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Elimination of L^1 Singularities on Hölder Peak Sets for CR Functions

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We deal in this article with those L^1_{loc} functions on a CR submanifold M of the euclidean complex space which are CR functions in the complement of some relatively closed subset $S \subset M$ and we prove, with suitable hypotheses on S and M, that those functions are actually restrictions of CR functions on M. Thus our results are of the form

$$L^1_{\text{loc}}(M) \cap CR(M \backslash S) \subset CR(M).$$

This can be viewed in the spirit of the classical Riemann theorem on elimination of singularities for holomorphic functions in $\mathbb{C}$, which may be written as follows

$$L^2_{\text{loc}}(\Omega) \cap \mathcal{O}(\Omega \backslash \{z_0\}) \subset \mathcal{O}(\Omega)$$

for some z_0 in an open set $\Omega \subset \mathbb{C}$.

A particular role will be played by characteristic submanifolds and by peak sets in M.

A connected submanifold $N \subset M$ is said to be *characteristic* if $\dim N < \dim M$ but $\dim_{CR} N = \dim_{CR} M$. A point of M which is not contained in any characteristic submanifold is said to be a *minimal point.*

A subset $S \subset M$ is said to be a C^λ peak set, $0 < \lambda < 1$, if there exists a non constant function $h \in C^\lambda(M) \cap CR(M)$ such that $S \equiv \{h = 1\}$, but $|h| < 1$ on $M \backslash S$.

We now state our results.

THEOREM 1. *Each minimal point p of a CR manifold of class $C^{2,\alpha}$, $0 < \alpha < 1$, has a neighbourhood M such that, if S is a C^λ peak set in M,*

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then

$$(1) \qquad L^1_{\mathrm{loc}}(M) \cap CR(M \backslash S) \subset CR(M).$$

Our proof can be sketched as follows. For any $f \in L^1_{\mathrm{loc}}(M) \cap CR(M \backslash S)$, $g \equiv (h-1)^{2/\lambda} f$ is in $CR(M)$ (Lemma 3). By minimality, g and $u \equiv (h-1)^{2/\lambda}$ extend holomorphically to an open wedge $\mathcal{W}$ over M. Then, by a careful study of Bishop's and Tumanov's constructions (next Proposition 5), one obtains that $u \not\equiv 0$ in $\mathcal{W}$ and that g/u, which is holomorphic in $\mathcal{W}$, has f as limit, in the L^1 sense, on M.

An anonymous referee indicated to us how to derive from Lemma 3 and from Proposition 5 below an analogue of (1) for a general complex vector field (not necesserely the operator $\overline{\partial}_b$), when there exists a hölder solution which peaks on S and S has measure 0. More precisely we shall prove the following:

PROPOSITION 3. *Let $L = \sum_{j=1}^{n} a_j(x) \dfrac{\partial}{\partial x_j}$ be a complex vector field, with C^1 coefficients, in the open set $\Omega \subset \mathbb{R}^n$, and let $h \in C^\lambda(\Omega)$, $0 < \lambda < 1$, be a solution of $Lh = 0$, such that $|h| \leq 1$ in Ω. If the maximum set $\{h = 1\}$ has vanishing Lebesgue measure, then each weak solution $f \in L^1_{\mathrm{loc}}(\Omega)$ of $Lf = 0$ in $\Omega \backslash \{h = 1\}$ is also a solution in Ω.*

On the other hand, Proposition 5, gives easily the next:

PROPOSITION 4. *In a CR manifold of class $C^{2,\alpha}$ each minimal point has a neighbourhood M such that any C^λ peak set in M has vanishing Lebesgue measure.*

These propositions obviosly imply Theorem 1. We present both proofs here.

In order to give a stronger version of Theorem 1 for real analytic manifolds (next Theorem 3), the minimality assumption in Theorem 1 can be considerably relaxed using the following:

PROPOSITION 1. *Let M be a CR submanifold of $\mathbb{C}^\nu$ and N an embedded characteristic submanifold of M. If M and N are of class C^1, then*

$$L^1_{\mathrm{loc}}(M) \cap CR(M \backslash N) \subset CR(M).$$

Further, if N has codimension 1 *and, conversely,*

$$L^\infty_{\mathrm{loc}}(M) \cap CR(M \backslash N) \subset CR(M)$$

holds, then N is characteristic.[3]

[3] This Proposition is valid not only for CR functions but holds in general for functions annihilating a vectorfield in the complement of a manifold which is tangent to it. In this form the

For optimal use of this proposition, it is convenient to disregard those characteristic manifolds which are not isolated but are contained in an isolated one of larger dimension. Therefore we define as *characteristic of maximal dimension* any characteristic manifold which is not contained in another one of higher dimension.

Assume for a moment that p is non minimal. The first part of Proposition 1 obviously implies that the conclusion of Theorem 1 still holds if p belongs to an isolated characteristic manifold of maximal dimension. This procedure can be carried further: Let $\mathcal{F}_0$ be the family of characteristic manifolds of maximal dimension in some neighbourhood of p. For instance $\mathcal{F}_0$ could consist of a sequence of isolated manifolds converging to some manifold. This situation escapes Theorem 1, but Proposition 1 is applicable twice: first for eliminating the sequence and then for eliminating the limit. This kind of argument can be pushed further:

Set $\mathcal{F}_n = \mathcal{F}_{n-1} \setminus \{\text{the isolated elements of } \mathcal{F}_{n-1}\}$, starting with $n = 1$. If $p \in \mathcal{F}_0$ but $\bigcap \mathcal{F}_n = \emptyset$, then Theorem 1 still holds. Thus we have showed that minimality in Theorem 1 can be weakened:

THEOREM 2. *If p does not belong to any perfect family of characteristic manifolds of maximal dimension, then the conclusion of Theorem* 1 *is still valid.*[4]

We consider now the real analytic case. The next Proposition generalizes a result of Khurumov [7].

