ANNALES DE L'INSTITUT FOURIER

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Annales de l'institut Fourier, tome 24, nº 3 (1974), p. 43-55

<http://www.numdam.org/item?id=AIF_1974__24_3_43_0>

Ann. Inst. Fourier, Grenoble
24,3 (1974), 43-55

UNIFORM MAPS INTO NORMED SPACES

by Zdeněk FROLIK

Thirteen properties of uniform spaces are shown to be equivalent. The most important properties seem to be those related to modules of uniformly continuous mappings into normed spaces (condition (3), (4) and (5) in Theorem 1 below), and to partitions of unity (condition (6)). The spaces satisfying these equivalent conditions seem to be useful for the theory of uniform spaces as well as for applications in analysis. Therefore we do not want to choose a name for them before the whole theory is developed, and basic applications are shown. In § 5 a short survey of these spaces is given.

If X is a uniform space we denote by αX the set X endowed with the finest uniformity which is topologically equivalent to X. The set of all uniformly continuous mappings of X into Y is denoted by U(X, Y). If Y = R, then we write simply U(X). If f is a real-valued function then $\operatorname{coz} f$ stands for $\{x \mid fx \neq 0\}$. A cozero set in a uniform space X is a set of the form $\operatorname{coz} f$, $f \in U(X)$. The collection of all cozero sets in X is denoted by cos X (by the way, this defines a functor into paved spaces).

THEOREM 1. – *The following conditions* (1) – (7) *on a uniform space* X *are equivalent :*

1) *If* M *is a metric space, and if* $f : X \to M$ *is uniformly continuous then so is* $f : X \to \alpha M$.

2) *If* M *is a metric space then* U(X, M) *is closed under taking the limits of continuously convergent sequences.*

3) U(X) *is inversion-closed, and for every norm space* B *the real module* U(X, B) *is a* U(X)*-module.*

4) *Condition* (3) *with* B *restricted to Banach spaces.*

5) *Condition* (3) *with* B *restricted to l-spaces,* i.e. $B = l(A)$ *for some* A.

6) *If* $f : X \to l(A)$ *is uniformly continuous,* $f = \Sigma\{f_a x \,.\, a \mid a \in A\}$, *then* $\{\operatorname{coz} f_a \mid a \in A\}$ *is a uniform cover of* X.

7) *If* $\mathfrak{U}$ *is a uniformly σ-discrete completely* coz(X)*-additive cover of* X, *then* $\mathfrak{U}$ *is a uniform cover of* X.

Proof. – It is self-evident that (3) implies (4), and (4) implies (5). Condition (7) implies (1) by a simple application of a non-trivial theorem of A.H. Stone [1], that every open cover of a metric space is refined by a uniformly σ-discrete open cover ; indeed, the preimage of such a cover under a uniformly continuous mapping is a uniformly σ-discrete completely coz-additive cover (every open set in a metric space is a cozero set in the metric uniformity). The implication (1) ⇒ (2) is almost obvious after the definitions are stated ; this is done in § 1. Implication (2) ⇒ (3) is verified in § 2 on modules of uniformly continuous mappings. The implications (5) ⇒ (6), and (6) ⇒ (7) are proved in § 3 devoted to the space $l(A)$ and partitions of unity.

1. Continuous convergence.

Let X be a set and let T be a topological space. A net $\{f_a : X \to T\}$ converges continuously to an $f : X \to T$ if for any net $\{x_b\}$ in X, and for each $x \in X$ such that $f_a x_b \to fx$ for each a, the net $\{f_a x_b\}$ converges to fx (with respect to the product order for indices).

If $\{f_a\}$ continuously converges to f, and if X is a topological space, and if all f_a are continuous then so is f.

This is easy to prove, and has been reproved by many authors, (the present one is not excluded).

Proof of (1) ⇒ (2) in Theorem 1. Assume that a sequence $\{f_n\}$ in U(X , M) converges continuously to f. Consider a uniformly continuous mapping h of X onto a metric space S such that all f_a factorize by h, i.e.

$$\begin{tikzcd} X \arrow[rr, "f_a"] \arrow[dr, "h"'] & & M \\ & S \arrow[ur, "g_a"'] & \end{tikzcd}$$

Thus $f_a = g_a \circ h$ for each a. Clearly we have $f = g \circ h$ for some continuous $g : S \to M$, i.e. uniformly continuous $g : \alpha S \to \alpha M$. By Condition (1) the mapping $h : X \to \alpha S$ is uniformly continuous, hence $f : X \to \alpha M$ is uniformly continuous, hence $f : X \to M$ is uniformly continuous.

