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On the Continuous Extension of Holomorphic Correspondences

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0. - Introduction

The category of holomorphic correspondences, which contains the inverses of proper holomorphic mappings, is a very natural object to consider in several complex variables. Rigidity properties of holomorphic correspondences are dramatically determined by their boundary behaviour. We refer to the papers of S. Pinchuk [Pi4] and E. Bedford and S. Bell [BB] where this principle is illustrated and the smooth extension studied under global boundary regularity assumptions.

Our aim in this paper is to establish some purely local boundary continuity properties for holomorphic correspondences and give some applications. The fundamental works of Henkin [H], Pinchuk [Pi1] and Diederich-Fornaess [DF1] have completely enlightened the Hölder-continuous extension phenomenon for proper holomorphic mappings onto bounded domains with strictly pseudoconvex or finite type boundaries. These results have been localized for proper holomorphic maps between bounded domains by Forstneric-Rosay [FR] and for any kind of holomorphic maps between non necessarily bounded domains by the authors [Be], [Su]. Although the proofs of these results are based on a rather simple strategy, it is not possible to follow it for holomorphic correspondences. Indeed, the technique is essentially the following: the derivatives of the map are controled via the asymptotic behaviour of the Kobayashi metric and an Hölder estimate is obtained by a standard integration argument. Both steps of this method do not work for multi-valued holomorphic mappings and we have therefore to elaborate a genuinely different approach.

Our proof is mainly based on a detailed study of the "attraction effect" of plurisubharmonic barrier functions on multi-valued analytic discs (see Section 3). The plurisubharmonic barrier functions also play an important role in estimating the distance to the boundaries (see Section 2). Thus, the boundaries which are considered in this paper admit good families of plurisubharmonic peak functions and, by the works of Fornaess-Sibony [FS] and Cho [Cho], this covers pseudoconvex and finite type ones.

A precise formulation of our results is given in Section 1, where the basic definitions and facts are also recalled. Some applications are given in the last section; we show for instance how our results lead to an elementary proof of a well-known theorem of Bell-Catlin [BC] which asserts that certain Cauchy-Riemann mappings are locally finite-to-one. We also prove that any proper holomorphic correspondence between algebraically bounded domains is algebraic.

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1. – Definitions, notations and results

a) Holomorphic correspondences and canonical defining functions.

Let D and G be two domains in $\mathbb{C}^n$ and let A be a complex purely n-dimensional subvariety contained in $D \times G$. We denote by $\pi_D: A \to D$ and $\pi_G: A \to G$ the natural projections. When the projection π_D is proper then $(\pi_G \circ \pi_D^{-1})(z)$ is a non-empty finite subset of G for any $z \in D$ and one may therefore consider the set-valued mapping:

$$\begin{aligned} F: \quad & D \to G \\ & z \mapsto (\pi_G \circ \pi_D^{-1})(z)\,. \end{aligned}$$

Such a map is called an holomorphic correspondence between D and G and the variety A is said to be the graph of F. More formally, one can treat the correspondence $F: D \to G$ as a triple $F = (\pi_D, A, \pi_G)$ where the projection π_D is proper. One says that F is irreducible if A is an irreducible variety and proper if both π_D and π_G are proper.

Since π_D is proper, there exists a complex subvariety $S_D \subset D$ and an integer m such that $F(z) = \{w^1(z), \dots, w^m(z)\}$ for any $z \in D$ and the w^j's are distinct holomorphic functions in a neighbourhood of z when $z \in D \setminus S_D$ (see, for instance, [Ch]). One then says that F is an m-valued correspondence. The set S_D is the branch-locus of F.

Let $\{\alpha^1, \dots, \alpha^m\}$ be a system of m points in $\mathbb{C}^n$ which are not necessarily distinct. For $(w, \xi) \in \mathbb{C}^n \times \mathbb{C}^n$, consider the polynomial $P_\alpha(w, \xi) =: \prod_{j=1,m} \langle \xi, w - \alpha^j \rangle$ where $\langle a, b \rangle = \sum_{j=1,n} a_j b_j$. It follows from the uniqueness principle that $P_\alpha(w, .) \equiv 0$ if and only if $w \in \{\alpha^1, \dots, \alpha^m\}$. Expanding P_α in powers of ξ, $P_\alpha(w, \xi) = \sum_{|I|=m} \Phi_I(w; \alpha) \xi^I$, one finds a family of polynomials $\Phi_I(w; \alpha)$ in $w = (w_1, \dots, w_n)$ whose common zeros in $\mathbb{C}^n$ coincide with the given system of points $\{\alpha^1, \dots, \alpha^m\}$. These polynomials are called canonical defining functions for the system $\{\alpha\}$. Renumbering the points α^j does not change P_α and, therefore, $\Phi_I(w; \alpha)$. Thus we have $\Phi_I(w; \alpha) = \sum_{|J| \le m} \varphi_{I,J}(\alpha) w^J$ where the $\varphi_{I,J}(\alpha)$ are polynomial in α_i^j (the coordinates of α^j) which are symmetric with respect to superscripts. Moreover these polynomials have integer coefficients which depend only on I, J. Now we consider an m-valued holomorphic correspondence $F = (\pi_D, A, \pi_G)$ with branch-locus S_D. For any $z \in D \setminus S_D$ we apply the above construction to the system $\alpha(z) =: \{\alpha^1(z), \dots, \alpha^m(z)\} = F(z)$. This leads to the functions

$$\Phi_I(z, w) = \sum_{|J| \leq m} \varphi_{IJ}(z) w^J, \quad |I| = m, \quad (z, w) \in \mathbb{C}^n \times \mathbb{C}^n \tag{1.1}$$

where $\varphi_{I,J}(z)$ stands for $\varphi_{I,J}(\alpha(z))$. One shows (see [Ch]) that the functions $\varphi_{I,J}(z)$ are holomorphic on D and that A is precisely the set of common zeros of the functions $\Phi_I(z, w)$ in $D \in \mathbb{C}^n$. These functions are called canonical defining functions for the correspondence $F = (\pi_D, A, \pi_G)$.

b) Continuous extension.

Let $F: D \to G$ be an holomorphic correspondence and let p be a boundary point of D. We say that F extends continuously to bD at point p if the canonical defining function for F extends continuously to bD at point p. If these canonical defining functions are Hölder-γ ($\gamma \in]0, 1[$) on $\overline{D} \cap U$ for some neighbourhood U of p, we say that the correspondence F extends Hölder-γ continuously at point p.

c) Cluster sets.

Let $F: D \to G$ be an m-valued holomorphic correspondence and let $S \subset bD$ be some subset of the boundary of D. We define the cluster set of F at S ($C_D(F, S)$) as follows:

$$C_D(F, S) = \Big\{ w \in \mathbb{C}^n \cup \infty \,\Big|\, \lim_j \operatorname{dist}(z^j, S) = 0 \text{ and } \lim_j \operatorname{dist}(F(z^j), w) = 0 \\ \text{for some sequence } (z^j) \text{ in } D \Big\}$$

where $\infty \in C_D(F, S)$ means that the set $\{F(z^j)\}$ is unbounded for some sequence (z^j) in D with $\lim_j \operatorname{dist}(z^j, S) = 0$. We note that F is proper if and only if $C_D(F, bD) \cap G = \emptyset$.

Let p be a point in $\overline{D}$ and (p_ν) be a sequence in D converging to p; let also $\{q^1, ..., q^m\}$ be a set points in $\overline{G}$ (the q^j's are not necessarily distinct). We say that the sequence $(F(p_\nu)) = \{w^1(p_\nu), ..., w^m(p_\nu)\}$ converges to $\{q^1, ..., q^m\}$,if after a possible reordering one has $\lim_j w^j(p_\nu) = q^j$ for $j = 1, ..., m$ or, in other words, if $(F(p_\nu))$ is converging to $\{q^1, ..., q^m\}$ in the sense of the Hausdorff convergence of sets.

d) Regular boundaries.

Let Ω ne a domain in $\mathbb{C}^n$. A point $p \in b\Omega$ is said to be (α, β)-regular $(0 < \alpha \leq 1 < \beta)$ if there exists a neighbourhood U of p and a family of functions $(\varphi_\xi)_{\xi \in U \cap b\Omega}$ which are p.s.h. on $\Omega \cap U$, continuous on $\overline{\Omega} \cap U$ and satisfy the following estimate:

$$-M|z - \xi|^\alpha \leq \varphi_\xi(z) \leq -|z - \xi|^\beta \tag{1.2}$$

on $\overline{\Omega} \cap U$. An open connected subset Γ of $b\Omega$ is said to be (α, β) regular if it is a $\mathcal{C}^2$-smooth real hypersurface whose points are (α, β) regular.

