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# Erratum to: Persistence of Coron's solution in nearly critical problems

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Formula (3.17) in our paper [1] is wrong. This erratum is devoted to give the right formula and to list the main changes that need to be made.

Results in Section 2 change as follows. In (2.1) we assume

$$\xi := \mu\tau, \quad \tau \in \mathbb{R}^N \text{ and } |\tau| < \delta^{-1}. \tag{1.1}$$

Lemma 2.2 becomes the following.

**Lemma 1.1.** *Let*

$$R_{\varepsilon,\mu}(x) := P_\varepsilon U_{\mu,\xi}(x) - U_{\mu,\xi}(x) + \alpha_N \mu^{\frac{N-2}{2}} H(x,\xi) + \alpha_N \frac{1}{\mu^{\frac{N-2}{2}}(1+|\tau|^2)^{\frac{N-2}{2}}} \varphi_\omega\left(\frac{x}{\varepsilon}\right).$$

*Then there exists a positive constant c such that for any $x \in \Omega \setminus \varepsilon\omega$*

$$\left|R_{\varepsilon,\mu}(x)\right| \le c\varepsilon^{\frac{N-2}{4}} \left(\frac{\varepsilon^{\frac{N-1}{2}}}{|x|^{N-2}} + \varepsilon\right) \qquad \text{if } N \ge 4, \tag{1.2}$$

$$\left|R_{\varepsilon,\mu}(x)\right| \le c\varepsilon^{\frac{1}{4}} \left(\frac{\varepsilon}{|x|} + \sqrt{\varepsilon}\right) \qquad \text{if } N = 3. \tag{1.3}$$

*Proof.* The function $R := R_{\varepsilon,\mu}$ solves $-\Delta R = 0$ in $\Omega \setminus \varepsilon\omega$ with

$$R(x) = \alpha_N \left[ -\frac{\mu^{\frac{N-2}{2}}}{(\mu^2 + |x-\xi|^2)^{\frac{N-2}{2}}} + \frac{\mu^{\frac{N-2}{2}}}{|x-\xi|^{N-2}} + \frac{1}{\mu^{\frac{N-2}{2}}(1+|\tau|^2)^{\frac{N-2}{2}}} \varphi_\omega\left(\frac{x}{\varepsilon}\right) \right], \ x \in \partial\Omega,$$

$$R(x) = \alpha_N \left[ -\frac{\mu^{\frac{N-2}{2}}}{(\mu^2 + |x-\xi|^2)^{\frac{N-2}{2}}} + \mu^{\frac{N-2}{2}} H(x,\xi) + \frac{1}{\mu^{\frac{N-2}{2}}(1+|\tau|^2)^{\frac{N-2}{2}}} \right], \ x \in \partial\varepsilon\omega.$$

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Therefore (1.2) and (1.3) follow, because

$$\varepsilon^{-\frac{N-2}{4}} R(x) = O\left(\varepsilon + \varepsilon^{\frac{N-2}{2}}\right), \; x \in \partial\Omega$$

and

$$\varepsilon^{-\frac{N-2}{4}} R(x) = O\left(\varepsilon^{-\frac{N-3}{2}}\right), \; x \in \partial\varepsilon\omega. \qquad \square$$

In Section 3 the function $\Psi$ defined in (3.17) becomes

$$\Psi(\tau, d) := -F(\tau)\frac{1}{d^{N-2}} - a_3 H(0,0) d^{N-2} + b_1\Lambda + b_2\Lambda \log d, \tag{1.4}$$

where

$$F(\tau) := \alpha_N^{p+1} c_\omega \frac{1}{(1+|\tau|^2)^{\frac{N-2}{2}}} \int_{\mathbb{R}^N} \frac{1}{|y+\tau|^{N-2}} \frac{1}{(1+|y^2)^{\frac{N+2}{2}}} dy. \tag{1.5}$$

The rest of the section remains unchanged.

In Section 4 the proof of Lemma (4.1) changes as follows. Estimate (4.7) becomes

$$\int_{\Omega\setminus\varepsilon\omega} U_{\mu,\xi}^{p+1} = \alpha_N^{p+1} \int_{\Omega\setminus\varepsilon\omega} \frac{\mu^N}{(\mu^2+|x-\xi|^2)^N} dx = \alpha_N^{p+1} \int_{\frac{\Omega\setminus\varepsilon\omega}{\mu}} \frac{1}{(1+|y-\tau|^2)^N} dy$$

$$= \alpha_N^{p+1} \int_{\mathbf{R}^N} \frac{1}{(1+|y|^2)^N} dy + O\left(\left(\frac{\varepsilon}{\mu}\right)^N + \mu^N\right).$$

Estimate (4.11) becomes

$$\int_{\Omega\setminus\varepsilon\omega} \left(P_\varepsilon U_{\mu,\xi} - U_{\mu,\xi}\right) U_{\mu,\xi}^p dx = \int_{\Omega\setminus\varepsilon\omega} R_{\varepsilon,\mu} U_{\mu,\xi}^p dx$$

$$- \alpha_N^{p+1} \int_{\Omega\setminus\varepsilon\omega} \left(\mu^{\frac{N-2}{2}} H(x,\xi) + \frac{1}{\mu^{\frac{N-2}{2}}(1+|\tau|^2)^{\frac{N-2}{2}}} \varphi_\omega\left(\frac{x}{\varepsilon}\right)\right) \frac{\mu^{\frac{N+2}{2}}}{(\mu^2+|x-\xi|^2)^{\frac{N+2}{2}}} dx.$$

Estimate (4.13) becomes

$$\begin{aligned}
&\frac{1}{\mu^{\frac{N-2}{2}}(1+|\tau|^2)^{\frac{N-2}{2}}}\int_{\Omega\setminus\varepsilon\omega}\varphi_\omega\left(\frac{x}{\varepsilon}\right)\frac{\mu^{\frac{N+2}{2}}}{(\mu^2+|x-\xi|^2)^{\frac{N+2}{2}}}dx\\
&=\frac{1}{(1+|\tau|^2)^{\frac{N-2}{2}}}\int_{\frac{\Omega\setminus\varepsilon\omega-\xi}{\mu}}\varphi_\omega\left(\frac{\mu}{\varepsilon}(y+\tau)\right)\frac{1}{(1+|y|^2)^{\frac{N+2}{2}}}dy\\
&=\left(\frac{\varepsilon}{\mu}\right)^{N-2}\frac{1}{(1+|\tau|^2)^{\frac{N-2}{2}}}\int_{\frac{\Omega\setminus\varepsilon\omega-\xi}{\mu}}f_\varepsilon(y)\frac{1}{|y+\tau|^{N-2}}\frac{1}{(1+|y|^2)^{\frac{N+2}{2}}}dy\\
&=\left(\frac{\varepsilon}{\mu}\right)^{N-2}\left(c_\omega\frac{1}{(1+|\tau|^2)^{\frac{N-2}{2}}}\int_{\mathbf{R}^N}\frac{1}{|y+\tau|^{N-2}}\frac{1}{(1+|y|^2)^{\frac{N+2}{2}}}dy+o(1)\right).
\end{aligned}$$

The rest of the proof remains unchanged.

## References

[1] M. MUSSO and A. PISTOIA, *Persistence of Coron's solution in nearly critical problems*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5) **6** (2007), 331–357.

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