PROPOSITION 2. *If M is a connected, real analytic CR submanifold of $\mathbb{C}^\nu$ and has at least one minimal point, then its characteristic manifolds of maximal dimension form a locally finite family.*

So we obtain, without need of proof, the following

THEOREM 3. *If the connected, real analytic CR manifold $M \subset \mathbb{C}^\nu$ has at least one minimal point, then* (1) *holds for any C^λ peak set S.*

Finally we discuss the hypotheses with an example which has a double purpose. On one hand it illustrates the role displayed by the measure assumption for the peak set in Proposition 3, on the other hand it shows how sharp is the hypothesis on the characteristic family containing p in Theorem 2.

THEOREM 4. *There exists a smooth, psuedoconvex hypersurface $M \subset \mathbb{C}^2$ with the following properties:*

(i) *M contains a closed set S, with positive Lebesgue measure and no interior point, which is both a peak set for a smooth, CR function and the union*

statement is probably known and can be proved along the line of our proof up to replace the entire approximation theorem by a standard mollification argument.

[4] A perfect set is a closed set without isolated points.

of all complex curves contained in M.[5]

(ii) $L^1(M) \cap CR(M\backslash S) \not\subset CR(M)$.[6]

(iii) $M\backslash S$ *is strictly pseudoconvex.*

REMARK 1. A C^1 submanifold M' of the complex euclidean space E' is said to be *generic* if $TM' + iTM' = E'$. It is well known that *any* CR *submanifold* M *of the euclidean space* E *is locally equivalent to a generic one*, i.e. for each $p \in M$ there exists a neighbourhood U in E, a generic manifold M' and a C^1 diffeomorphism $\Psi : M \cap U \to M'$ which is CR together with its inverse.

Indeed one can fix a complex supplement of $E' \equiv TM + iTM$ and consider the corresponding projection $\pi : E \to E'$. For suitably small U, $\Psi \equiv \pi_{|M\cap U}$ is a diffeomorphism onto a generic submanifold $M' \subset E'$ and, since π is holomorphic, Ψ is a CR map. Thus Ψ^{-1} is also CR.

So, since all of our statements are local, *we shall implicitly assume that the given* CR *manifold is generic.*

A particular case of Theorem 1 has been proved by Kytmanov [9]. There the absence of characteristic submanifolds is replaced by a non vanishing hypothesis on the Levi form. In this case one can use a construction due to Bogges and Polking ([2]) of a particularly nice family of analytic discs similar to the family Φ in our Proposition 5.

In our case we use Tumanov's method ([15], [16]) but the "nice family" is not furnished there and some work must be spent for its construction.

Elimination of L^1 singularities of CR functions is of some interest in various problems as for instance the study of rational functions on compact subsets of boundaries ([10]).

After Harvey and Polking's article [4], much has been done about eliminable or removable singularities of CR functions by Henkin, Lupacciolu, Stout, Kytmanov and many other authors. A result in our spirit is a Rado theorem for CR functions which are continuous on a hypersurface whose Levi form vanishes at most on a suitably thin set. This has been established by J.P. Rosay and E.L. Stout [13] but has no intersection with our results. For a wide survey we refer to Henkin [5] and Stout [14]. We are indebted to E.M. Chirka, A. Schiaffino and Y.V. Khurumov for useful conversations.

[5] A characteristic manifold here is necessarily a regular, complex curve.

[6] In fact we prove more i.e. $L^\infty(M) \cap CR(M\backslash S) \not\subset CR(M)$.

1. - Proof of Proposition 1

We first need the following, simple result:

LEMMA 1. *Let g be a real C^1 function on the generic C^1 submanifold $M \subset \mathbb{C}^\nu$ such that g and dg do not vanish together, $f \in L^1_{\text{loc}}(M) \cap CR(M)$ and ψ a smooth (ν, ν')-form in $\mathbb{C}^\nu$ with $\operatorname{supp}\psi \cap M$ compact and $\nu + \nu' + 1 = \dim M$. Then for almost all $0 < \varepsilon \ll 1$ we have*

$$\int_{g<\varepsilon} f\, \mathrm{d}\psi = \int_{g=\varepsilon} f\psi.$$

PROOF. $\operatorname{supp}\psi$ can be taken so small in order that there exists a sequence of holomorphic polynomials f_n converging to f in $L^1(\operatorname{supp}\psi)$, (see [9], Lemma 2). The formula holds for f_n, for all n and $\varepsilon > 0$. Since $f \in L^1(\operatorname{supp}\psi \cap \{g = \varepsilon\})$, then, by the Fubini Theorem, for almost all ε and after taking a subsequence, the restrictions of f_n to $\operatorname{supp}\psi \cap \{g = \varepsilon\}$ converge to f in L^1 (see also [9]). Obviously on the left side we have convergence too. □

PROOF OF PROPOSITION 1. Let $z = z(t, \tau)$ be a local parametric representation of M, with $t \in \mathbb{R}^{\mu-d}$, $\tau \in \mathbb{R}^d$, $|\tau| < R$, and let N be given by $\{z(t, 0), t \in \mathbb{R}^{\mu-d}\}$, $\mu = \dim M$. Since N is characteristic, the z_j's can be reordered so that $dz_1, \ldots, dz_d$ depend linearly on $d\tau_1, \ldots, d\tau_d$, $dz_{d+1}, \ldots, dz_\nu$, at the points of N. This can be verified taking M and N linear. Thus we have

$$dz_\alpha = \sum_{\beta=1}^{d} (a_{\alpha\beta} d\tau_\beta + \tau_\beta b_{\alpha\beta}) + \sum_{j=d+1}^{\nu} a_{\alpha j} dz_j, \quad \alpha \le d$$

where $a_{\alpha\beta}$, $a_{\alpha j}$ are continuous functions and $b_{\alpha\beta}$ continuous 1-forms. Thus, if $\pi^\varepsilon : M \to M$ is the map $(t, \tau) \mapsto (t, \varepsilon\tau)$ and $dz \equiv dz_1 \wedge \ldots \wedge dz_\nu$, we obtain

$$\pi^\varepsilon_* dz = \varepsilon^d \chi_\varepsilon \tag{2}$$

where χ_ε is a ν-form with coefficients depending continuously on t, τ, ε.