It is worth to emphasize that (α, β)-regularity is known to be satisfied for finite type boundaries. To be precise the following holds:

i) If $b\Omega$ is $\mathcal{C}^2$-strictly pseudoconvex at point p, then p is $(1, 2)$-regular.
ii) If $\Omega \subset \mathbb{C}^2$ and $b\Omega$ is smooth, pseudoconvex and of type $2k$ at point p, then p is $(1, 2k)$ regular (see [FS]).
iii) If $\Omega \subset \mathbb{C}^n$ and $b\Omega$ is smooth, convex and of type $2k$ at point p, then p is $(\alpha, 2k)$-regular for any $\alpha \in]0, 1[$ (see [FS]).
iv) If $\Omega \subset \mathbb{C}^n$ and $b\Omega$ is smooth, pseudoconvex of type $2k$ at point p, then p is $(1, \beta)$-regular for some $\beta \geq 1$ (see [DF1]) for real analytic boundaries and [Cho] for smooth ones).

We are now in order to state our main result:

THEOREM A. *Let D, D' be two domains in $\mathbb{C}^n$ which are not necessarily bounded. Let $F: D \to D'$ be a holomorphic correspondence. Assume that $\Gamma \subset bD$ and $\Gamma' \subset bD'$ are open pieces of the boundaries which are respectively (α, β)-regular and (α', β')-regular. If the cluster set $C_D(F, \Gamma)$ is contained in Γ' then F continuously extends to $D \cup \Gamma$. Moreover, F extends as a Hölder-continuous correspondence to $\Gamma \setminus \overline{S}_D$ (which is an open dense subset of Γ).*

Actually, we shall obtain Theorem A from the following more concrete result which will be proved in Section 4. It is actually this theorem which generalizes the results about the local continuous extension of holomorphic mappings:

THEOREM A$'$. *Let D, D' be two domains in $\mathbb{C}^n$ which are not necessarily bounded. Assume that $\Gamma \subset bD$ and $\Gamma' \subset bD'$ are open pieces of the boundaries which are respectively (α, β)-regular and (α', β')-regular. Let $F: D \to D'$ be a holomorphic correspondence such that the cluster set $C_D(F, \Gamma)$ does not intersect D'. Suppose that p is a point in Γ and that there exists a sequence (p_ν) in D converging to p such that the sequence $\{F(p_\nu)\}$ converges to a subset $\{q^1, \ldots, q^m\}$ of Γ'. Then F continuously extends to p.*

As we have already mentioned, finite type boundaries are regular. We may therefore state the following global version of Theorem A:

THEOREM B. *Let $F: D \to D'$ be a proper holomorphic correspondence between bounded domains in $\mathbb{C}^n$. Suppose that D is pseudoconvex with $\mathcal{C}^2$ boundary and D' is pseudoconvex with $\mathcal{C}^\infty$ boundary of finite type in the sense of d'Angelo. Then F extends continuously to $\overline{D}$ and Hölder-continuously to $\overline{D} \setminus \overline{S}_D$.*

Various applications of our results will be described in Section 5.

2. – Boundary distance equivalence

We will denote by $\mathbb{B}$ the unit ball of $\mathbb{C}^n$. From now and on one assumes that the holomorphic correspondence F which is considered in Theorems A, A$'$ and B is m-valued for some $m \in \mathbb{Z}_+$; we set $F(z) = \{w^1(z), \ldots, w^m(z)\}$.

The aim of this section is to establish the following.

LEMMA 2.1. *Under the hypothesis of Theorem A′, there exist constants $C_1 > 0$ and $r, r' > 0$ such that for every $z \in D \cap (p + r\mathbb{B})$ the following holds: if for some $j = 1, \dots m$ one has $w^j(z) \in D' \cap \bigcup_{i=1,\dots,m}(q^i + r'\mathbb{B})$ then*

$$\operatorname{dist}(w^j(z), \Gamma') \leq C_1 \operatorname{dist}(z, \Gamma)^\alpha .$$

Our proof makes use of the family (1.2) of p.s.h. barrier functions and is based on the ideas of [Su].

Given $\varepsilon \geq 0$ we set $D^\varepsilon = \{z \in U \cap D \mid \varphi_p(z) > -\varepsilon\}$. It follows from (1.2) that there exists an $\varepsilon_0 > 0$ such that for every $\varepsilon \in (0, \varepsilon_0]$ one has $\overline{D^\varepsilon} \subset U$, where U is an (open) neighbourhood of p such that the functions (1.2) are defined on $\overline{D} \cap U$ and $U \cap \partial D = U \cap \Gamma$.

First, we show that the correspondence F is locally finite.

LEMMA 2.2. *For every $\varepsilon \in (0, \varepsilon_0/2]$ and $w \in D'$ the intersection $(\pi_D \circ \pi_{D'}^{-1})(w) \cap D^\varepsilon$ is at most finite.*

PROOF. Since the projection π_D is proper, for any $w \in D'$ the set $V = (\pi_D \circ \pi_D'^{-1})(w)$ is an analytic subset of D. One may assume that the intersection $V^\varepsilon = V \cap D^\varepsilon$ is not empty. Since the cluster set $C_D(F\,;\,\Gamma)$ does not intersect D', the set V^ε has no limit points on Γ. Therefore, the boundary $\partial \overline{V^\varepsilon} \setminus V$ of the analytic variety V^ε is contained in $S^\varepsilon = \{z \in U \cap D \mid \varphi_p(z) = -\varepsilon\}$. Now if we assume that $\dim V^\varepsilon \geq 1$, it follows from the maximum principle for p.s.h. functions on analytic varieties that φ_p is constant on V^ε; this is impossible. Thus, $\dim V^\varepsilon = 0$ for any $\varepsilon \in (0, \varepsilon_0]$. Now assume that V^ε is infinite for certain $\varepsilon \in (0, \varepsilon_0/2]$; then there exists an $a \in S^\varepsilon$ which is a limit point for V^ε. But then a is an interior limit point for a 0-dimensional variety V^{ε_0}: a contradiction. □

We now introduce a family of psh barrier functions which will play a fundamental role in the proof of Lemma 2.1. Fix an $\varepsilon \in (0, \varepsilon_0/2]$ and $\varepsilon_1 \geq 0$ such that $D \cap (p + \varepsilon_1\overline{\mathbb{B}})$ is contained in D^ε. We denote by $\pi(z) \in \Gamma$ the orthogonal projection of $z \in U \cap D$ to Γ: without loss of generality one may assume that $\operatorname{dist}(z, \pi(z)) = \operatorname{dist}(z, \Gamma)$ for every $z \in D \cap U$. Fix $\delta > 0$ such that $\varepsilon_1/50 < \delta < 2\delta < \varepsilon_1/10$ and consider the compact corona $K = \overline{D} \cap (p + 2\delta\overline{\mathbb{B}}) \setminus (p + \delta\mathbb{B})$. It follows from (1.2) that for $\varepsilon_2 < \varepsilon_1/100$ small enough there exists a $\tau > 0$ such that

$$\max\{\varphi_\zeta(z) \mid z \in K, \zeta \in \partial D \cap (p + \varepsilon_2\overline{\mathbb{B}})\} < -\tau < -\tau/2$$
$$< \min\{\varphi_\zeta(z) \mid z \in D \cap (p + \varepsilon_2\overline{\mathbb{B}}), \zeta \in \partial D \cap (p + \varepsilon_2\overline{\mathbb{B}})\}.$$

Consider a smooth convex non-decreasing function $\theta(t)$ such that $\theta(t) = -\tau$ for $t \leq -\tau$, $\theta(t) = t$ for $t \geq -\tau/2$ and set $\rho_\zeta(z) = \tau^{-1}(\theta \circ \varphi_\zeta)(z)$. Then $\rho_\zeta \mid K = -1$ if $\xi \in \partial D \cap (p + \varepsilon_2\overline{\mathbb{B}})$ and we may extend the function $\rho_\zeta(z)$ to D setting $\rho_\zeta(z) = -1$ for $z \in D \setminus (p + 2\partial\overline{\mathbb{B}})$. Thus, for every $\zeta \in \Gamma \cap (p + \varepsilon_2\mathbb{B})$ we get a negative continuous p.s.h. function $\rho_\zeta(z)$ on D with the properties:

i) $\rho_\zeta(z) = -1$ on $D \setminus (p + \delta\mathbb{B})$;
ii) $\rho_\zeta(z) = \tau^{-1}\varphi_\zeta(z)$ on $D \cap (p + \varepsilon_2\mathbb{B})$.