An important example of continuous convergence is the following.

Example. – Let $\{f_a : X \to T\}$ be a net, T be a uniform space, and let X be a topological space, the topology of X being projectively generated by all f_a. If $\{f_a\}$ locally uniformly converges to $f : X \to T$, then $\{f_a\}$ converges to f continuously.

That means, if each point of X has a neighborhood U such that $f_a \mid U$ converges to $f \mid U$ in the topology of uniform convergence, then $\{f_a\}$ converges to f continuously. The proof is evident.

2. Modules of uniformly continuous maps.

Let X be a uniform space, and let B be a real normed space. The set $U(X, B)$ of uniformly continuous mappings of X into B is a real linear space with the operations defined pointwise. Let $U^b(X, B)$ be the subset of $U(X, B)$ consisting of all bounded mappings endowed with the sup-norm. Thus $f \in U^b(X, B)$ if and only if $f \in U(X, B)$ and

$$\| f \| = \sup \{ \| fx \|_B \mid x \in X \} < \infty$$

PROPOSITION. – $U^b(X, B)$ *is a* $U^b(X)$*-module (with the natural operation).*

Proof. – We use $U^b(X)$ for $U^b(X, R)$, when R is the space of real numbers. If $f \in U(X)$ and if $g \in U(X, B)$, then

$$f \,.\, g = \{x \to fx \,.\, gx \mid x \in X\} : X \to B$$

In general, $f \,.\, g$ does not belong to $U(X, B)$. However, if f and g are bounded, then $f \,.\, g \in U^b(X, B)$. Apply the following identity :

$$(f \,.\, g)\, x - (f \,.\, g)\, y = fx(gx - gy) + gy(fx - fy) \;.$$

This concludes the proof.

LEMMA 1. – *Condition* (2) *implies that* $U(X, B)$ *is a* $U(X)$ *module.*

Proof. – Let $f \in U(X)$, $f \geqq 1$, $g \in U(X, B)$, $h = f \,.\, g$. Let

$$f_n = \min(n, f)$$

and

$$g_n x = \begin{cases} gx & \text{if} \quad \| gx \| \leqq n \\ n \cdot \dfrac{gx}{\| gx \|} & \text{if} \quad \| gx \| > n \;. \end{cases}$$

By Proposition $h_n = f_n \cdot g_n \in U^b(X, B)$. Assume that $h_n x_k \underset{k}{\to} h_n x$ for each n. The sequence $\{h_n x\}$ converges to hx, $\{f_n x\}$ converges to fx and $\{g_n x\}$ to gx. It follows by an easy argument that h_n converges continuously to h.

LEMMA 2. – *Condition* (2) *with* $M = R$ *implies that* X *is inversion-closed, that means, if* $g \in U(X)$, $gx \neq 0$ *for each* x *then* $1/g \in U(X)$.

Proof. – Define $\{g_n\}$ by the formula in the proof of Lemma 1. Clearly $\{g_n\}$ converges to g continuously.

Proof of (2) $\Rightarrow$ (3). Combine Lemmas 1 and 2.

3. Partitions of unity and $l(A)$.

For any set A let $l(A)$ be a linear subspace of R^A of all $z = \{z_a\}$ such that

$$\| z \| = \Sigma\, \{ | z_a | \mid a \in A\} < \infty$$

Then $\{z \to \|z\|\}$ is a norm on $l(\mathrm{A})$, and $l(\mathrm{A})$ with this norm is a Banach space. Denote by b, $b \in \mathrm{A}$, the element $z = \{z_a\}$ such that $z_a = 1$ if $a = b$, and $z_a = 0$ otherwise, and denote by π_b the restriction of the projection, i.e. $\pi_b z = z_b$. In this notation for any z we have

$$z = \Sigma \{\pi_a z \,.\, a \mid a \in \mathrm{A}\}$$

Furthermore, we denote by $\mathrm{S}(\mathrm{A})$ the unit sphere in $l(\mathrm{A})$, i.e.