Let be $f \in L^1_{\text{loc}}(M) \cap CR(M \backslash N)$.

We must prove

$$\int_M f\, \mathrm{d}z \wedge \mathrm{d}\psi = 0 \tag{3}$$

for all smooth compactly supported forms ψ of degree $\mu - \nu - 1$. Since $f \in L^1(M)$, we have for the integral in (3)

$$\int_M = \lim_{\varepsilon \downarrow 0} \int_{M_\varepsilon}, \quad \text{with} \quad M_\varepsilon \equiv \{z(t, \tau), \varepsilon < |\tau| < R\}$$

and thanks to lemma 1, with $g = |\tau|$, we obtain

$$\int_{M_\varepsilon} f\,\mathrm{d}z \wedge \mathrm{d}\psi = \int_{N_\varepsilon} f\,\mathrm{d}z \wedge \psi, \quad \text{with} \quad N_\varepsilon \equiv \{z(t,\tau), |\tau| = \varepsilon\},$$

for almost all $\varepsilon > 0$.

Thus it remains to prove

$$\lim_{\varepsilon\to 0} \int_{N_\varepsilon} f\,\mathrm{d}z \wedge \psi = 0, \tag{4}$$

when ε varies in a full measure subset of $(0, R)$.

We introduce polar coordinates in $\mathbb{R}^d$ and write $d\tau = |\tau|^{d-1} d|\tau| \wedge \omega_{d-1}$ where ω_{d-1} is the pull-back to $\mathbb{R}^d \backslash \{0\}$ of the standard measure on S^{d-1} via the map $\tau \mapsto \tau/|\tau|$. We have $\pi_*^\varepsilon \omega_{d-1} = \omega_{d-1}$.

After shrinking M so that $f \in L^1(M)$ but still $\operatorname{supp}\psi \subset\subset M$, we have

$$\|f\|_{L_1} = \int_{|\tau|<R} \int_{t\in\mathbb{R}^{\mu-d}} |f|\,\mathrm{d}t \wedge \mathrm{d}\tau = \left| \int_0^R \varepsilon^{d-1}\,\mathrm{d}\varepsilon \int_{N_\varepsilon} |f|\,\mathrm{d}t \wedge \omega_{d-1} \right| =$$

$$= \left| \int_0^R \varepsilon^{d-1}\,\mathrm{d}\varepsilon \int_{N_1} |f^\varepsilon|\,\mathrm{d}t \wedge \omega_{d-1} \right|, \quad \text{with} \quad f^\varepsilon(t,\tau) \equiv f(t, \varepsilon\tau).$$

We can conclude that $g : \varepsilon \mapsto \varepsilon^{d-1} \left| \int_{N_1} |f^\varepsilon|\,\mathrm{d}t \wedge \omega_{d-1} \right|$ is a function in $L^1(0, R)$.

On the other hand we have from (2)

$$\int_{N_\varepsilon} f\,\mathrm{d}z \wedge \psi = \varepsilon^d \int_{N_1} f^\varepsilon \chi_\varepsilon \wedge \pi_*^\varepsilon \psi.$$

Since $dt \wedge \omega_{d-1}$ is a positive maximal form on N_1, we have $\chi_\varepsilon \wedge \pi_*^\varepsilon \psi = b_\varepsilon dt \wedge \omega_{d-1}$ where b_ε is a continuous, bounded function of t, τ, ε. Thus

$$\left| \int_{N_\varepsilon} f\,\mathrm{d}z \wedge \psi \right| \le C\varepsilon g(\varepsilon) \quad \text{for a.a.} \quad \varepsilon \in (0, R).$$

Since the limit of the left side exists, for $\varepsilon \downarrow 0$ in a full measure subset of $(0, R)$, and $g \in L^1(0, R)$, we obtain (4) as we wanted.

For the second part, since the statement is local, we can assume that N divides M into two components and observe that the function which is equal to 1 on one of them and 0 on the other, is a CR function if and only if N is characteristic. $\square$

2. - Proof of Theorem 1

Theorem 1 depends on the construction of analytic discs which will be done in Proposition 5 below.

D will be the unit disc and Γ its boundary.

We must study some properties of the analytic discs of 0 defect at some point of M.

Roughly speaking an analytic disc $\varphi : \overline{D} \to \mathbb{C}^\nu$, with boundary $\varphi\Gamma \subset M$ at $p = \varphi(1)$ has defect 0 if $\tilde{\varphi}D$ fills an open wedge over M, when $\tilde{\varphi}$ varies among all small perturbations of φ, keeping $\tilde{\varphi}\Gamma \subset M$ and $p = \tilde{\varphi}(1)$.

Let p be a minimal point of a generic real submanifold $M \subset \mathbb{C}^\nu$ of codimension m, CR dimension n and class $C^{2,\alpha}$, $0 < \alpha < 1$. Thus $n + m = \nu$. Choose coordinates $(w, z) \in \mathbb{C}^{n+m}$ so that $p = (0,0)$, and M, near p, can be given the form

$$x = k(w, y), \quad (w, y) \in B_\rho^{2n} \times B_\rho^m, \tag{5}$$

where B_ρ^{2n}, B_ρ^m are the usual balls, $k(0,0) = \mathrm{d}k(0,0) = 0$.

Fix β with $0 < \beta < \alpha$ and let W be the Banach space of $C^{1,\beta}$ maps $w : \overline{D} \to \mathbb{C}_w^n$ which are holomorphic in D and such that $w(1) = 0$. Set $W_\delta \equiv \{w \in W, \|w\|_{1,\beta} < \delta\}$.