Fix $\varepsilon_3 \in (0, \varepsilon_2/2)$ such that $\pi(z) \in \Gamma \cap (p+\varepsilon_2\mathbb{B})$ for any $z \in D \cap (\rho+\varepsilon_3\mathbb{B})$. For every $a \in D \cap (\rho + \varepsilon_3\mathbb{B})$ we consider the function

$$\psi_a(w) = \begin{cases} \sup\{\rho_{\pi(a)}(z) \mid z \in (\pi_D \circ \pi_{D'}^{-1}(w)\} & \text{if } w \in F(D^\varepsilon)\,; \\ -1 & \text{if } w \in D' \setminus F(D^\varepsilon)\,. \end{cases}$$

LEMMA 2.3. *The function $\psi_a(w)$ is continuous negative and p.s.h. on D'.*

PROOF. Since the restriction $F\colon D^\varepsilon \to F(D^\varepsilon)$ is finite and π_D is proper, for every point $(z, w) \in A = \Gamma_F$ with $z \in D^\varepsilon$ there exist (small enough) neighbourhoods Ω_D and $\Omega_{D'}$ of (z, w) in $\mathbb{C}^{2n}$ such that $\Omega_{D'} \subset \Omega_D$ and the restrictions $\pi_D \mid A \cap \Omega_D$ and $\pi_{D'} \mid A \cap \Omega_{D'}$ are proper. In particular, $F(D^\varepsilon)$ is open.

Let ω be an interior point of $F(D^\varepsilon)$ and $(\omega_k)_k \subset F(D^\varepsilon)$ be a sequence converging to ω. Let $\xi \in D^\varepsilon \cap (\pi_D \circ \pi_{D'}^{-1}(\omega)$ be such that $\rho_{\pi(a)}(\xi) = \psi_a(\omega)$; we denote by ξ_k a point in $D^\varepsilon \cap (\pi_D \circ \pi_{D'}^{-1}(\omega_k)$ with $\rho_{\pi(a)}(\xi_k) = \psi_a(\omega_k)$ for $k = 1, 2, \ldots$. Let ζ be a limit point of the sequence $(\xi_k)_k$. Since the cluster set $C_D(F\,;\,\Gamma)$ does not intersect D', one has $\zeta \in D^\varepsilon \cup S^\varepsilon$. Hence, if we set $(\pi_{D'} o \pi_D^{-1})(\xi_k) = \{w^1(\xi_k), \ldots, \omega^m(\xi_k)\}$ and $(\pi_{D'} \circ \pi_D^{-1})(\zeta) = \{w^1(\zeta), \ldots, w^m(\zeta)\}$, we will have $\lim_k w^j(\xi_k) = w^j(\zeta)$ for every j (after a possible renumeration of w^j, s) (see [Ch]), and, therefore, $\zeta \in (\pi_D \circ \pi_{D'}^{-1}(\omega)$. Thus, $\rho_{\pi(a)}(\zeta) \leq \rho_{\pi(a)}(\xi) = \psi_a(\omega)$. Assume that $\rho_{\pi(a)}(\zeta)$ is not equal to $\rho_{\pi(a)}(\xi)$. Fix a neighbourhood Ω of ξ small enough such that $(\pi_D \circ \pi_{D'}^{-1})(\omega) \cap \Omega = \{\xi\}$. Let $(\xi_{k_\nu})_\nu$ be a subsequence converging to ζ. Since $F \mid \Omega$ is open, for any ν sufficiently large there is η_ν in $(\pi_D \circ \pi_{D'}^{-1})(\omega_{k_\nu}) \cap \Omega$; then we get by continuity $\rho_{\pi(a)}(\xi_{k_\nu}) < \rho_{\pi(a)}(\eta_\nu)$: this contradicts the choice of $(\xi_k)_k$. Thus, ψ_a is continuous on $F(D^\varepsilon)$.

If $\omega \in D'$ is a boundary point for $F(D^\varepsilon)$, then $\psi_a(\omega) = -1$. For sequences $(\omega_k)_k \subset F(D^\varepsilon)$, $(\xi_k)_k \subset D^\varepsilon$, and ξ, ζ defined as above, the same argument shows that $\zeta \in S^\varepsilon$ and $\rho_{\pi(a)}(\zeta) = \psi_a(\omega) = -1$. This gives the continuity of ψ_a on D'.

It remains to verify the subaveraging property. Let $\omega \in F(D^\varepsilon)$. Choose ξ in $(\pi_D \circ \pi_{D'}^{-1})(\omega) \cap D^\varepsilon$ such that $\rho_{\pi(a)}(\xi) = \psi_a(\omega)$ and neighbourhoods $\Omega_D \supset \Omega_{D'}$ of (ξ, ω) such that the restrictions $\pi_D \mid A \cap \Omega_D$ and $\pi_{D'} \mid A \cap \Omega_{D'}$ are proper. One may suppose Ω_D to be small enough such that one has $(\pi_D \circ \pi_{D'}^{-1}(\omega) \cap \pi_D(A \cap \Omega_D) = \{\xi\}$. The function

$$u_a(w) = \max\{(\rho_{\pi(a)} \circ \pi_D)(z, w) \mid (z, w) \in A \cap \Omega_{D'} \cap (\pi_{D'}^{-1})(w)\}$$

is negative continuous on $\pi_{D'}(A \cap \Omega_{D'})$ and p.s.h. on $\pi_{D'}(A \cap \Omega_{D'}) \setminus V$, where V is a complex variety of dimension $\leq n - 1$. By the removal singularities theorem $u_a(w)$ is p.s.h. on $\pi_{D'}(A \cap \Omega_{D'})$. Since $u_a \leq \psi_a$ on $\pi_{D'}(A \cap \Omega_{D'})$, $u_a(\omega) = \psi_a(\omega)$ and u_a satisfies the subaveraging property, the function ψ_a has the subaveraging property at ω.

If $\omega \in D'$ is a boundary point of $F(D^\varepsilon)$, the same arguments give the existense of ζ in $S^\varepsilon \cap D \cap (\pi_D \circ \pi_{D'}^{-1})(\omega)$ such that $\rho_{\pi(a)}(\zeta) = \psi_a(\omega) = -1$. For $u_a(w)$ defined as above, we get similarly $u_a(w) \leq \psi_a(w)$ for $w \in \Omega \cap F(D^\varepsilon)$, where Ω is a neighbourhood of ω. But $u_a = \psi_a = -1$ on $\Omega \setminus F(D^\varepsilon)$ and we conclude. □

We shall need the following refined version of the Hopf lemma (see [Co], [PiT]).

LEMMA 2.4. *Let $\Omega \subset\subset \mathbb{C}^n$ be a domain with a boundary of class C^2 and let $K \subset\subset \Omega$ be a compact subset with non-empty interior, $L > 0$ be a constant. Then there exists constant $C(K, L) > 0$ such that any negative p.s.h. function u on Ω with $u(z) < -L$ for every $z \in K$, satisfies the estimate*

$$|u(z)| \geq C \ \mathrm{dist}\,(z, \partial\Omega) \quad \text{for} \quad z \in \Omega\,.$$

Fix an $r' > 0$ small enough such that $\partial D' \cap (q^i + r'\mathbb{B}) = \Gamma' \cap (q^i + r'\mathbb{B})$ for $i = 1, \ldots, m$. For any $i = 1, \ldots, m$ fix also a compact K_i in $F(D^{\varepsilon_2}) \cap (q^i + r'\mathbb{B})$ with non-empty interior (this is possible since $F(D^{\varepsilon_2})$ is open by Lemma 2.2 and $q^i \in C_D(F; p)$). One has

$$\max_{w \in K_i} \psi_a(w) = \max\{\rho_{\pi(a)}(z) \mid z \in (\pi_D \circ \pi_{D'}^{-1})(K_i) \cap D^\varepsilon\}\,.$$

It follows from the condition on the cluster set that the pull-back $F^{-1}(K_i) = (\pi_D \circ \pi_{D'}^{-1})(K_i)$ has no limit points on $U \cap \partial D$; hence, the closure K_i' of $(\pi_D \circ \pi_{D'}^{-1})(K_i) \cap D^\varepsilon$ is compact in $D \cap U$. If $z \in K_i'$, it follows from (1.2) that

$$\begin{aligned} \max\{\varphi_{\pi(a)}(z) \mid z \in K_i'\} &\leq -\ \min\{|z - \zeta|^\beta \mid z \in K_i',\, \zeta \in \partial D \cap (p + \varepsilon_2\mathbb{B})\} \\ &= -\ \mathrm{dist}\,(K_i', \partial D \cap (p + \varepsilon_2\mathbb{B}))^\beta\,. \end{aligned}$$

Since the last quantity is smaller than a constant $-N < 0$, one can set

$$\max_{z \in K_i'} \rho_{\pi(a)}(z) \leq -\tau^{-1}N = -2L$$

where $L > 0$ does not depend on a and i. Now it follows from Lemma 2.4 that $|\psi_a(w)| \geq C \ \mathrm{dist}\,(w, \Gamma')$ for $w \in D' \cap \bigcup_{i=1,\ldots,m}(q^i + r'\mathbb{B})$ with $C > 0$ independent of a. Let a be in $(p + \varepsilon_3\mathbb{B}) \cap D$. If for some $j = 1, \ldots, m$ one has $w^j(a) \subset D' \cap \bigcup_{i=1,\ldots,m}(q^i + r'\mathbb{B})$, we get

$$\begin{aligned} \mathrm{dist}\,(w^j(a), \Gamma') &\leq C|\psi_a(w^j(a))| \leq C|\rho_{\pi(a)}(a)| \\ &\leq C|a - \pi(a)|^\alpha = C \ \mathrm{dist}\,(a, \Gamma)^\alpha\,. \end{aligned}$$

This proves Lemma 2.1.