$$\mathrm{S}(\mathrm{A}) = \{z \mid \|z\| = 1\} \,.$$

Let X be a set, and let $\{f_a \mid a \in \mathrm{A}\}$ be a family or real valued functions such that $\Sigma\{|f_a x| \mid a \in \mathrm{A}\} < \infty$ for each a. Thus there exists a unique $f : \mathrm{X} \to l(\mathrm{A})$ such that

$$fx = \Sigma \{f_a x \,.\, a \mid a \in \mathrm{A}\} \,,$$

i.e. $f_a = \pi_a \circ f$.

LEMMA 3. – *If* $f : \mathrm{X} \to l(\mathrm{A})$ *is uniformly continuous, then the family* $\{\pi_a \circ f \mid a \in \mathrm{A}\}$ *is uniformly equicontinuous.*

Proof. – The mapping

$$k = f - f : \mathrm{X} \times \mathrm{X} \to l(\mathrm{A})$$

defined by $k\langle x, y\rangle = fx - fy$ is uniformly continuous, and

$$\|fx - fy\| = \Sigma\{|f_a x - f_a y| \mid a \in \mathrm{A}\} \,,$$

and hence

$$\|fx - fy\| \geqq |f_a x - f_a y|$$

for each a. This concludes the proof.

A partition of unity on X is a mapping f of X into some $\mathrm{S}(\mathrm{A})$ such that $\pi_a \circ f \geqq 0$ for each $a \in \mathrm{A}$.

LEMMA 4. – *Condition* (5) *implies* (6).

Proof. – Assume that $f : \mathrm{X} \to l(\mathrm{A})$ is uniformly continuous, $fx \neq 0$ for each x, and $f_a = \pi_a \circ f \geqq 0$ for each a. Hence

$$\mathcal{U} = \{\operatorname{coz} f_a \mid a \in \mathrm{A}\}$$

is a cover of X. We shall show that $\mathcal{U}$ is a uniform cover. First consider the partition of unity

$$\mathrm{F} = 1/\|f\| \cdot f = \Sigma\{\mathrm{F}_a x \cdot a \mid a \in \mathrm{A}\}$$

By Condition (5), F is uniformly continuous.

α) For each x put

$$hx = \sup\{\mathrm{F}_a x \mid a \in \mathrm{A}\}$$

Then

$$h = \{x \rightarrow hx\} : \mathrm{X} \rightarrow \mathrm{R}$$

is uniformly continuous.

Indeed, for each x in X choose an a_x in A such that $\mathrm{F}_{a_x} x > hx - r$, where r is a given positive real number. Choose a uniform pseudometric d on X such that $d < x, y > < 1$ implies $| \mathrm{F}_a x - \mathrm{F}_a y | < r$ for each a. Now if $d < x, y > < 1$ then

$$| hx - hy | \leqslant 2r$$

Indeed

$$hx \leqslant h_{a_x} x + r \leqq h_{a_x} y + 2r \leqq hy + 2r$$

β) Consider the mapping

$$\mathrm{G} = 1/h \cdot \mathrm{F}$$

By Condition (5), G is uniformly continuous. Let $\mathcal{V}$ be a uniform cover of X such that the diameter of G[V] is less than 1/2 for each V in $\mathcal{V}$. We shall show that $\mathcal{V}$ refines $\mathcal{U}$. Let $\mathrm{V} \in \mathcal{V}$, and let $x \in \mathrm{V}$. We shall show that for any $x \in \mathrm{V}$

$$\mathrm{V} \subset \operatorname{coz} f_{a_x} ,$$

where a_x is the index chosen in α with $r = 1/3\,(hx)$. We have $\| \mathrm{G}x \| > 1/hx \,.\, \mathrm{F}_{a_x} x > 2/3$. If $y \in \mathrm{V} - \operatorname{coz} f_{a_x}$, then

$$\| \mathrm{G}y - \mathrm{G}x \| = \Sigma \left| \frac{\mathrm{F}_a x}{hx_2} - \frac{\mathrm{F}_a y}{hy} \right| \geqq \left| \frac{\mathrm{F}_{a_x} x}{hx} - \frac{\mathrm{F}_{a_x} y}{hy} \right| = \frac{\mathrm{F}_{a_x} x}{hx} > \frac{2}{3} \cdot$$

which contradicts the assumption that the diameter of G[V] is less than 1/2.