A small analytic disc φ, near $(0,0)$ in $\mathbb{C}^{n+m}$ of class $C^{1,\beta}$, attached to M, can be written in the form

$$\varphi(\varsigma) = [w^0 + w(\varsigma), z(\varsigma) + iy^0], \quad y(1) = 0$$

where (w^0, y^0, w) is chosen arbitrarily in $B_\delta^{2n} \times B_\delta^m \times W_\delta$ and the z-component is uniquely determined by the condition $\varphi\Gamma \subset M$ via the Bishop construction. Hence we can write $z(\varsigma) = z(w^0, y^0, w|\varsigma)$, $\varphi(\varsigma) = \varphi(w^0, y^0, w|\varsigma)$.

φ depends C^1 on the parameters (w^0, y^0, w) (see [16]) and, as they vary in $B_\delta^{2n} \times B_\delta^m \times W_\delta$, φ describes a neighbourhood of zero in the set of all analytic discs satisfying $\varphi\Gamma \subset M$.

DEFINITION. *The disc $\varphi(w^0, y^0, w|\cdot)$ is said to have defect 0 if the map*

$$W \ni w \mapsto \left.\frac{dx(w^0, y^0, w|\lambda)}{d\lambda}\right|_{\lambda=1} \in \mathbb{R}^m$$

has surjective differential at w.

By semicontinuity of the rank, 0 defect discs are an open set and the main point in [15] is that at a minimal point p there are arbitrarily small discs φ of 0 defect with $\varphi(1) = p$.

By minimality of p there exists a neighbourhood V of p in M and an open wedge $\mathcal{W}$ over V such that functions in $CR(M)$ extend holomorphically to $\mathcal{W}$ ([15]). This extension must be viewed in a continuous or L^1 sense if the given function is continuous or L^1 respectively and is obtained by covering $\mathcal{W}$ with small perturbations ψ of φ, still satisfying $\psi\Gamma \subset V$, and, for each $h \in CR(V)$, extending $h \circ \psi : \Gamma \to \mathbb{C}$ inside the unit disc. In particular, if h is a peak function then its extension to $\mathcal{W}$, which is still called h, satisfies $|h| \leq 1$ and hence $|h| < 1$ in $\mathcal{W}$ for otherwise $h \equiv 1$ because $\mathcal{W}$ is open.

LEMMA 2. *Let p be a minimal point of a generic manifold M. If φ, $\varphi\Gamma \subset M$, has defect 0 and is sufficiently small, then for any peak function h the modulus of the extension of $h \circ \varphi$ is strictly smaller than 1 in D and $h[\varphi(\varsigma)] \neq 1$ almost everywhere in Γ.*

PROOF. For $\lambda < 1$ very close to 1 the point $\varphi(\lambda)$ belongs to $\mathcal{W}$; thus the modulus of the extension of $h \circ \varphi$ is not identically 1 in D and, since it is ≤ 1 at the boundary, it is < 1 in D and $h[\varphi(\varsigma)] \neq 1$ a.e. on Γ. □

In Prop. 5 we want to construct a particular family of discs of defect 0. For we need some more information about the Bishop map $B_\delta^{2n} \times B_\delta^m \times W_\delta \ni (w^0, y^0, w) \mapsto z(w^0, y^0, w|\cdot)$ considered above.

We shall use the Hilbert transform as the continuous linear map $T : C^{1,\beta}(\Gamma) \to C^{1,\beta}(\Gamma)$ defined on real functions by the property that $f + iTf$ is the boundary value of a holomorphic function in the disc, and $Tf(1) = 0$.

For given (w^0, y^0, w), the trace on Γ of $z(\varsigma) \equiv z(w^0, y^0, w|\varsigma)$ is defined by the equation $z(\varsigma) = k[w^0 + w(\varsigma), y(\varsigma)] + iy(\varsigma)$, $|\varsigma| = 1$, where $y(\varsigma)$ is the solution of Bishop's equation $y = Tk(w^0 + w, y) + y^0$. In D z is defined by its trace on Γ via Poisson formula. Thus the jacobian matrix $D_{y_0}y$ is the solution of $D_{y_0}y = Tk_y(w^0 + w, y) + 1$. By the continuity of T and the condition $dk(0,0) = 0$ we conclude that $D_{y_0}y$ has non vanishing determinant if $0 < \delta \ll 1$.

We are now in a position to prove the following

PROPOSITION 5. *Let p be a minimal point of a generic real submanifold $M \subset \mathbb{C}^\nu$ of codimension m and class $C^{2,\alpha}$, $0 < \alpha < 1$, and let B be the ball of $\mathbb{R}^{2\nu-m-1}$. There exists a C^1 map $\Phi : B \times \overline{D} \to \mathbb{C}^\nu$ with the following properties*

(i) *$\Phi(0,1) = p$, $\Phi(B, \Gamma) \subset M$ and $\Phi(b, \cdot)$ is holomorphic in D, $\forall b \in B$.*

(ii) *The map $\Phi_{|B\times\Gamma} : B \times \Gamma \to M$ has non vanishing jacobian.*

(iii) *For each $v \in L^1_{\text{loc}}(M) \cap CR(M)$ there exists $\tilde{v} \in L^1(B \times D)$, holomorphic with respect to $\varsigma \in D$, such that $v \circ \Phi_{B\times\Gamma}$ is the boundary value of $\tilde{v}$ on*

$B \times \Gamma$ in L^1 sense.[7] *In particular, for almost all $b \in B$, $\tilde{v}(b, \cdot)$ belongs to the Hardy class H^1.*

(iv) *If v is also continuous then $\tilde{v} \in C^0(B \times \overline{D})$ and $\tilde{v} = v \circ \Phi$ on $B \times \Gamma$.*

(v) *If in addition v is a peak function then $|\tilde{v}| < 1$ in $B \times D$.*

(vi) *If S is a hölder peak set in M, then, for any fixed $b \in B$, the set $\{\varsigma \in \Gamma, \Phi(b, \varsigma) \in S\}$ has Lebesgue measure 0 in Γ.*

PROOF. The statement is local thus M can be assumed to be given by (5) and ρ will be reduced if necessary. Since $(0,0)$ is minimal, for any $\delta > 0$, we can fix $w \in W_\delta$ such that $[w(\cdot), z(0, 0, w|\cdot)]$ has defect 0. Thanks to the openness of the set of these discs, we can also assume $w_n'(\varsigma) \neq 0$ for $|\varsigma| = 1$ and that those properties are still fullfilled by $[w(\cdot), z(w^0, y^0, w|\cdot)]$ when $w^0 \in B_\delta^{2n}$, $y^0 \in B_\delta^m$, after reducing δ. Using the notation $w_* = (w_1, \dots, w_{n-1})$, we set, for $|t| < \delta$,

$$w^t(\varsigma) = [w_*(\varsigma), w_n(\varsigma) + t\varsigma w_n'(\lambda\varsigma)],$$

$\lambda < 1$ will be chosen later very close to 1.