If the correspondence $F\colon D \to D'$ is proper, the proof is much easier; moreover it suffices to use the Diederich-Fornaess exhaustion p.s.h. function instead of the family (1.2) and we do not need the assumption on ∂D to be (α, β)-regular. Thus we have:

LEMMA 2.5. *In the hypothesis of Theorem B, there exist constant* $0 < \eta < 1$ *and* $C > 0$ *such that for every* $z \in D$ *one has*

$$\operatorname{dist}(F(z), \partial D') \leq C_1 \operatorname{dist}(z, \partial D).$$

For the proof, see, for instance, [BB1], [Pi5].

3. – The contraction of multi-valued holomorphic discs near p.s.h barriers

Since we deal with multi-valued mappings, the standard technique based on estimates of the Kobayashi infinitesimal metric cannot be used directly for getting boundary continuity properties. This is due to the fact that the Kobayashi metric is not decreasing by proper holomorphic correspondences. The following assertion is of crucial importance for our construction, it basically describes how a multi-valued analytic disc behaves when its "center" is close to some plurisubharmonic peak point.

LEMMA 3.1. *Let* $\Omega \subset \mathbb{C}^n$ *be a bounded domain* ($n \geq 1$), *let* $\mathbb{B}_p$ *be the unit ball of* $\mathbb{C}^p (p \geq 1)$ *and* $F: \mathbb{B} \to \Omega$ *be an* m*-valued holomorphic correspondence. Let* Λ *be a* $\mathbb{C}$*-linear form on* $\mathbb{C}^n$. *Assume that there exists a function* φ, *continuous on* $\overline{\Omega}$, *p.s.h. on* Ω *and satisfying the following:*

i) $\varphi \leq 1$ *on* Ω *and* $\varphi(\omega^0) = 1$ *for some* $\omega^0 \in b\Omega$;
ii) there are constants $\alpha, \beta, r, A, B > 0$ *such that*

$$1 - A|w - \omega^0|^\alpha \leq \varphi(w) \leq 1 - B|w - \omega^0|^\beta$$

for $|w - \omega^0| \leq r$.

Then there exists an $\varepsilon_0 = \varepsilon_0(r, \alpha, \beta, A, B) \leq \min(r, 1)$ *such that for any* $w^0 \in F(0)$ *satisfying* $|w^0 - \omega^0| = \operatorname{dist}(w^0, \partial\Omega) = \varepsilon < \varepsilon_0$ *the following estimate holds:*

$$\begin{gathered} \forall z \in \mathbb{B}_p \quad , \quad \exists w \in F(z) \text{ s.t.} \\ |\Lambda(w) - \Lambda(w^0)| \leq C(1 - |z|)^{-2p/m} \varepsilon^{\tau/m} \end{gathered}$$

where $\tau = \min(\frac{\alpha}{2}, \frac{\alpha}{2\beta})$ *and* C *is a positive constant which depends only on* A, B, m, β, $\|\Lambda\|$ *and the diameter* M *of* Ω.

PROOF. Consider a function $\varphi'(w)$ defined by $\max\{\varphi(w), 1 - Br^\beta/2\}$ on $\{|w - \omega^0| \leq r\}$ and by $1 - Br^\beta/2$ on $\Omega \backslash \{|w - \omega^0| \leq r\}$. It follows from (ii) that the function φ' is p.s.h. on Ω and coincides with φ if $|w - \omega^0| < \varepsilon_1$ for $\varepsilon_1 < (B/2A)^{1/\alpha} r^{\beta/\alpha}$. Set $\sigma(z) =: \max\{\varphi(w) \mid w \in F(z)\}$ and let $\varepsilon_0 < \min\{\varepsilon_1, r, 1\}$.

Then σ is a p.s.h. function on $\mathbb{B}_p$ and it follows from (ii) that $\sigma(0) \geq 1 - A\varepsilon^\alpha$. If μ denotes the normalized Lebesgue's measure on $\mathbb{B}_p$ one has:

$$\sigma(0) \leq \int_{\{\sigma \geq 1-\varepsilon^{\alpha/2}\}} \sigma d\mu + \int_{\{\sigma < 1-\varepsilon^{\alpha/2}\}} \sigma d\mu \tag{3.1}$$

hence

$$1 - A\varepsilon^\alpha \leq (1 - \mu\{\sigma < 1 - \varepsilon^{\alpha/2}\}) + (1 - \varepsilon^{\alpha/2})\mu\{\sigma < 1 - \varepsilon^{\alpha/2}\} \tag{3.2}$$

and

$$\mu\{\sigma < 1 - \varepsilon^{\alpha/2}\} \leq A\varepsilon^{\alpha/2} . \tag{3.3}$$

Let us now define a holomorphic function g on $\mathbb{B}_p$ by $g(z) = \prod_{w \in F(z)} \Lambda(w - \omega^0)$. Using (ii) one sees that if $\varepsilon < \varepsilon_0 < (B/2)^{2/\alpha} r^{2\beta/\alpha}$ then for any z with $\sigma(z) \geq 1 - \varepsilon^{\alpha/2}$ there exists $w \in F(z) \cap \{|w - \omega^0| < r\}$ such that $\sigma(z) = \varphi(w)$ and $|w - \omega^0| \leq B^{-1/\beta}\varepsilon^{\alpha/2\beta}$. Thus we have:

$$\sigma(z) \geq 1 - \varepsilon^{\alpha/2} \Rightarrow |g(z)| \leq (M\|\Lambda\|)^{m-1} B^{-1/\beta} \varepsilon^{\alpha/2\beta} . \tag{3.4}$$

By the subaveraging property one has for any $z' \in \mathbb{B}_p$:

$$\begin{aligned} |g(z')| &\leq \left(\mu\{|z - z'| \leq 1 - |z'|\}\right)^{-1} \int_{|z-z'|\leq 1-|z'|} |g| d\mu \\ &\leq (1 - |z'|)^{-2p} \left[\int_{\{\sigma(z) < 1-\varepsilon^{\alpha/2}\}} |g(z)| d\mu + \int_{\{\sigma(z) \geq 1-\varepsilon^{\alpha/2}\}} |g(z)| d\mu\right\} . \end{aligned} \tag{3.5}$$

Then from (3.3), (3.4) and (3.5) we get:

$$|g(z')| \leq C_0(1 - |z'|)^{-2p}\varepsilon^\tau ; \ \forall z' \in \mathbb{B}_p \tag{3.6}$$

where C_0 depends only on $A, B, \|\Lambda\|, M,$ and β and $\tau =: \min(\frac{\alpha}{2}, \frac{\alpha}{2\beta})$.

By the definition of g it follows from (3.6) that for any $z' \in \mathbb{B}_p$ there exists $w \in F(z')$ such that $|\Lambda(w - \omega^0)| \leq C_0^{1/m}(1 - |z'|)^{-2p/m}\varepsilon^{\tau/m}$ and the conclusion follows since $|\Lambda(w - w^0)| \leq \|\Lambda\|\varepsilon + |\Lambda(w - \omega^0)|$ □

In principle, this lemma allows to prove our main results in the case of bounded domains. In order to use it in the case of unbounded domains we need the following localization lemma. Its proof uses some ideas of [Si2].

LEMMA 3.2. *Let Ω be a domain in $\mathbb{C}^n (n \geq 1)$ and $\mathbb{B}_p$ be the unit ball in $\mathbb{C}^p (p \geq 1)$. Let $\{a_j, 1 \leq j \leq \ell\} \subset b\Omega$ be a collection of ℓ points on $b\Omega$ and let $\delta = \min_{j\neq k} |a_j - a_k| > 0$. Assume that there exist ℓ functions $\varphi_j (1 \leq j \leq \ell)$ such that:*

i) *φ_j is p.s.h. and continuous on Ω;*
ii) *$-\infty < -K_j < \varphi_j < 0$ on Ω;*
iii) *$\varphi_j - |z|^2$ is p.s.h. on $\Omega \cap \{|z - a_j| < \tau\}$*

where $0 < \tau < \frac{\delta}{2}$ and $K_j > 0$ are some constants. Then for every $0 < R < \tau$ there exist some constants $0 < R_1 < R_2 < R$ and $0 < r < 1$ such that, for any m-valued holomorphic correspondence $F: \mathbb{B}_p \to \Omega$:

$$F(0) \subset \bigcup_{1\leq j\leq \ell} B(a_j, R_1) \Rightarrow F(\{|t| \leq r\}) \subset \bigcup_{1\leq j\leq \ell} B(a_j, R_2).$$

PROOF. Let us first show that it suffices to consider the case $p = 1$. If $u_0 \in \mathbb{B}_p$ and $|u_0| < \tilde{r} < r$ then we may find a sequence (u_k) in $\mathbb{B}_p$ which avoids the branch locus of F and converges to u_0. Consider the sequence of m-valued analytic discs f_k defined by $f_k(t) = F(t\frac{u_k}{|u_k|})$ for $t \in \mathbb{B}_1 =: \Delta$. Applying the lemma to the f_k's and passing to the limit we find that $F(u_0) \subset \bigcup_{1\leq j\leq \ell} B(a_j, \tilde{R}_2)$ where $R_2 < \tilde{R}_2 < R$.

