unipotent one-parameter subgroups of H, and let F, ρ, f, g_0 be as in the statement of Lemma 5. As unique ergodicity and minimality are not affected by conjugation we may replace μ by $g_0^{-1}\mu$ and H with $g_0 H g_0^{-1}$ and assume that g_0 is trivial and that $f = \rho(g_0)f$.

Let ν be an ergodic component of μ with respect to V. By Lemma 5, ν can be assumed to be invariant under a conjugate of F.

Let H' be the cocompact normal subgroup of H given by Corollary 6. Let $\hat{\mathbf{G}}$ denote the product of all noncompact factors of $\mathbf{G}$. $\hat{\mathbf{G}}$ is either semisimple without compact factors, or trivial; in the latter case G is compact, and the theorem is obvious. So assume that $\hat{\mathbf{G}}$ is nontrivial, and let $\theta : \mathbf{G} \to \hat{\mathbf{G}}$ denote the projection, so K is a compact normal subgroup of G, where $\mathbf{K} = \ker\theta$. Then $\theta(H')$ acts minimally on $\hat{G}/\theta(\Gamma)$ by Proposition 4, and therefore KH' acts minimally on G/Γ. By part 6 of Lemma 5, the orbit of H' in G/Γ is contained in a closed subset $L\Gamma$, where $\mathbf{L} = \{g \in \mathbf{G} : \rho(g)f = f\}$, a subgroup of $\mathbf{G}$ which contains H'

and normalizes $\mathbf{F}$. Therefore $KL\Gamma$ is a closed subset of G/Γ containing $KH'\Gamma$, and this implies that $KL\Gamma/\Gamma = G/\Gamma$, and by connectedness of G and discreteness of Γ, that $\mathbf{KL} = \mathbf{G}$.

By Step 1, Γ projects densely on every compact factor of G. Dani's version of the Borel density theorem (see [D1]) states that if an algebraic group acts algebraically on an algebraic variety V, and λ is a measure on V, then the pointwise fixer of the support of λ is a normal cocompact algebraic subgroup of the group preserving λ. Using this theorem with V the quotient of G by the Zariski closure of Γ we see that Γ is Zariski dense in $\mathbf{G}$. Now $\rho(G)f = \rho(KL)f = \rho(K)f$ is a compact subset of X in which $\rho(\Gamma)f$ is discrete, by part 6 of Lemma 5, and therefore finite. Therefore $\rho(\Gamma)f$ is equal to its Zariski closure, and by the connectedness of G, we have that $\rho(G)f = f$, that is, F is normal in G.

By part 3 of Lemma 5 we see that $F\Gamma$ is closed in G/Γ and therefore the projection of Γ in G/F is discrete. Since F contains all the Ad-unipotent one parameter subgroups of H, we obtain by Step 2 that G/F does not contain a noncompact semisimple subgroup, and is therefore compact. Hence by Step 1, $F = G$. This implies that ν, and hence also μ, is G-invariant. □

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