We consider first the jacobian of the map $B_\delta^1 \times \Gamma \to \mathbb{C}_{w_n}$ given by $(t, \theta) \mapsto w_n^t(e^{i\theta})$. Define E by $\frac{i}{2} dw_n^t \wedge d\overline{w}_n^t = E(t, \theta) dt \wedge d\theta$ so that $E(0, \theta) = \Re[w_n'(e^{i\theta})\overline{w}_n'(\lambda e^{i\theta})]$ and choose now $\lambda < 1$ very close to 1 in order to have $E(0, \theta) \neq 0$ $\forall\theta$ and thus, reducing δ, $E \neq 0$ on $B_\delta^1 \times \Gamma$. The map $B_\delta^{2n-2} \times B_\delta^1 \times \Gamma \to \mathbb{C}^n$ given by $(w_*^0, t, \theta) \mapsto (w_*^0 + w_*(e^{i\theta}), w_n^t(e^{i\theta}))$ has obviously nonvanishing jacobian too.

Now set $\tilde{w}_n^t(\varsigma) = w_n^t(\varsigma) - w_n^t(1)$ and consider, for any $(w_*^0, t, y^0) \in B_\delta^{2n-2} \times B_\delta^1 \times B_\delta^m$, the analytic disc

$$\varphi(w_*^0, t, y^0|\varsigma) \equiv \{(w_*^0 + w_*(\varsigma), w_n^t(\varsigma)), z[(w_*^0, w_n^t(1)), y^0, (w_*, \tilde{w}_n^t)|\varsigma]\}$$

which has 0 defect if $|t| \ll 1$.

For $\varsigma = e^{i\theta}$ the jacobian of the corresponding map $B_\delta^{2n-2} \times B_\delta^1 \times B_\delta^m \times \Gamma \to M$ is $\pm E \cdot \det D_{y_0} y$ and does not vanish as we observed before.

We only have to put $b \equiv (w_*^0, t, y^0)$, $B \subset B_\delta^{2n-2} \times B_\delta^1 \times B_\delta^m$, and obtain that $\Phi(b, \varsigma) \equiv \varphi(b|\varsigma)$ fullfills (i) and (ii) by construction. (iv) follows from the Baouendi-Treves approximation theorem ([1]), (v) and (vi) from lemma 2 and the fact that $\Phi(b, \cdot)$ is a disc of defect 0 for all $b \in B$. To show (iii) one notices that, if v_n is the sequence of polynomials given by the Baouendi-Treves approximation operator, then $v_n \to v$ in $L^1_{\text{loc}}(M)$. The fact that $\Phi|_{B\times\Gamma}$ has non vanishing jacobian implies that $v_n \circ \Phi|_{B\times\Gamma}$ converges in $L^1(B \times \Gamma)$ and the limit $\tilde{v}$ is ς-holomorphic in $B \times D$. □

REMARK 2. A family Φ satisfying the condition (i)–(iv) of the proposition can be trivially constructed for any CR manifold. The crucial point in the

[7] We mean $\lim_{\rho\uparrow 1} \int_{(b,\varsigma)\in B\times\Gamma} |\tilde{v}(b, \rho\varsigma) - v[\Phi(b, \varsigma)]| \, db \wedge d\varsigma / i\varsigma = 0$.

proposition are the conditions (v) and (vi) for which the minimality displays a crucial role. For example if M were the hyperplane $\{z \in \mathbb{C}^{\nu},\ y_{\nu} = 0\}$, then for any closed set $C \subset \mathbb{R}_{x_{\nu}}$, the set $\mathbb{C}^{\nu-1} \times C$ is a peak set and thus it can have interior points. Hence the conclusions of Proposition 5 and of Theorem 1 cannot hold ((v) and (vi) cannot be satisfied).

One could think that, in the last example, Proposition 5 and Theorem 1 do not hold because analytic discs with boundary on this manifold lie completely in the manifold and that a family Φ with the properties (i)–(iv) of Prop. 5 but such that $\Phi(b, D) \cap M = \emptyset$, $\forall b \in B$, would also satisfy condition (v) and consequently Th. 1 holds as soon as such a family exists.

This is false as the next example shows

EXAMPLE. $M \equiv \left\{z \in \mathbb{C}^3,\ |z| = 1,\ y_3 = 0;\ |x_3| < \frac{1}{2},\ |z_2| < \frac{1}{2}\right\}$. We have $\dim M = 4$, $\dim_{CR} M = 1$. M is fibered by a family of characteristic submanifolds $N_t \equiv M \cap \{x_3 = t\}$ with $|t| < \frac{1}{2}$, thus no point of M is minimal.

M is also fibered by the boundaries of analytic discs whose interiors are disjoint with M. These discs are

$$\Phi(z_2, x_3|\varsigma) = (\varsigma(1 - |z_2|^2 - x_3^2)^{1/2}, z_2, x_3),\ |\varsigma| \leq 1.$$

We have

$$\Phi(z_2, x_3|\varsigma) \notin M, \quad \forall |z_2| < \frac{1}{2},\ |x_3| < \frac{1}{2}, \quad |\varsigma| < 1.$$

Nevertheless since every function $f(x_3)$ is a CR function, M has peak sets with interior points and thus Prop. 5 and Th. 1 do not hold for M.