We now assume that $p = 1$ and proceed in three steps.

FIRST STEP. Construction of some p.s.h. function u_{z_0} associated to $z_0 \in \Omega$.

Let $0 < R_2' < \frac{R}{2}$ and $0 < \alpha < \frac{R^2}{8}$. Assume that $z_0 \in \Omega \cap B(a_j, R_2')$. Consider a smooth increasing function χ on $\mathbb{R}^+$ such that $\chi(x) = x$ for $0 \leq x \leq \alpha$ and $\chi(x) = \frac{3\alpha}{2}$ for $x \geq 2\alpha$. Let us set $\varphi =: \sum_{1\leq j\leq \ell} \varphi_j$, then φ is a continuous p.s.h. function on Ω such that:

i) $-K < \varphi < 0$ on Ω and
ii) $\varphi - |z|^2$ is p.s.h. on $\bigcup_{1\leq j\leq \ell} B(a_j, R)$.

We now set $u_{z_0}(z) = e^{M\varphi(z)}\chi(|z - z_0|^2)$.

Since $\overline{\partial}\partial \log u_{z_0}$ is positive outside $\Omega \cap \{\alpha \leq |z - z_0|^2 \leq 2\alpha\}$ which is contained in $\Omega \cap B(a_j, R)$, the assertion (ii) shows that $\log u_{z_0}$ is p.s.h. on Ω for some sufficiently large M. This choice of M is clearly independant of z_0.

SECOND STEP. There exists a constant $A > 0$ such that for any m-valued holomorphic correspondence $F: \Delta \to \Omega$, any $\varepsilon \in]0, \alpha]$ and any $j \in \{1, \dots, \ell\}$:

$$z_0 \in F(0) \cap B(a_j, R_2') \Rightarrow \forall t \in \Delta_{A\varepsilon^m}, \exists z \in F(t) \; s.t. \; |z - z_0| \leq \varepsilon.$$

Consider the function u_{z_0} which is given by the first step. Let us define a function v_{z_0} on Δ by setting $v_{z_0}(t) = \prod_{z\in F(t)} u_{z_0}(z)$. The function v_{z_0} is a well-defined continuous function on Δ, moreover $0 \leq v_{z_0}(t) \leq 1$ for any $t \in \Delta$. Let $t_0 \in \Delta$ be some point which does not belong to the branch locus of F. Denote as $F_1, \dots, F_m$ the m branches of F which are defined on some neighbourhood U of t_0. Then $\log v_{z_0}(t) = \sum_{1\leq j\leq m} \log u_{z_0} o F_j(t)$ on U and this shows that $\log v_{z_0}$ is subharmonic outside of the branch locus of F. As $\log v_{z_0}$ is continuous we see that it is actually subharmonic on Δ.

Let us temporarily admit the following.

CLAIM. Let $f\colon \Delta \to \mathbb{C}^n$ be an m-valued holomorphic correspondence such that $0 \in f(0)$. Then $\prod_{z\in f(t)} |z|^2 \lesssim |t|^2$ on a neighbourhood of the origin in Δ.

Thus we have:

$$(3.7)\qquad \frac{v_{z_0}(t)}{|t|^2} \leq \frac{\prod_{z\in f(t)} \chi(|z-z_0|^2)}{|t|^2} \leq \frac{\prod_{z\in f(t)}(|z-z_0|^2)}{|t|^2} \lesssim 1 \quad \text{on}\Delta - \{0\}.$$

Since $|t|^{-2} v_{z_0}(t) = e^{\log v_{z_0} - \log |t|^2}$ is subharmonic on $\Delta - \{0\}$, it follows from (3.7) that the function $\frac{v_{z_0}(t)}{|t|^2}$ extends to some subharmonic function of Δ which, by the maximum principle, is taking values in $[0, 1]$. Then we have:

$$(3.8)\qquad \sup_{|t|\leq r} v_{z_0}(t) \leq r^2$$

for any $r \in [0, 1]$.

Let t be any point in Δ such that $|z - z_0| \geq \varepsilon$ for every $z \in F(t)$ where $\varepsilon \in]0, \alpha]$. Then, according to (i) and the definition of v_{z_0}, we have: $v_{z_0}(t) \geq (e^{-MK}\varepsilon^2)^m$. Thus, from (3.8), we get $|t|^2 \geq A^2\varepsilon^{2m}$ where $A^2 = e^{-MK}$. We have shown that $|t| < A\varepsilon^m$ implies that $|z - z_0| < \varepsilon$ for some $z \in F(t)$.

THIRD STEP. The localization argument.

Let us fix R_1 and R_2 such that $0 < R_2 < R_2'$ and $0 < R_1 < \frac{R_2}{2}$. Assume that the conclusion of the lemma is not true, then we may find a sequence of m-valued holomorphic correspondences $F_p\colon \Delta \to \Omega$ such that $F_p(0) \subset \bigcup_{1\leq j\leq \ell} B(a_j, R_1)$ and a sequence of point (t_p) in Δ such that $\lim t_p = 0$ and the intersection of $F_p(t_p)$ with $\bigcup_{j=1,\ell}\{R_2 \leq |z - a_j| \leq R_2'\}$ is not empty. After taking a subsequence, we may assume that $R_2 \leq |z_p - a_{j_0}| \leq R_2'$ for $z_p \in F_p(t_p)$ and $j_0 \in \{1, \dots, \ell\}$. We set $\tilde{F}_p(t) = F_p\left(\frac{t+t_p}{1-\bar{t}_p t}\right)$. Clearly, $(\tilde{F}_p)$ is a sequence of m-valued holomorphic correspondences from Δ to Ω. Fix $0 < \varepsilon < \min(\alpha, \frac{R_2}{2})$, then by the second step there exists $\tilde{z}_p \in \tilde{F}_p(-t_p) \cap \{|z_p - z| \leq \varepsilon\}$ when p is big enough. We have therefore $|\tilde{z}_p - a_{j_0}| \geq R_2 - \varepsilon \geq \frac{R_2}{2} > R_1$ and, for any $j \neq j_0$, $|\tilde{z}_p - a_j| \geq |a_j - a_{j_0}| - R_2' - \varepsilon \geq \delta - R_2' - \frac{R_2}{2} \geq \delta - \frac{3}{4}R \geq \frac{R}{2} > R_1$. Thus $\tilde{z}_p$ does not belong to $\bigcup_{1\leq j\leq \ell} B(a_j, R_1)$; since $\tilde{z}_p \in F_p(0)$ we have reach a contradiction.

Let us now prove the claim:

Without any loss of generality we may assume that f is irreducible and $f(0) = 0$. Let us denote by $f_1, \dots, f_n$ the coordinates of f. Each f_j is an m_j-valued holomorphic function on some neighbourhood of $t = 0$ and may therefore be defined by $f_j(t^{m_j}) = g_j(t)$ where g_j is an holomorphic function near the origin. It follows that $|f_j(t)| \lesssim |t|^{1/m_j}$ for $|t|$ small enough. Let us set $m' = \max\{m_1, \dots, m_n\}$ then $\prod_{z \in f(t)} |z|^2 \lesssim |t|^{2m/m'}$ and the claim follows since $m' \leq m$. $\square$

4. – Boundary continuity

We are now in order to proceed with the "integration" argument and prove the following:

LEMMA 4.1. *Assume that the hypothesis of Theorem A are fullfilled. Let $p \in \Gamma$ and (p_ν) be a sequence in D converging to p. Suppose that $\lim_\nu w^j(p_\nu) = q^j \in \Gamma'$ for $j = 1, \dots, m$ where we set $F(z) = \{w^1(z), \dots, w^m(z)\}$. Then, for any sequence (p'_ν) in D converging to p, the sequence $F(p'_\nu)$ is converging to $\{q^1, \dots, q^m\}$. That is, $\lim_\nu w^j(p'_\nu) = q^j_\nu$, after a possible reordering of superscripts (the point q^j are not necessarily distinct).*

REMARK 1 Roughly speaking, the above lemma says that the cluster set of $F(z_\nu)$ for (z_ν) converging to p does not depend on the sequence (z_ν).