LEMMA 5. – *If* $\{\mathrm{U}_a \mid a \in \mathrm{A}\}$ *is uniformly σ-discrete completely coz-additive cover of* X, *then there exists a uniformly continuous mapping f of* X *into $l(\mathrm{A})$ such that*

$$U_a = \operatorname{coz} \pi_a \circ f$$

for each a.

Proof. – Choose a cover $\{A_n\}$ of A such that each $\{U_a \mid a \in A_n\}$ is uniformly discrete. The sets $G_n = \cup \{U_a \mid a \in A_n\}$ are cozero sets, and hence we may choose uniformly continuous functions g_n such that $G_n = \operatorname{coz} g_n$, $0 \leqq g_n \leqq 2^{-n}$. For $a \in A_n$ let

$$f_a x = \begin{cases} g_n x & \text{if} \quad x \in U_a \\ 0 & \text{otherwise} \end{cases}$$

Put

$$f_n x = \Sigma \{ f_a x \cdot a \mid a \in A_n \} .$$

We shall check that each f_n is uniformly continuous. Then $f = \Sigma \{f_n\}$ will be uniformly continuous because of the uniform convergence of the series.

Fix n. Choose a uniformly continuous pseudometric d on X such that the distance of each pair U_a, $U_{a'}$ is 1 at least, and $d < x , y > < \frac{1}{2}$ implies $| g_n x - g_n y | < \epsilon$. If $d < x , y > < 1$, then

$$f_n x - f_n y = f_a x - f_a y = g_n x - g_n y$$

for some a, and hence

$$| f_a x - f_a y | = | g_n x - g_n y | < \epsilon .$$

This concludes the proof.

Proof of (6) $\Rightarrow$ (7). Lemma 5. This concludes the proof of Theorem 1.

4. Inversion-closed spaces.

Recall that a uniform space X is called inversion-closed (or U(X) is called inversion-closed) if for each $f \in U(X)$, $fx \neq 0$ for each x, $1/f$ is uniformly continuous. Usefulness of inversion-closed spaces will be indicated below. We begin with the following list of equivalent conditions.

THEOREM 2. – *The following conditions on a space* X *are equivalent* :

a) X *is inversion-closed.*

b) *If the preimage of cozero sets under an* $f : X \to R$ *are cozero sets in* X, *then* $f \in U(X)$.

c) *If* $f : X \to R - (0))$ *is uniformly continuous then so is* $f : X \to \alpha(R - (0))$.

c′) *If* $f : X \to]\!]0, \to 1[\![$ *is uniformly continuous then so is* $f : X \to \alpha\,]\!]0, \to [\![$.

Proof. – We shall prove a) ⇒ b) ⇒ c) ⇒ c′) ⇒ a). It is easy to see that c′) implies a) ($1/fx = \{y \to 1/y\} \circ f$), since $\alpha\,]\!]0, \to [\![$ is a subspace of $\alpha(R - (0))$ and $]\!]0, \to [\![$ is a subspace of $R - (0)$, evidently c) implies c′). Assume b) and let $f : X \to R - (0)$ be uniformly continuous. Hence f satisfies the assumption in b) (with R replaced by $R - (0)$), and hence each $g \circ f : X \to R$, with $g : R - (0) \to R$ continuous, satisfies the assumption in b), and hence by b) each $g \circ f$ is uniformly continuous. Since $\alpha(R - (0))$ is projectively generated by all g (see the subsequent remark), this implies that $f : X \to \alpha(R - (0))$ is uniformly continuous. It remains to show that a) implies b). First observe that a) implies that if z_1 and z_2 are disjoint zero-sets in X, then there exists $g \in U(X)$ which is 0 on z_1, 1 on z_2, and $0 \leqslant g \leqslant 1$. To do that, choose $f_{z'} \in U(X)$, $f_i \geqq 0$, such that

$$z_i = \{x \mid f_i x = 0\} .$$

Put $g = \dfrac{f_1}{f_1 + f_2}$ ·

Now assuming a), for any bounded function $f : X \to R$ satisfying the assumption in b), and for any $r > 0$, by a standard procedure one can construct a $g \in U(X)$ with $|f - g| < r$. This shows that bounded functions satisfy b). Now let f be an unbounded function. We may and shall assume that $f \geqq 1$. Then $1/f$ is bounded, hence uniformly continuous, hence f is uniformly continuous because U(X) is inversion-closed. This concludes the proof.