This shows that for the validity of (v) in Proposition 5 (and hence of Th. 1) it is necessary that the discs in the family have defect 0. Roughly speaking it must be possible, keeping their boundary on M and a point fixed, to perturb them so much to fill an open set.

Before proving Theorem 1 we need also two lemmas from [9]. For sake of completeness we also give the proofs, which are very short.

LEMMA 3. *Let* $L = -\sum_{j=1}^{n} a_j(x) \dfrac{\partial}{\partial x_j}$ *be a complex vector field, with* C^1 *coefficients, in the open set* $\Omega \subset \mathbb{R}^n$, *and* $u \in C^{\lambda}(\Omega)$, $0 < \lambda < 1$, *a solution of* $Lu = 0$. *Then, for each* $f \in L^1_{\mathrm{loc}}(\Omega)$ *which solves* $Lf = 0$ *weakly in* $\Omega \backslash \{u = 0\}$, *we have* $L(u^{2/\lambda} f) = 0$ *in* Ω.

PROOF. Set d for the distance from $\{u = 0\}$. We have $u^{2/\lambda} < Cd^2$. Let $\psi \in C_0^{\infty}(\Omega)$ be an arbitrary test function. Setting, as usual, $L^*\psi = \sum_{j=1}^{n} \partial(a_j\psi)/\partial x_j$, we must prove that $\int_{\Omega} u^{2/\lambda} f L^*\psi \, dx = 0$. For each δ, $0 < \delta \ll 1$, we can choose a smooth function χ_δ with compact support contained in $\Omega \cap \{d < 3\delta\}$ and equal to 1 in a neighbourhood of $\operatorname{supp}\psi \cap \{d < \delta\}$, such that $|d\chi_\delta| < C/\delta$. Since $L(u^{2/\lambda} f) = 0$ in a neighbourhood of the support of $(1-\chi_\delta)\psi$, we have

$$\left|\int_\Omega u^{2/\lambda} f\, L^*\psi\, dx\right| = \left|\int_\Omega u^{2/\lambda} f\, L^*(\chi_\delta\psi)\, dx\right| \le C_1\delta \int_{\operatorname{supp}\psi} |f|\, dx$$

with C_1 independent of δ which is arbitrarily small. □

The next lemma concerns Hardy classes in the disc.

LEMMA 4. *If $g \in H^1$, $u \in \mathcal{O}(D)$, $\Re u^\mu > 0$ in D for some $\mu > 0$, and $g = uf$ on Γ with $f \in L^1(\Gamma)$, then $gu^{-1} \in H^1$.*

PROOF. We can apply Smirnov's theorem ($H^p \cap L^s \subset H^s$ for $p < s$, see [3] p. 65 or [8] p. 102), indeed if gu^{-1} were in H^p for some $p > 0$, then $(gu^{-1})_\Gamma = f \in L^1(\Gamma)$ would imply $gu^{-1} \in H^1$. For all $0 < q < \mu$ we have $|u|^{-q} \le C_q \Re u^{-q}$ which is harmonic, thus $u^{-1} \in H^q$. Note $\|\cdot\|_{s,r}$ for norms on $|z| = r < 1$. For fixed $0 < p < 1$, the Hölder inequality with exponents $1/p$, $1/(1-p)$, applied to $g^p u^{-p}$ gives $\|gu^{-1}\|_{p,r} \le \|g\|_{1,r}\|u^{-1}\|_{\frac{p}{1-p},r}$ thus, for $p < \mu/(1+\mu)$, we have $q \equiv \dfrac{p}{1-p} < \mu$ and hence $gu^{-1} \in H^p$. □

PROOF OF THEOREM 1. By remark 1 we can assume M to be generic. We shall first apply Lemma 3, in which we set $u = 1 - h$, and the family Φ of discs constructed in Proposition 5. Since Φ has non vanishing jacobian on $B \times \Gamma$, $U \equiv \Phi(B \times \Gamma)$ is an open, relatively compact neighbourhood of p in M and, for $u \equiv (1-h)^{2/\lambda}$, $uf \in L^1(U) \cap CR(U)$. Thus, according to Prop. 5 (iii), we have a ς-holomorphic function $\tilde{g} \in L^1(B \times D)$ such that, for almost all $b \in B$, $\tilde{g}(b,\cdot)$ has $uf \circ \Psi(b,\cdot)$ as boundary value in H^1 sense. We can now apply (iv), (v) of Prop. 5 to h and conclude that we have a ς-holomorphic $\tilde{h} \in C^0(B \times \overline{D})$ such that $\tilde{h} = h \circ \phi$ on $B \times \Gamma$ and $|\tilde{h}| < 1$ on $B \times D$. Thus, if we define $\tilde{u} \equiv (1-\tilde{h})^{2/\lambda}$, $\tilde{u}^{\lambda/2}$ has positive real part on $B \times D$.

Using the non vanishing property of the jacobian of $\Phi_{|B\times\Gamma}$ we can affirm that $f \circ \Phi(b,\cdot)$ is in $L^1(\Gamma)$ for almost all $b \in B$ and is the boundary value of $\tilde{g}\tilde{u}^{-1}(b,\cdot)$. Thus lemma 4 applies to $\tilde{g}(b,\cdot)$ and $\tilde{u}(b,\cdot)$ and we conclude that $\tilde{g}\tilde{u}^{-1}(b,\cdot) \in H^1$ for a.a. $b \in B$. Now, since $f \circ \Phi_{|B\times\Gamma} \in L^1(B \times \Gamma)$, by the Lebesgue and Fubini theorems we have $f \circ \Phi_{|B\times\Gamma} = \lim\limits_{r\uparrow 1} (\tilde{g}\tilde{u}^{-1})_{|B\times\Gamma_r}$ in the L^1 sense, where Γ_r is the circle $|\varsigma| = r$. If g_n and u_n are sequences of holomorphic polynomials converging respectively to uf in $L^1(M)$ and to u in $C^0(M)$, then those sequences converge on $\Phi(B \times \overline{D})$ in L^1 and uniform sense respectively and we have $\tilde{g} = g \circ \Phi$, $\tilde{u} = u \circ \Phi$. By reasons of continuity $\Phi_{|B\times\Gamma_r}$ has a non vanishing jacobian if $r < 1$ is very close to 1 and thus $M_r \equiv \Phi(B \times \Gamma_r)$ is a C^1-smooth manifold. By the previous remark, we have

$$f = \lim_{r\uparrow 1} (gu^{-1})_{|M_r} \quad \text{in} \quad L^1 \quad \text{sense}^8.$$