For $\zeta \in \Gamma$ we denote by $\overline{n}(\zeta)$ the unit vector of real inward normal to Γ at ζ; there exists a neighbourhood U of p in $\mathbb{C}^n$ such that $D \cap U$ is filled by the rays $\{\zeta + t\overline{n}(\zeta), t \in \mathbb{R}^+\}$ when ζ runs over U. Now let d_ν denote $|p_\nu - p'_\nu|$, δ_ν (resp. (δ'_ν) be $\operatorname{dist}(p_\nu, \partial D)$ (respectively $\operatorname{dist}(p'_\nu, \partial D)$) and ζ_ν (respectively ζ'_ν) in Γ be the orthogonal projection of p_ν (respectively p'_ν) to Γ. For any ν large enough we consider a piecewise linear path π_ν in D which consists of 3 linear segment: the segment $[\zeta_\nu, a_\nu]$ in $\{\zeta_\nu + t\overline{n}(\zeta_\nu) t \in \mathbb{R}^+\}$ of length $\delta_\nu + d_\nu$, the segment $[\zeta'_\nu, a'_\nu]$ in $\{\zeta'_\nu + t\overline{n}(\zeta'_\nu) t \in \mathbb{R}^+\}$ of length $\delta'_\nu + d_\nu$ and the segment $[a_\nu, a'_\nu]$.

Considering that π_ν is oriented from p_ν to p'_ν, we denote by π^x_ν the portion of π_ν which consists in the points lying between p_ν and x, for any $x \in \pi_\nu$. We may assume that $q^j \neq q^k$ for $j \neq k$, $j, k \leq \ell \leq m$. Since Γ' is regular, one easily shows that the hypothesis of Lemma 3.2 are satisfied at points $q^j (j \leq \ell)$. Let R_1, R_2 and r be the positive constants which are associated to these points by Lemma 3.2. Recall that $R_1 < R_2 < 2\delta$ where $\delta = \min\{\operatorname{dist}(q^j, q^k)\,;\ j \neq k, k, j \leq \ell\}$. Since $F(p_\nu)$ is converging to $\{q^1, \dots, q^m\}$, we may assume that $F(p_\nu) \subset \bigcup_{j=1,\ell} B(q^j, R_1)$ for every ν.

We now argue by contradiction, assuming that $F(p'_\nu)$ is not converging to $\{q^1, \dots, q^m\}$. Then, we may find $\tilde{R}_1 \in]0, R_1[$ and $p''_\nu \in \pi_\nu$ such that

$$F(\pi_\nu^{p''_\nu}) \subset \bigcup_{j=1,\ell} \overline{B}(q^j, \tilde{R}_1) \text{ and } \max\{\operatorname{dist}(w, \{q^1, \dots, q^\ell\}) | w \in F(p''_\nu)\} = \tilde{R}_1 .$$

Thus, after taking some subsequence, $F(p''_\nu)$ is converging to $\{q'_1, \dots, q'_m\} \subset \Gamma'$ where $\operatorname{dist}(q'^{j_0}, \{q^1, \dots, q^\ell\}) = \tilde{R}_1$ for some j_0. We then pick a $\mathbb{C}$-linear form

Λ on $\mathbb{C}^n$ such that $|\Lambda(q'^{j_0}) - \Lambda(q^j)| \geq \varepsilon > 0$ for $1 \leq j \leq \ell$. For every $z \in D$ we may consider the m-valued holomorphic correspondence defined on $\mathbb{B}_1$ by $f_z(u) = F(z + urd(z))$ where $r \in]0, 1[$ is given by Lemma 3.2 and $d(z) = \operatorname{dist}(z, bD)$. It follows from Lemma 3.2 that $f_z(\mathbb{B}_1)$ is contained in $\bigcup_{1\leq j\leq \ell} B(q^j, R_2)$ for every $z \in \pi_\nu^{p''_\nu}$. This will allow us to apply Lemma 3.1 to these correspondences in order to establish our "integration" argument.

Assume that $p''_\nu \in [a'_\nu, p'_\nu]$. Then we replace p'_ν by p''_ν but keep the notation p'_ν. Without any loss of generality we may assume that $r = 1$.

• We parametrize $[\zeta_\nu, a_\nu]$ as follows: $\lambda_\nu: [0, \delta_\nu + d_\nu] \to D$ with $\lambda_\nu: t \mapsto t\overline{n}(\zeta_\nu)$, so that $\zeta_\nu = \lambda_\nu(0)$, $p_\nu = \lambda_\nu(\delta_\nu)$ and $a_\nu = \lambda_\nu(\delta_\nu + d_\nu)$. Note that $\operatorname{dist}(\lambda_\nu(t), \partial D) = t$ when ν is large enough. Consider a sequence $t_k = (\delta_\nu + d_\nu)/2^k, k \geq 1$ and denote by N_ν the integer such that $t_{N_\nu+1} \leq \delta_\nu \leq t_{N_\nu}$. First, we will show the following

$$\begin{aligned} &\text{For any } k \geq 0 \text{ and } w_k \in F(\lambda_\nu(t_k)) \text{ there exists } w_{k+1} \in F(\lambda_\nu(t_{k+1})) \\ &\text{such that } |\Lambda(w_{k+1}) - \Lambda(w_k)| \leq C(\delta_\nu + d_\nu)^{\alpha\tau/m}(\frac{1}{2})^{\alpha\tau k/m} \\ &\text{with } C > 0 \text{ independent of } k, \nu. \end{aligned} \tag{4.1}$$

Let us consider an m-valued holomorphic correspondence f_k, defined on the unit disc Δ of $\mathbb{C}$ by $f_k(u) = F(\lambda_\nu(t_k)+\lambda_\nu(t_k)u)$. It follows from Lemma 2.1 that

$$\operatorname{dist}(f_k(o), \Gamma') = \operatorname{dist}(F(\lambda_\nu(t_k)), \Gamma') \leq C \operatorname{dist}(\lambda_\nu(t_k), \Gamma)^\alpha = Ct_k^\alpha .$$

Fix $w_k \in f_k(0) = F(\lambda_\nu(t_k))$. Then Lemma 3.1 implies the existence of $w_{k+1} \in f_k(-1/2) = F(\lambda_\nu(t_{k+1}))$ such that

$$\begin{aligned} |\Lambda(w_{k+1}) - \Lambda(w_k)| &\lesssim (1/2)^{-2/m} \operatorname{dist}(w_k, \Gamma')^{\tau/m} \lesssim \\ (t_k)^{\alpha\tau/m} &= \big((\delta_\nu + d_\nu)/2^k\big)^{\alpha\tau/m} = (\frac{1}{2})^{\frac{k\alpha\tau}{m}}(\delta_\nu + d_\nu)^{\alpha\tau/m} . \end{aligned}$$

Thus (4.1) is proved.

Quite similarly, using the correspondence $f_{N_\nu}: \Delta \to D'$ defined by $f_{N_\nu}(z) = F(\lambda_\nu(t_{N_\nu}) + u\lambda_\nu(t_{N_\nu}))$, we get the existence of $w_\nu \in f_{N_\nu}\big((\delta_\nu - t_{N_\nu})/t_{N_\nu}\big) = F(\lambda_\nu(\delta_\nu)) = F(p_\nu)$, such that

$$\begin{aligned} |\Lambda(w_\nu) - \Lambda(w_{N_\nu})| &\lesssim (1 - |(\delta_\nu - t_{N_\nu})/t_{N_\nu}|)^{-2/m} t_{N_\nu}^{\alpha\tau/m} \\ &= (\delta_\nu/t_{N_\nu})^{-2/m}(\delta_\nu + d_\nu)^{\alpha\tau/m}/(2^{\alpha\tau/m})^{N_\nu} \\ &\lesssim (\delta_\nu + d_\nu)^{\alpha\tau/m}/(2^{\alpha\tau/m})^{N_\nu} \end{aligned} \tag{4.2}$$

(by choice, $\delta_\nu/t_{N_\nu} \geq \frac{1}{2}$). Thus, we get from (4.1) and (4.2):

$$\begin{aligned} |\Lambda(w_\nu) - \Lambda(\lambda_\nu(d_\nu + \delta_\nu))| &\lesssim (d_\nu + \delta_\nu)^{\alpha\tau/m} \sum_{k=0}^{N_\nu} (1/2^{\alpha\tau/m})^k \\ &\lesssim (\delta_\nu + d_\nu)^{\alpha\tau/m} \end{aligned} \tag{4.3}$$

and we have therefore proved the following:

$$\text{(4.4)} \qquad \begin{aligned} &\forall w_0 \in F(a_\nu), \exists\, w_\nu \in F(p_\nu) \text{s.t.:} \\ &|\Lambda(w_\nu) - \Lambda(w_0)| \lesssim \operatorname{dist}(a_\nu, \Gamma)^{\alpha\tau/m} . \end{aligned}$$

• Now we consider the segment $[a_\nu, a'_\nu]$.