Remark. – In the proof we used the fact that $\alpha(R - (0))$ is projectively generated by all continuous functions, i.e. that for every open cover of $R - (0)$ there exists a continuous function h such that $\mathfrak{U}$ is refined by the preimage of an r-cover. It is enough to do that just for $]\!]0, \rightarrow[\![$. For each x let gx be the supremum of all $r > 0$ such that $]\!]x, r[\![$ is in some member of $\mathfrak{U}$. Define f as follows :

$$fy = \begin{cases} \inf\{gx \mid 1 \leqslant x \leqslant y\} & \text{for} \quad y \geqq 1 , \\ \inf\{gx \mid y \leqslant x \leqslant 1\} & \text{for} \quad y \leqq 1 . \end{cases}$$

Since on compact sets $fy > 0$, f is positive, and it is easy to see that f is continuous. Put

$$hy = \left| \int_1^y 1/fx \; dx \right| .$$

It is easy to check that h has the required property for any $r < 1$.

Note. – Every continuous form on $U^b(X)$ can be represented by a finitely additive measure on the smallest algebra containing coz X if X is inversion-closed. Only if it holds in the following sense : there exists a Y which is inversion closed and such that $U^b(X) = U^b(Y)$. This is the classical theorem of A.D. Alexandrov.

Along the lines of Theorem 2 we can add the following characterizations of the spaces in Theorem 1.

THEOREM 3. – *Each of the following conditions* (8) – (10) *is equivalent to conditions in Theorem 1.*

8) *If* B *is a normed space and* $f : X \rightarrow B - (0)$ *is uniformly continuous then so is* $f : X \rightarrow \alpha(B - (0))$.

9) (8) *with* B *Banach spaces.*

(10) (8) *with* $B = l(A)$.

Proof. – Clearly (1) implies (8), (8) implies (9), (9) implies (10). It remains to show that (10) implies one of the conditions (1) – (7). We shall show that (10) implies (5). First, (10) implies that X is inversion-closed (with A a singleton) by Theorem 2. Now let $f \in U(X)$, $g \in U(X, l(A))$. If $fx \neq 0 \neq gx$ for each x, then

$$g : X \rightarrow \alpha(l(A) - (0))$$

is uniformly continuous, hence

$$f \cdot g : X \to \alpha(l(A) - (0))$$

is uniformly continuous, hence $f \cdot g : X \to l(A)$

is uniformly continuous. For the general case, let $A' = A \cup (a)$, where $a \notin A$. Then $l(A) \subset l(A')$. Consider the map $h = g + a$. Then h has no root, hence $f \cdot h : X \to l(A')$ is uniformly continuous, hence

$$f \cdot (g + a) - f \cdot a = f \cdot g$$

is uniformly continuous. We have proved (5) for non-vanishing elements of U(X). Condition (5) now follows by a routine argument. This concludes the proof.

The last set of characterizations is given in the concluding.

THEOREM 4. – *Each of the following conditions* (11) – (13) *is equivalent to the conditions* (1) – (10) :

11) *For any normed space* B, *the linear space* U(X , B) *is scalar inversion-closed,* i.e. *if* $f : X \to B \in U$ *does not vanish, then the map*

$$\{x \to 1/\| fx \|^2 \circ fx\} : X \to B$$

is uniformly continuous.

12) *Condition* (11) *with* B *restricted to Banach spaces.*

13) *Condition* (11) *with* B *restricted to spaces* l(A).