In particular, if ψ is a smooth (ν, ν')-form, with $\nu+\nu'+1 = \dim M$ and $\operatorname{supp}\psi \cap M$ is compact, we have

$$\int_M f\, \mathrm{d}\psi = \lim_{r \uparrow 1} \int_{M_r} g u^{-1}\, \mathrm{d}\psi.$$

We shall be done if we prove

$$\int_{M_r} g u^{-1}\, \mathrm{d}\psi = 0 \qquad \text{for} \quad r < 1 \quad \text{close to } 1. \tag{6}$$

Since $u_n \circ \Phi$ converges uniformly on $B \times D$ to a function which is bounded away from 0 on Γ_r, for $r < 1$ very close to 1, so we have, for $n > n(r)$,

$$|u_n| > C(r) \quad \text{on} \quad M_r \cap \operatorname{supp}\psi.$$

By continuous dependence of the constant in the Stiltijes-Vitali theorem ([6]), $g_n \circ \Phi$ converges uniformly on compact subsets of $B \times D$, thus g_n converges uniformly to g on M_r. Hence, for $n \to \infty$, we have

$$\int_{M_r} g_n u_n^{-1}\, \mathrm{d}\psi \to \int_{M_r} g u^{-1}\, \mathrm{d}\psi$$

but since $g_n u_n^{-1}$ is holomorphic in a neighbourhood of M_r the first integral vanishes. This gives (6). The proof is complete. □

We noticed in the introduction that Th. 1 follows immediately also from Prop. 3 and 4. We shall now show that these are consequences of Lemma 3 and Proposition 5 respectively.

PROOF OF PROPOSITION 3. The statement being local we can assume $f \in L^1(\Omega)$. Set $S = \{h = 1\}$ and $S_\varepsilon = \{x \in \Omega, s.t. |h - 1| < \varepsilon\}$, by hypothesis $m(S_\varepsilon) \to O$ when $\varepsilon \downarrow 0$.

As m goes to infinity, the sequence $u_m = [1 - (h+1)^m/2^m]^{2/\lambda}$ tends to 1, uniformly on $\Omega \backslash S_\varepsilon$. Since $(h+1)^m/2^m$ is also a C^λ solution of $Lu = 0$ which peaks on S, $L(u_m f)$ vanishes by Lemma 3. We have

$$\int_\Omega |(1 - u_m) f|\, \mathrm{d}x = \int_{S_\varepsilon} + \int_{\Omega \backslash S_\varepsilon} \leq (1 + 2^{2/\lambda}) \int_{S_\varepsilon} |f|\, \mathrm{d}x + \sup_{\Omega \backslash S_\varepsilon} |1 - u_m| \cdot \|f\|_{L_1},$$

and hence $u_m f \to f$ in $L_1(\Omega)$. But then, for any test function $\psi \in C_0^\infty(\Omega)$, we

[8] I.e. $\lim_{r \uparrow 1} \int_{B \times \Gamma} |f[\Phi(b, \varsigma)] - g u^{-1}[\Phi(b, r\varsigma)]|\, \mathrm{d}b \wedge \mathrm{d}\varsigma / i\varsigma = 0.$

obtain, with $L^*\psi = -\sum_{j=1}^{n} \partial(a_j\psi)/\partial x_j$,

$$\int_\Omega Lf\,\psi\,\mathrm{d}x = \int_\Omega f\,L^*\psi\,\mathrm{d}x = \lim_{m\to\infty} \int_\Omega u_m f\,L^*\psi\,\mathrm{d}x = 0.$$

PROOF OF PROPOSITION 4. The statement will follow from Prop. 5 (vi). The neighborhood mentioned in the conclusion is $\Phi(B\times\Gamma)$ of Prop. 5. Since the restriction of Φ to $B\times\Gamma$ is a diffeomorphism, if $S\cap\Phi(B\times\Gamma)$ had positive measure, the same could be said for the pre-image $\Sigma\subset B\times\Gamma$ of S by this map. This contradicts Prop. 5 (vi) which says that $\Sigma\cap(\{b\}\times\Gamma)$ has 1-dimensional measure 0 for all $b\in B$. □

3. - Proof of Theorem 3

As we have seen in the introduction, Theorem 3 is an obvious consequence of Theorem 2 and Proposition 2.

PROOF OF PROPOSITION 2. For $z\in M$, let $H_z\equiv T_z\cap iT_z$ be the (real representative of) the complex tangent space of M at z. First we disregard the analyticity assumption on M and consider the germs of all smooth vectorfields $v(z)$ such that $v(z)\in H_z$. Next, for each $z\in M$, we consider the space $V_z\subset T_zM$ spanned by those vectorfields and all their brakets of any order. We have obtained a smooth, involutive distribution V of tangent subspaces to M. Its dimension varies with z between $2\dim_{CR} M$ and $\dim M$. V has in general no integral manifold through a given point, but if it has one, of dimension $<\dim M$, then this is a characteristic submanifold because $V_z\supset H_z$. Assume now for a moment that V has an integral submanifold through each point of M and, conversely, let N be a characteristic submanifold.