By construction we have $|p_\nu - a_\nu| = d_\nu$, $|p'_\nu - a'_\nu| = d_\nu$ and for any $z \in [a_\nu, a'_\nu]$ one has $\operatorname{dist}(z, \Gamma) \approx d_\nu$. Let us divide $[a_\nu, a'_\nu]$ in 4 parts of equal lengths by the points c_j, $j = 0, 1, 2, 3$. Let us consider a correspondence $f_j: \Delta \to D'$, $f_j(u) = F(c_j + u d_\nu)$; then $f_j\left(\frac{c_{j+1}-c_j}{d_\nu}\right) = F(c_{j+1})$. Fix $w_j \in f_j(0) = F(c_j)$. Since $\operatorname{dist}(c_j, \Gamma) \lesssim d_\nu$, we get by Lemma 3.1 that there exists $w_{j+1} \in f_j\left(\frac{c_{j+1}-c_j}{d_\nu}\right)$ such that

$$|\Lambda(w_j) - \Lambda(w_{j+1})| \lesssim \left(1 - \left|\frac{c_{j+1} - c_j}{d_\nu}\right|\right)^{-2/m} d_\nu^{\alpha\tau/m} .$$

Since $\left|\frac{c_{j+1}-c_j}{d_\nu}\right| \leq \frac{1}{4}\frac{|a_\nu - a'_\nu|}{d_\nu} \leq \frac{3}{4}$, we get $|\Lambda(w_j) - \Lambda(w_{j+1})| \lesssim d_\nu^{\alpha\tau/m}$, this implies that:

$$\text{(4.5)} \qquad \begin{aligned} &\forall w'_\nu \in F(a'_\nu), \exists w_\nu \in F(a_\nu) \text{s.t.:} \\ &|\Lambda(w'_\nu) - \Lambda(w_\nu)| \lesssim (d_\nu)^{\alpha\tau/m} . \end{aligned}$$

• Finally we proceed with $[p'_\nu, a'_\nu]$ in the same way than for $[a_\nu, p_\nu]$. We get:

$$\text{(4.6)} \qquad \begin{aligned} &\forall w'_\nu \in F(p'_\nu), \exists w_0 \in F(a'_\nu) \text{s.t.:} \\ &|\Lambda(w'_\nu) - \Lambda(w'_0)| \lesssim \operatorname{dist}(a'_\nu, \Gamma)^{\alpha\tau/m} . \end{aligned}$$

We are now in order to finish the proof of Lemma 4.1. Let us consider a sequence (w'_ν), $w'_\nu \in F(p'_\nu)$, such that $\lim_\nu w'_\nu = q'^{j_0}$. From (4.6), (4.5) and (4.4) we get the existence of $w_\nu \in F(p_\nu)$ such that:

$$\text{(4.7)} \qquad |\Lambda(w_\nu) - \Lambda(w'_\nu)| \lesssim [d_\nu + \max(\delta_\nu, \delta'_\nu)]^{\alpha\tau/m} .$$

This is a contradiction since, after taking a subsequence, w_ν is converging to $q^{i_0} \in \{q^1, \dots, q^\ell\}$ and Λ has been chosen such that $|\Lambda(q^i) - \Lambda(q'^{j_0})| \geq \varepsilon > 0$ for every $i \in \{1, \dots, m\}$.

When p''_ν belongs to segment $[a_\nu, a'_\nu]$ or to the segment $[p_\nu, a_\nu]$ one replace a'_ν or p_ν by p''_ν and the above procedure remains valid, with the only modification that we deal with at most two segments instead of three. □

We are now in order to prove the main theorems. Let us first recall some elementary properties of canonical defining functions.

LEMMA 4.2. *Let* $\alpha_\nu = \{\alpha_\nu^1, \dots, \alpha_\nu^m\}$ *be a sequence of systems of m points in a bounded domain* Ω *in* $\mathbb{C}^n$, *converging to* $\alpha = \{\alpha^1, \dots, \alpha^m\}$, *that is* $\alpha_\nu^j \to \alpha^j$ *as* $\nu \to \infty$, $j = 1, \dots, m$. *Then* $\phi_I(w; \alpha_j) \to \phi_I(w; \alpha)$, $|I| = m$, *uniformly on compact subsets in* $\mathbb{C}^n$. *Conversely, if the polynomials* $\phi_I(w; \alpha_j)$ *converge in this way to a polynomial* $\phi_I(w)$, $|I| = m$, *then the sequence of systems* α_ν *has, under a suitable ordering of points, a limit* $\alpha = \{\alpha^1, \dots, \alpha^m\}$ *and* $\phi_I(w) = \phi_I(w; \alpha)$, $|I| = m$.

PROOF. see ([Ch], page 46). □

This lemma together with Lemma 4.1 immediately implies that the coefficients of the canonical defining functions (1.1) extend continuously to p, or in other words that the correspondence $F: D \to D'$ extends continuously to p, under the assumptions of Theorem A$'$.

Recal that $S_D \subset D$ denotes the singular locus of the correspondence F (in particular, $\pi_D: A \setminus \pi_D^{-1}(S_D) \to D$ is locally biholomorphic). The structure of S_D is described by the following:

LEMMA 4.3. *Let A, the graph of some holomorphic correspondence, be defined by the canonical functions* Φ_I. *Then* $S_D = \pi_D(V)$, *where* $V \subset A$ *is a complex subvariety of A of codimension* ≥ 1 *of the form*

$$V = \left\{(z, w) \in A \mid \operatorname{rank} \frac{\partial \phi_I}{\partial w}(z, w) < n\right\}.$$

The proof is contained in [Ch], page 50.

In order to get the Hölder continuity, we will establish the following:

LEMMA 4.4. *Let* $F: D \to G$ *be a holomorphic correspondence between domains* $D, G \subset \mathbb{C}^n$, *which continuously extends up to the boundary in a neighbourhood of* $p \in D$. *Then* S_D *is set of commons zeros of functions which are holomorphic in D and continuous up to D near p.*

PROOF. We just follow the standard exception of variables argument (see [Ch], page 30), keeping in mind the boundary continuity of the defining functions.

Without any loss of generality one assumes that G is the polydisc $\Delta_n \subset \mathbb{C}^n$. We represent the projection $\pi_D: A \to D$ in the form $\pi_D = \pi_n \circ \cdots \circ \pi_1$, where

$$\mathbb{C}_z^n \times \mathbb{C}_w^n \xrightarrow{\pi_1} \mathbb{C}^n \times \mathbb{C}^{n-1} \xrightarrow{\pi_2} \cdots \xrightarrow{\pi_n} \mathbb{C}^n.$$

Since $\pi_D: A \to D$ is proper, all the projections $\pi_i: (\pi_{i-1} \circ \cdots \circ \pi_1)(A) \to (\pi_i \circ \cdots \circ \pi_1)(A)$ are proper.

Set $w = (w', w_n) = (w_1, \dots, w_{n-1}, w_n)$ and $\pi_1: (z, w) \mapsto (z, w')$. It follows from (1.1) and Lemma 4.3 that V is a set of common zeros of the functions $f_j(z, w)$, $j = 1, \dots, \ell$, which are polynomials in w with coefficients holomorphic in z on D and continuous up to ∂D near p. The degree of f_1 in w_n will be assumed maximal, and is denoted by $d (d > 0$, since π_1 is proper).

We replace the system $f_2, \dots, f_\ell$ by the one-parameter family of functions $F_t = \sum_2^\ell f_k t^k$, $t \in \mathbb{C}$; the zero sets in $D \times \Delta_n$ of these families coincide. As it is well-known, the polynomials f_1 and F_t in one variable w_n have common zeros iff the resultant $R(f_1, F_t)$ vanishes. Therefore, $\pi_1(V) \subset \{(z, w') \in D \times \Delta_{n-1} \mid R(f_1, F_t) \equiv 0 \text{ in } t\}$. We write the resultant (being the corresponding determinant in the coefficients of f_1 and F_t) in powers of t: $R = \sum_0^{\ell d} R_k(z, w')t^k$. The condition $R \equiv 0$ in t is equivalent to $R_k(z, w') = 0$ for all k, hence $\pi_1(V) \subset \{(z, w') \in D \times \Delta_{n-1} \mid R_k(z, w') = 0, k = 0, \dots, \ell d\}$.

Conversely, suppose that every $R_k(a, b') = 0$ for $(a, b') \in D \times \Delta_{n-1}$ fixed, and let $b_n^1, \dots, b_n^d \in \mathbb{C}$ be the roots of the polynomial $f_1(a, b', w_n)$. The resultant $R(f_1(a, b', \cdot), F_t(a, b', \cdot)) \equiv 0$ in t, hence for each $t \in \mathbb{C}$ the polynomial $F_t(a, b', w_n)$ vanishes at at least one of the points b_n^j, that is $\bigcup_{j=1}^\alpha \{t \mid F_t(a, b', b_n^j) = 0\} = \mathbb{C}_t$.