Proof. – Condition (5) implies Condition (11), because $\{x \to \| fx \|\} : X \to R$ is uniformly continuous. Obviously (11) implies (12), and (12) implies (13). We shall prove that (13) implies (5). Clearly U(X) is inversion-closed (R is l(A) with A a singleton). Next let $f \in U(X)$, $f \geqq 1$, and let $g \in U(X, l(A))$. Embed l(A) into $l(A')$, where $A' = A \cup (a)$, $a \notin A$, and consider $h = g + a$. Clearly $\| gx \| \geqq | a | = 1$ for each x, and hence $1/h$ $(= 1/\| f \|^2 \cdot f)$ is bounded. Also $1/f$ is bounded, and hence $1/f \cdot 1/h$ is uniformly continuous by Proposition in § 2. Again by Condition (13), $f \cdot h$ is uniformly continuous, hence $f \cdot g = f \cdot h - f$, a is uniformly continuous. Now if $f \in U(X)$, then we apply what is already proved to $1 + \max(0, f)$, $1 - \min(0, f)$, and then we get general f by simple computation. This concludes the proof.

5. Remarks.

Property (1) was introduced by A. Hager [1] (under the name m-fine), who restricted his attention just to separable uniform spaces. J. Vilimovsky [1], [2] studied (1) in quite general categories with α any coreflection, and proved that property (1) is coreflective. This means that in the setting of this note for each uniform space X there exists a finer space mX with property (1) such that if $f : \mathrm{Y} \to \mathrm{X}$ is uniformly continuous, and if Y has (1), then $f : \mathrm{Y} \to m\mathrm{X}$ is uniformly continuous. The proof of coreflectivity in uniform spaces is usually quite simple. It is enough to show that the class of spaces is closed under inductive generation. (For example inversion-closed spaces are coreflective). However the description of the coreflection may be a difficult problem. In the case of property (1), the coreflection was described by M. Rice [1] and the present author [1], [2]. Rice has a basis for uniform covers of mX all covers $\mathcal{U}$ such that there exists a countable cover $\mathcal{V}$ of X by cozero sets in X such that each $\mathcal{U}/\mathrm{V}$, $\mathrm{V} \in \mathcal{V}$ is a uniform cover of the subspace V of X. The present author has the covers in (7) for a basis. The setting of (1) led the present author to introduce metrically determined functors. A functor F of uniform spaces is called metrically determined if FX is projectively generated by all maps $f : \mathrm{FX} \to \mathrm{FM}$ where $f : \mathrm{X} \to \mathrm{M}$ is uniformly continuous and M is metric, and a complete family of projections of a space into spaces in a class (e.g. metric) [1].

Property (2) is a modification of the author's definition of measurable spaces [3] (pointwise convergence is replaced by continuous convergence).

Properties (3) – (6) seem to be new. I believe that the further investigations of these properties may bring interesting results. Perhaps one should try to strenghten the conditions to get spaces investigated by the author in [4] ; i.e. to add the property that a uniformly locally uniformly continuous real valued function is uniformly continuous.

Conditions (8) – (13) are seemingly new.

It may be interesting to know more about the following (obviously coreflective) property (weakening of (3)).

U(X , B) is a U(X)-module for every normed space B.

Concluding remarks.

a) If X has the property that for every $f \in U(X), f : X \to \alpha R \in U$, then $U(X)$ is an algebra, and the converse is not true, because e.g. the elements of $U(R)$ do not grow faster than a multiple of $\{x \to x\}$ when x tends to infinity.

b) If X has the properties (1) – (13), in particular the properties (11) – (13), then one can show that

$$f \in U(X, B) \quad \text{implies} \quad f \in U(X, \alpha B) , \tag{*}$$

and this implies that

$$U(X, B) \quad \text{is a} \quad U(X) - \text{module.}$$

If we add to the last property that X is inversion-closed then X has the properties (1) – (13).

One can show that (*) for Banach spaces is really weaker than (1) – (13).

c) It follows from Theorem 1, property (7), and the description of coreflection corresponding to (1), that this coreflection has for its basis the covers of the form

$$\{\operatorname{coz} f_a \mid a \in A\}$$

where $f : X \to l(A)$ is uniformly continuous.

d) The results of this paper are needed for a theory of vector measures on uniform spaces.

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Manuscrit reçu le 19 juin 1973
accepté par G. Choquet.

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