Since the property of tangency to N for vectorfields is preserved under the braket operation, we obtain $T_zN\supset V_z$ for all $z\in N$ and thus N is a union of integral manifolds of V. We can conclude that a connected, embedded manifold $L\subset M$, with $\dim L<\dim M$, is characteristic if and only if it is a union of integral manifolds of V. We introduce now the real analyticity of M and use Nagano's Theorem [12] which gives us the following information

(a) *V has an integral submanifold through each point of M*

(b) *V is locally generated by a finite number of local, real analytic vectorfields.*

Thus the union of all characteristic submanifolds of M coincides with the following set

$$C\equiv\{z\in M,\ \dim V_z<\dim M\}.$$

Since M has a minimal point, (b) ensures that C is a real analytic subspace of M of dimension $<\dim M$ and hence, according to Lojasiewicz [11], C is a real analytic stratification. In particular C is a locally finite union of disjoint, embedded manifolds of dimension $<\dim M$. Each of those manifolds is characteristic and no submanifold of higher dimension containing one of them can be contained in C. Thus they are exactly all the characteristic submanifolds of maximal dimension of M. $\square$

4. - Proof of Theorem 4

Let A be an open subset, of measure ≤ 1 of the interval $I \equiv \{|x_1| < 1\}$, containing $I \cap \mathbb{Q}$. Set $C = I \backslash A$. C can be assumed to be compact. Let $\gamma(x_1) < 1$ be a real, smooth function, positive on A, zero in the complement of A. Let $A_n = (a_n, b_n)$ be the n-th connected component of A, π_n its characteristic function and λ_n, c_n positive constants which will be determined later. Set

$$g(x_1) = \sum \lambda_n \pi_n(x_1)\gamma(x_1), \quad h(z) = h(z_1) = \sum c_n \pi_n(x_1) h_n(z_1),$$

with $h_n(z_1) = e^{[(z_1-a_n)(z_1-b_n)]^{-1}}$ and $z = (z_1, z_2)$. h is holomorphic in $(A + iI) \times D$. The hypersurface M will be

$$\Phi(z) \equiv -y_1 + g(x_1)|z_2|^2 = 0, \quad x_1 \in I, z_1 \in D. \tag{7}$$

We have $M = z(I \times D)$, with $z(x_1, z_2) \equiv (x_1 + ig(x_1)|z_2|^2, z_2)$. Set

$$S = z(C \times D).$$

We shall choose λ_n and c_n in order to satisfy our requirements. Let the λ_n's be bounded so that g, and hence M, is smooth. Observe that the set $\{z_1 | \Re h_n(z_1) > 0\}$ is a neighbourhood of $(A_n \times \{0\})$ in $\mathbb{C}$ and it is bounded by algebraic real curves. Thus the λ_n's can be taken so small to have $\Re h > 0$ on $M \backslash S$ independent of the c_n's which will be chosen now.

The functions $\pi_n h_n$ are smooth on M, vanish with all their derivatives on S and have bounded C^k norms $\|\pi_n h_n\|_k$ there. Thus, for any fixed k, $\sum c_n^k \pi(x_1) h_n(z_1)$ is C^k on M as soon as $c_n^k \|\pi_n h_n\|_k \to O$ when $n \to \infty$. We choose such a positive double sequence imposing also $c_n^k \leq c_{n-1}^k$ so that we have $h \in C^\infty(M)$ taking $c_n = c_n^n$ (indeed $h \in C^k$, $\forall k$). Notice that h is defined on $M \backslash S$ as a trace of a function holomorphic on $(A + iI) \times D$ and this is a neighbourhood of $M \backslash S$. Thus $h \in CR(M \backslash S)$ and hence $h \in CR(M)$ because h is smooth and $M \backslash S$ is dense in M. Since h vanishes on S, e^{-h} is our peak function. Hence (i) is proved.

To prove (ii) set $f = 1 - |z_2|$ on S, $f = O$ on $M \backslash S$. Obviously

$f \in L^\infty(M) \cap CR(M \backslash S)$. It is sufficient to prove that

$$J_\varepsilon \equiv \int_M f d\psi^\varepsilon \wedge dz \neq O \tag{8}$$

with $dz = dz_1 \wedge dz_2$, for some $\psi^\varepsilon \in C_0^\infty(M)$. Choose $\chi_\varepsilon \in C_0^\infty(I)$, $\chi_\varepsilon(t) = 1$ for $|t| < 1 - \varepsilon$, $|\chi'_\varepsilon| < C/\varepsilon$ and set $\psi^\varepsilon = \bar{z}_2 \chi_\varepsilon(|z_2|)\chi_\varepsilon(x_1) \in C_0^\infty(M)$. We have on S

$$d\psi^\varepsilon \wedge dz = \chi_\varepsilon(x_1)[\chi_\varepsilon(|z_2|) + |z_2|\chi'_\varepsilon(|z_2|)/2] dx_1 \wedge dz_2 \wedge d\bar{z}_2.$$

Thus

$$J_\varepsilon = \left(\int_C \chi_\varepsilon(x_1) dx_1 \right) \left(\int_D (1 - |z_2|)\chi_\varepsilon(|z_2|) dz_2 \wedge d\bar{z}_2 + \right.$$

$$\left. + \frac{1}{2} \int_D (1 - |z_2|)\chi'_\varepsilon(|z_2|) dz_2 \wedge d\bar{z}_2 \right).$$

The last integral is estimated by $2C\pi\varepsilon$ thus, for $\varepsilon \to O$, we have

$$J_\varepsilon \to mesC \int_D (1 - |z_2|) dz_2 \wedge d\bar{z}_2 \neq O$$

which gives (8) for small ε.

Finally we prove (iii). The Levi form of the function Φ appearing in (7), computed on the complex tangent vector $(-\phi_{z_2}, \phi_{z_1})$, is

$$g(1 + gg''|z_2|^4 + 2g'^2|z_2|^4 - g'|z_2|^2)/4$$

which vanishes on S and is positive on $M \backslash S$ as soon as all constants λ_n (and hence g) are multiplied by the same, positive, small factor. This proves pseudoconvexity of M and (iii). □

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