On the other hand, $F_t(a, b', b_n^j)$ is a polynomial in t, and if it does not vanish identically, then the number of its roots is finite. Hence, for some j_0 the polynomial $\sum_2^\ell f_k(a, b', b_n^{j_0})t^k \equiv 0$ in t, that is $f_k(a, b', b_n^{j_0}) = 0$, $k = 2, \dots, \ell$. Hence $(a, b') \in \pi_1(V)$ and

$$\pi_1(V) = \{(z, w') \in D \times \Delta_{n-1} \mid R_k(z, w') = 0, k = 0, \dots, \ell d\}.$$

Here $R_k(z, w')$ are polynomials in w' with coefficients, holomorphic in z on D and continuous up to ∂D near p (by construction). Now one may apply the same argument to π_2, etc ..., and we conclude. □

We now complete the proof of Theorem A. Since F is continuous up to Γ, as a corollary of Lemma 4.4 we get that $\Gamma \setminus \overline{S}_D$ is an open dense subset of Γ. Fix $p \in \Gamma \setminus \overline{S}_D$. Then there is a neighbourhood U of p such that $(\pi_{D'} o \pi_D^{-1})(z) = \{F^1(z), \dots, F^m(z)\}$ for $z \in U \cap D$, where every $F^j: U \cap D \to D'$ is an holomorphic mapping on $U \cap D$ which is continuous up to Γ.

Since Γ' is (α', β')-regular, we have (see [DF1] for the bounded case and [Su] for the general one):

LEMMA 4.5. *There exists a neighbourhood V' of Γ' such that for any $w \in V' \cap D'$ and $Y \in \mathbb{C}^n$ one has*

$$K_{D'}(w, Y) \geq C_2|Y|)/ \operatorname{dist}(w, \Gamma')^{\alpha'/\beta'}$$

where $K_{D'}$ denotes the Kobayashi infinitesimal metric on D' and $C_2 > 0$ is constant.

On the other hand, one has an evident

LEMMA 4.6. *Given $z \in D \cap U$ and $X \in \mathbb{C}^n$ one has*

$$K_{D\cap U}(z, Y) \leq C_3|Y|)/ \operatorname{dist}(z, \Gamma).$$

By continuity, one may assume that $F^j(z) \in D' \cap V'$, $j = 1, \dots, m$, $\forall z \in D \cap U$. We have

$$\begin{aligned} K_{D\cap U}(z, X) &\geq K_{D'}(F^j(z), dF^j(z)X) \\ &\geq C_2 \operatorname{dist}(F^j(z), \Gamma')^{-\alpha'/\beta'}|dF^j(z)X| \\ &\geq C_1^{-\alpha'/\beta'} \operatorname{dist}(z, \Gamma')^{-\alpha\alpha'/\beta'}|dF^j(z)X| \end{aligned}$$

where C_1 if from Lemma 2.1. Thus, we get by Lemma 4.6 that

$$|dF^j(z)X| \leq C_1^{-\alpha'/\beta'} C_2 C_3^{-1} \operatorname{dist}(z,\Gamma)^{-1+\alpha\alpha'/\beta'}|X|.$$

Now the standard integration argument implies that

$$|F^j(z') - F^j(z'')| \leq C_1^{-\alpha'/\beta'} C_2 C_3^{-1} |z' - z''|^{\alpha\alpha'/\beta'}$$

for any $z', z'' \in U \cap \overline{D}$. This means that the coefficients of the canonical defining functions (1.1) are Hölder $\alpha\alpha'/\beta'$ continuous on any compact subset of $\Gamma \setminus \overline{S}_D$. □

The proof of Theorem B is quite similar, we merely use Lemma 2.5 instead of Lemma 2.1. Note that, in general, we cannot extend the coefficients to $\overline{S}_D$, since the constant C_3 depends on U, that is on the distance from $p \in \Gamma \setminus \overline{S}_D$ to $\overline{S}_D$.

5. – Some applications

In this section we give several different applications of our results.

PROPOSITION 5.1. *Suppose that $F: \Gamma \to \Gamma'$ is a continuous CR mapping between pseudoconvex hypersurfaces in $\mathbb{C}^n$. Suppose further that Γ_1 is C^∞ of finite type in the sense of D'Angelo, Γ_2 is of class C^2 and does not contain any one dimensional analytic varieties. If $F(z_0) = w_0$ and $F^{-1}(w_0)$ is a compact subset of Γ_1, then F is a finite-to-one mapping in a neighbourhood of $F^{-1}(w_0)$, which itself is necessearily a finite set.*

This result was obtained by [BC] for the case where Γ' is of class C^∞ (they proved also that in this case F also in C^∞). Their proof is based on the Bergman projector technique and essentially uses deep PDI results on regularity of the Bergman projection and the assumption that Γ' is of class C^∞. Our approach provides a quite elementary proof of this assertion.

PROOF. It follows by [T] that F extends holomorphically to the pseudoconvex side D of Γ; since Γ' does not contain complex varieties of positive dimension, F maps D to the pseudoconvex side D' of Γ' (see [B]). As it was shown in [BC], one may find arbitrarily small neighbourhoods V of w_0 and U of $F^{-1}(w_0)$ such that $F: D \cap U \to D' \cap V$ is proper. Then it follows from Theorem B that the proper holomorphic correspondence $F^{-1}: D' \cap V \to D \cap U$ extends continuously to Γ' and, therefore, $F^{-1}(w)$ is finite for any $w \in \Gamma \cap U$. □

The second assertion may be considered as version of the classical Poincaré-Alexander theorem for analytic varieties.

PROPOSITION 5.2. *Let D and D' be bounded domains in $\mathbb{C}^n (n > 1)$ with strictly pseudoconvex simply connected real analytic boundaries. Let Ω be a subdomain of D such that $\partial\Omega$ contains an open piece Γ of ∂D and let $F: \Omega \to D'$ be an m-valued holomorphic corespondence.*

Suppose that the cluster set $C_\Omega(F; \Gamma)$ is contained in $\partial D'$. Then the graph of F is the union of m graphs of restrictions to Ω of (global) biholomorphisms between D and D'. In particular, if F is irreducible, then $m = 1$.

This assertion generalizes many well-known results [Al], [BB1], [Pi4] and is new even in the classical case where D and D' coincide with the unit ball of $\mathbb{C}^n$.

PROOF. It follows from Theorem A that F extends continuously to Γ; as it was shown in Section 4, there is a point $p \in \Gamma$ and a neighbourhood U of p in $\mathbb{C}^n$ such that $F \mid D \cap U$ "splits", that is the graph of F is an union of holomorphic in $D \cap U$ mappings $F^{(j)}: D \cap U \to D'$, continuous up to Γ. It follows from [PiT] that every $F^{(j)}$ is a local smooth CR diffeomorphism on $\Gamma \cap U'$, if a neighbourhood U' of p is small enough. Therefore, $F^{(j)}$ extends holomorphically past p [L], [Pi2], and, moreover, extends analytically along any path in ∂D (see [Pi2], [Pi3]); since ∂D is simply connected, the extended mapping is holomorphic in a neighbourhood of ∂D and, therefore, extends to D. The extended mapping $F^{(j)}: D \to D'$ is proper and, therefore, biholomorphic in view of the simply connecteness condition (see [Pi3]). □

Our next assertion concerns the mapping problem for algebraically bounded domains. A domain $D \subset c\mathbb{C}^n$ is said to be **algebraically bounded** if $D = \{z \in \mathbb{C}^n \mid p(z, \bar{z}) < 0\}$, where p is a real polynomial in $\mathbb{C}^n$ and $dp \neq 0$ in a neighbourhood of ∂D. We say that a holomorphic correspondence $f: D \to D'$ between domains D, D' in $\mathbb{C}^n$ is **algebraic**, if the graph of f is contained in a complex n-dimensional algebraic variety V in $\mathbb{C}^n \times \mathbb{C}^n$.

PROPOSITION 5.3. *Let $F: D \to D'$ be a proper holomorphic correspondence between algebraically bounded domains in $\mathbb{C}^n (n > 1)$. Then F is algebraic.*

PROOF. First we prove that there exists $p \in \partial D$ such that every coefficient in (1.1) extends holomorphically past p. If D is not pseudoconvex this is evident. If D is pseudoconvex, D' is necessarily pseudoconvex as well; since they are of finite type [DF1], it follows from Theorem B that F extends continuously to ∂D; so we may take $p \in \partial D$ suc h that f "splits" in a neighbourhood U of p to holomorphic on $D \cap U$ mappings $F^{(j)}$, $j = 1, \dots, m$, which are continuous up to ∂D.

Since F^{-1}: $D' \to D$ also extends continuously to $\partial D'$, the pull-back $F^{-1}(F(p))$ is finite. Therefore, it follows by [BC] that any $F^{(j)}$ extends C^∞ smoothly up to ∂D near p and $f^{(j)}: D \cap U_j \to D' \cap (F^{(j)}(p) + r\mathbb{B})$ is proper for a suitable neighbourhood U_j of p and $r > 0$ (see [B]). But then every $F^{(j)}$ extends holomorphically past p in view of [DF2].

Now one may choose a Levi-non-degenerate point $p' \in \partial D$ close enough to p such that any $F^{(j)}$ is biholomorphic at p'; hence $F^{(j)}(p')$ is a Levi-non-degenerate point in $\partial D'$ and $F^{(j)}$ extends to an algebraic mapping by the Webster theorem [W]; therefore, F is algebraic by the uniqueness theorem. □

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